

Bruno Touschek  
Memorial Lectures 2010  
LNF

Nicola Cabibbo and  
Frascati

Mario Greco - November 30th, 2010

# BTML - 2009

Nicola Cabibbo:

Chiral Symmetry and Particle Physics





Frascati, December 2009





Frascati, December 2009





Liceo Touschek, January 2009





Liceo Touschek, January 2009

# Early times and 60's

Thesis with Bruno Touschek on w.i. (1958)

Collaboration with Raoul Gatto on  $e^+e^-$  physics.

- Pion form factor (Phys.Rev. Lett. 4, March '60)

Theoretical work on nucleon structure<sup>1,2</sup> has revealed the important role of the photon-( $\pi$ -pion) vertices in the theory of the nucleon form factors. In particular the  $\gamma$ - $2\pi$  vertex is thought to be the most important one contributing to the isotopic vector part of the structure, and the  $\gamma$ - $3\pi$  vertex the most important one for the isotopic scalar part. Recent technical developments showing the feasibility of colliding beam experiments<sup>3</sup> make it appealing to think of possible direct measurements of the photon-pion vertices through processes of the sort

$$e^+ + e^- \rightarrow n \text{ pions.} \quad (1)$$

We shall here discuss the reactions (1) in their lowest electromagnetic approximation. Examination of the higher order terms in the electromagnetic coupling constant may become necessary when detailed experiments are carried out.

Let us consider the reaction (1) in the center-of-mass frame. In the lowest electromagnetic approximation only the  $^3S_1$  and  $^3D_1$  states for the initial positron-electron state will contribute to (1) and the final  $n$ -pion state must have parity minus, charge-conjugation quantum number minus, total angular momentum one, and total isotopic spin one if  $n$  is even, zero if  $n$  is odd. In particular the process is forbidden in this

order if the final pions are all neutrals. The  $S$ -matrix element for (1) is given by

$$S_{fi} = \delta(p^{(1)} + p^{(2)} + \dots + p^{(n)} - e^{(+)} - e^{(-)}) \\ \times \frac{2\pi e}{K^2} (\bar{v}(e^+) \gamma_\nu u(e^-)) \langle p^{(1)}, p^{(2)}, \dots, p^{(n)} | j_\nu(0) | 0 \rangle, \quad (2)$$

where  $p^{(1)}, p^{(2)}, \dots, p^{(n)}$  are the final pion momentum four-vectors,  $e^{(+)}$  and  $e^{(-)}$  the positron and electron momenta, respectively,  $K = e^{(+)} + e^{(-)}$ ,  $\bar{v}(e^+)$  and  $u(e^-)$  are the Dirac spinors, and the matrix element of the electric current operator  $j_\nu(x)$  is taken between the vacuum state and the final state of  $n$  outgoing pions.

We define

$$J_\nu(p^{(1)}, p^{(2)}, \dots, p^{(n)}) \\ = (2\pi)^{3n/2} \langle p^{(1)}, p^{(2)}, \dots, p^{(n)} | j_\nu(0) | 0 \rangle. \quad (3)$$

It follows from gauge invariance that, in the center-of-mass system for (1),  $J_\nu$  has only the space component  $\vec{J}$ . From parity conservation  $\vec{J}$  is a polar vector formed from the  $n$  vectors  $\vec{p}^{(1)}, \vec{p}^{(2)}, \dots, \vec{p}^{(n)}$  if  $n$  is even, an axial vector formed from these vectors if  $n$  is odd. Inserting (3) into (2) one finds for the total cross section

$$\sigma = \frac{\alpha}{32E^4 (2\pi)^{3n-5}} \int d\vec{p}^{(1)} d\vec{p}^{(2)} \dots d\vec{p}^{(n)} \delta(\omega^{(1)} + \omega^{(2)} + \dots + \omega^{(n)} - 2E) \delta(\vec{p}^{(1)} + \vec{p}^{(2)} + \dots + \vec{p}^{(n)}) |\vec{J}|^2 \sin^2 \theta, \quad (4)$$

where  $\alpha$  is the fine structure constant (another factor  $\alpha$  is contained in  $|\vec{J}|^2$ ),  $E$  is the electron (or positron) energy,  $\omega^{(i)}$  is the energy of the  $i$ th pion, and  $\theta$  is the angle between the vector  $\vec{J}$  and the line of collision. The dependence on  $\theta$ , as  $\sin^2 \theta$ , is therefore a direct consequence of gauge invariance independent of any knowledge of the  $\gamma$ -pion vertices. In the case of two final pions, with only one vector available,  $\vec{J}$  must be proportional to  $\vec{p}^{(1)} = -\vec{p}^{(2)}$  and the angular dis-

form  $\sin^2 \theta$ . In the case of three final pions,  $\vec{J}$  must be proportional to the only available axial vector  $\vec{p}^{(1)} \times \vec{p}^{(2)} = -\vec{p}^{(1)} \times \vec{p}^{(3)}$ , etc., and therefore the distribution of the angle  $\theta$  between the normal to the plane in which the three pions are produced and the line of collision is uniquely given by  $\sin^2 \theta$ .

The final state of two pions (necessarily one positive, one negative) produced according to (1)

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- Symmetry b. muon and electron (PRL 5, Aug. '60)



<sup>4</sup>A more detailed account of these processes will be published elsewhere.

<sup>5</sup>Samuel K. Allison, *Revs. Modern Phys.* **30**, 1137 (1958).

<sup>6</sup>H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Clarendon Press, Oxford, 1952).

<sup>7</sup>T. B. Day, *Bull. Am. Phys. Soc.* **5**, 225 (1960) and University of Maryland Physics Department Technical Report No. 175 (unpublished).

<sup>8</sup>A. A. Evett, *J. Chem. Phys.* **24**, 150 (1956).

<sup>9</sup>E. A. Mason and J. T. Vanderslice, *J. Chem. Phys.* **27**, 917 (1957). We would like to express our appreciation to Professor Mason of the University of Maryland Institute of Molecular Physics for several informative discussions about these molecules.

<sup>10</sup>See reference 5, Chap. X.

<sup>11</sup>H. Hellman, *Einführung in die Quantenchemie* (Franz Deuticke, Leipzig, 1937), p. 285; R. P. Feyn-

man, *Quantum Electrodynamics* (Wiley, New York, 1948), 3rd ed.

<sup>12</sup>A closely related process wherein the meson shifts states, keeping the same principal quantum number, while the nearby electron is virtually excited, is found not to be important here and will be discussed in reference 4. This polarization-capture process was considered in another connection by M. A. Ruderman, *Phys. Rev.* **118**, 1632 (1960). The transition rates for radiation are negligible for these highly excited states.

<sup>13</sup>G. Careri, U. Fasali, and F. S. Goeta, *Nuovo cimento* **15**, 774 (1960).

<sup>14</sup>G. A. Baker, Jr., *Phys. Rev.* **117**, 1130 (1960).

<sup>15</sup>M. Demeur, *Nuclear Phys.* **1**, 518 (1958).

<sup>16</sup>The dipole matrix elements involved in the external Auger rates increase with increasing  $l$  for fixed initial and final principal quantum numbers. See H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957), p. 264.

## SYMMETRY BETWEEN MUON AND ELECTRON

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(Received July 13, 1960)

As is well known, muons and electrons appear to have identical couplings. Their masses are, however, different. Such a situation seems rather peculiar and has recently received much attention.<sup>1</sup> In this note we shall (1) define a formal operation of muon-electron symmetry; (2) show how the total Lagrangian, excluding weak couplings, can be written in a form exhibiting such a symmetry, if electromagnetic coupling is minimal; (3) show that it is impossible to satisfy such a symmetry when universal weak interactions are included, if only one neutrino exists; (4) show that it is possible to have such a symmetry in a two-neutrino theory; (5) point out the close connection of muon-electron symmetry to a principle forbidding the transformation of muons into electrons.

The present investigation is related to some recent papers<sup>2-4</sup> dealing with the elimination of particular muon-electron couplings. Of the above points, (2) is already contained in reference 3. We shall also make use of the general theorem of reference 4.

We first define a formal operation of muon-

electron symmetry. We introduce a two-dimensional  $e-\mu$  space, which we call  $L$  space (lepton space). The  $e-\mu$  symmetry, or  $L$  symmetry, is performed by an unitary operator  $S$ , such that

$$S^{-1}\psi S = \sigma_1\psi, \quad (1)$$

where  $\psi$  is a vector in  $L$  space describing the electron and muon fields and  $\sigma_1$  is a Pauli matrix in the usual notation. In the representation in which the components of  $\psi$  are  $e$  and  $\mu$  the operation (1) just amounts to the substitution  $e \leftrightarrow \mu$ .

A general renormalizable Lagrangian, excluding weak interactions, can be written as<sup>5,6</sup>

$$\mathcal{L} = -\bar{\psi}[\gamma^\mu \partial_\mu + \gamma_5 D](\psi) + \mathcal{L}_\gamma + \mathcal{L}_S, \quad (2)$$

where  $\mathcal{L}_\gamma$  is the free-photon Lagrangian,  $\mathcal{L}_S$  is the strong Lagrangian that we assume does not contain  $e$  or  $\mu$ , and  $A, B, C, D$  are Hermitian matrices in  $L$  space.<sup>5</sup> The requirement of invariance under  $L$  symmetry implies that  $A, B, C, D$  all commute with  $\sigma_1$ .

A theorem, whose proof can be found in reference 4, states the existence of a nonsingular



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Staff at LNF in spring 1961.

- “The Bible”: Cabibbo and Gatto, P.R.124, 1577, '61  
 $e^+ e^- \rightarrow a+b+\dots+c$

## Electron-Positron Colliding Beam Experiments

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(Received June 8, 1961)

Possible experiments with high-energy colliding beams of electrons and positrons are discussed. The role of the proposed two-photon resonance and of the three-photon resonance or bound state is investigated in connection with electron-positron annihilation into pions. The existence of a three-photon bound state would give rise to a very large cross section for annihilation into  $e^+e^-$ . A discussion of the possible resonances is given based on consideration of the relevant widths as compared to the experimental energy resolution. Annihilation into baryon-antibaryon pairs is investigated and polarization effects arising from the nonreal character of the form factors on the absorptive cut are examined. The density matrix for annihilation into pairs of vector mesons

is calculated. A discussion of the limits from unitarity to the annihilation cross sections is given for processes going through the one-photon channel. The cross section for annihilation into pairs of spin-one mesons is rather large. The typical angular correlations at the vector-meson decay are discussed.

A neutral weakly interacting vector meson would give rise to a strong resonant peak if it is coupled with lepton pairs. Effects of the local weak interactions are also examined. The explicit relation between the  $\delta$  corrections to the photon propagator due to strong interactions and the cross section for annihilation into strongly interacting particles is given.

## INTRODUCTION

A PROPOSAL for electron-electron colliding beams was made some time ago at Stanford by Barber, Gittelman, O'Neill, Panofsky, and Richter, and an experiment on electron-electron scattering based on such a proposal is being carried out.<sup>1</sup> Projects for electron-positron colliding beams are also under development at Stanford<sup>2</sup> and at Frascati.<sup>3</sup> The project at Frascati is intended to obtain high-energy ( $>1$  Bev) electron-positron colliding beams.<sup>4</sup> We have already discussed possible experiments with  $e^+e^-$  colliding beams.<sup>4</sup> In this note we shall present a more detailed discussion of possible electron-positron experiments and of the theoretical questions connected with them.

Like electron-electron experiments, electron-positron experiments can test the validity of quantum electrodynamics at small distances.<sup>4a</sup> They present, however, some very typical features that sufficiently justify the effort to produce electron-positron colliding beams. Most of the annihilation processes of  $e^+e^-$  take place through the conversion of the pair into a virtual photon of mass equal to the total center-of-mass energy. The photon then converts into the final products. These reactions proceed through a state of well-defined

quantum numbers, and as consequence the possible initial and final states are essentially limited. The interaction of the final particles with the virtual photon is directly measured in the experiment. The virtual photon four-momentum is timelike in these experiments, in contrast, for instance, to electron scattering on nucleons where the four-momentum of the transferred virtual photon is spacelike. Form factors of strongly interacting particles can thus be measured for timelike values of the momentum, in a region where they have, in general, an imaginary part. Electron-positron annihilations in flight offer the possibility of carrying out a Panofsky program, of a systematic exploration of the spectrum of elementary particles by observing their production by the intermediate virtual gammas. Unstable particles with the same quantum numbers as the intermediate photon can be produced singly as resonant states that soon after decay. At the appropriate energy there would appear resonance peaks in the production cross section for the final decay products.

## 1. GENERAL CONSIDERATIONS

1.1. We consider a reaction of the kind

$$e^+ + e^- \rightarrow a + b + \dots + c, \quad (1)$$

where  $a, b, \dots, c$  are strongly interacting particles. At the lowest electromagnetic order we assume that the reaction goes through the one-photon channel shown

<sup>1</sup> W. Barber, R. Gittelman, E. E. O'Neill, W. K. H. Panofsky, and W. C. Richter (to be published); G. E. O'Neill, *Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, CERN, 1959* (CERN, Geneva, 1959), p. 125; W. K. H. Panofsky, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester Interscience*

# Theory vs Experiments.

- Application of  $SU(3)$  to weak and e.m. transitions.

Cabibbo and Gatto, N.C. 21, 872, 1961.



### Consequences of Unitary Symmetry for Weak and Electromagnetic Transitions.

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Laboratori Nazionali del CNEN - Frascati*

(ricevuto il 14 Agosto 1961)

Recent papers <sup>(1)</sup> have dealt with the introduction of unitary symmetry, i.e. invariance under the three-dimensional unitary group, as a convenient approximation in the theory of strong interaction. The strong-interaction Lagrangian is assumed to consist of a part invariant under the unitary group, plus a « correction » breaking the invariance.

In this note we shall examine the properties that follow, for weak and electromagnetic amplitudes, from this hypothesis, together with the violent assumption that the symmetry-breaking « corrections » can be neglected. We do not know under what conditions this last hypothesis can be applied. It might be applicable in the high-energy region — or, better, we know for sure that it is not generally applicable at low energies.

The consequences of violent assumptions are usually far-reaching. Thus we find that the  $K^0$  (or  $\bar{K}^0$ ) electromagnetic form factors must be zero, the charged  $K$  form factors must be equal to the charged pion form-factors; there are simple stringent relations between the form factors of the baryons (already given by COLEMAN and GLASHOW <sup>(2)</sup>), of the vector mesons, between the electromagnetic amplitudes from vector to pseudoscalar mesons, between the amplitudes of  $\chi^0$  decay and  $\pi^0$  decay, and between weak interaction amplitudes. These relations depend in part on the particular representation adopted for the baryons.

For instance according to the « eightfold way » the  $\Lambda$  form-factors are one-half of the neutron form factors. They must instead be equal in the Sakata represen-

(\*) S. CHAWA: *Prog. Theor. Phys.*, **21**, 202 (1959); Y. YAMAGUCHI: *Prog. Theor. Phys. Suppl.*

# Theory vs Experiments.

- Application of  $SU(3)$  to weak and e.m. transitions.

Cabibbo and Gatto, N.C. 21, 872, 1961.

- Coherent Bremsstr. in crystals (Diambrini et al. 1961)

Cabibbo, Da Prato, De Franceschi and Mosco,  
PRL 9, 270, 1962 and PRL 9, 435, 1962.

3050 (1958)]. Proton-polarization experiments in cerium magnesium nitrate have been reported by M. Abraham, M. A. H. McCasland, and F. N. H. Robinson, *Phys. Rev. Letters* **2**, 449 (1959); and by M. Dorphini

and A. Abragam, *Suppl. Helv. Phys. Acta*, **VI**, 143 (1961).

<sup>4</sup>J. Hutton and B. V. Rollin, *Proc. Roy. Soc. (London)* **A193**, 222 (1948).

## NEW METHOD FOR PRODUCING AND ANALYZING LINEARLY POLARIZED GAMMA-RAY BEAMS

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and

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(Received August 6, 1962)

In this note we propose a new method for the production and the analysis of a polarized beam of high-energy gamma rays. The method is based on the interference effects which are observable in high-energy electron pair production on crystals. As a consequence of these the absorption rate (inverse of the mean free path) of very high-energy photons in crystal matter depends on their linear polarization. This suggests the possibility of using a thick crystal for the polarization and analysis of high-energy gamma rays. The polarization is effected by preferential absorption of the unwanted polarization component, and the analysis by transmission measurements, as in the case of a polaroid filter for visible light. Extensive theoretical work on the interference effects of high-energy electrodynamic processes in crystals has been done by Overall,<sup>1</sup> and refined experiments have given results which are in excellent agreement with the theory.<sup>2</sup>

Another method for the production of linearly polarized gamma rays by means of interference effects in a crystal is based on bremsstrahlung.<sup>3</sup> In this case the theory is also in excellent agreement with the experimental results obtained with electrons of ~1 GeV.<sup>4</sup>

As a polarimeter the device we propose is perhaps unique in the high-energy region, where the application of other methods based on the angular distribution in pair production and elastic photo-

The absorption of high-energy photons is mainly due to electron pair production, a process which is already known to give interference effects.<sup>1,2</sup> Let us consider the case of a cubic crystal, where the momentum  $\vec{k}$  of the incoming gamma rays is in the (001) plane and makes a small angle  $\alpha$  with the (110) axis. The (001) plane is then a symmetry plane. We find that the total cross section for pair production depends in this situation on the linear polarization of the gamma rays.

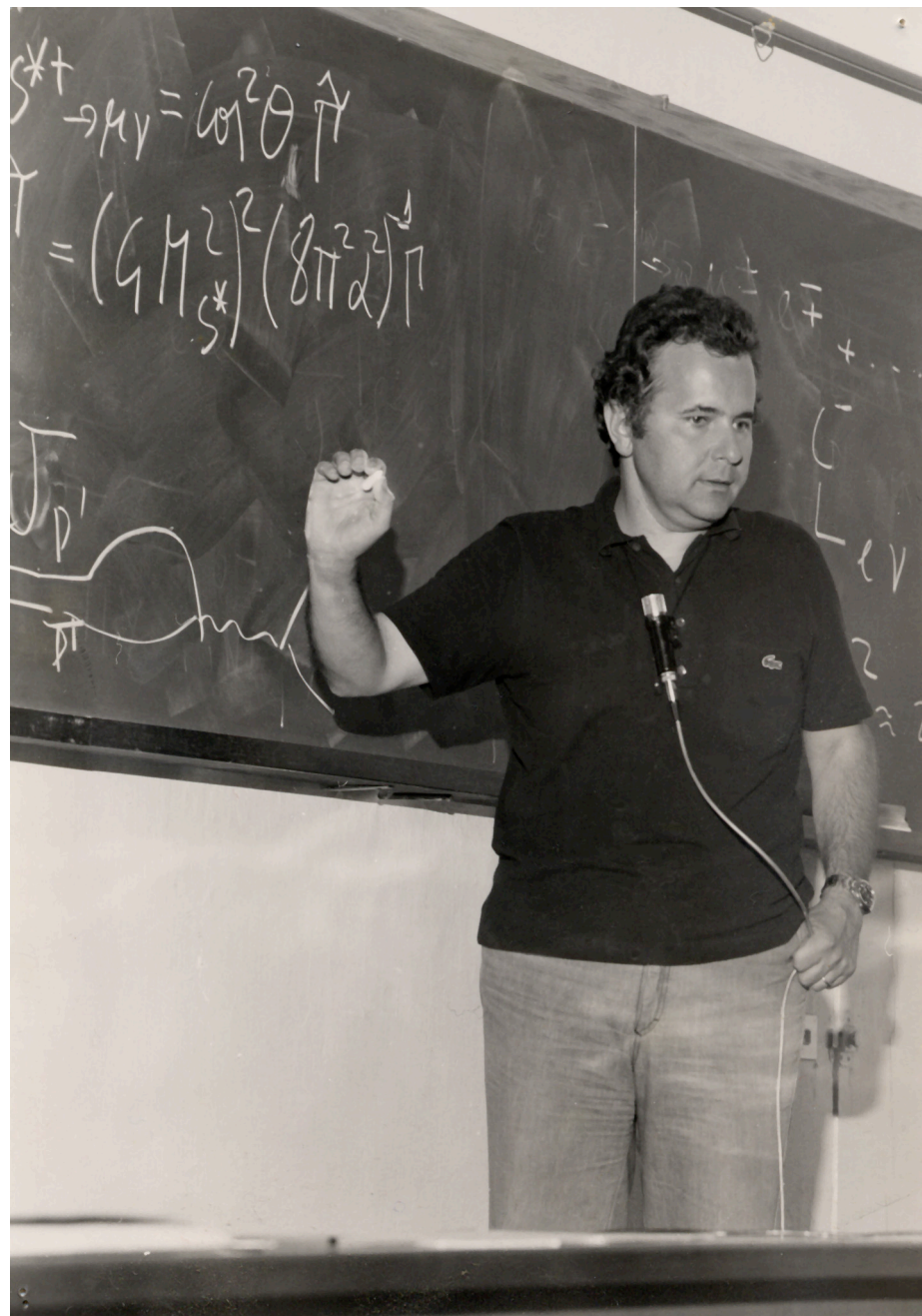
Let us denote by  $\Sigma^{\parallel}$  and  $\Sigma^{\perp}$  the total cross sections per unit volume of the crystal for gamma rays which are linearly polarized in the (001) plane and orthogonal to it. The two polarization components will be absorbed with different mean free paths; i.e., after having penetrated a thickness  $x$  of the crystal the intensities of the two components will be reduced according to\*

$$\begin{aligned} I^{\parallel}(x) &= I^{\parallel}(0) \exp[-\Sigma^{\parallel} x], \\ I^{\perp}(x) &= I^{\perp}(0) \exp[-\Sigma^{\perp} x]. \end{aligned} \quad (1)$$

If the beam was originally unpolarized [ $I^{\parallel}(0) = I^{\perp}(0)$ ], we now have a polarization:

$$\begin{aligned} P(x) &= [I^{\parallel}(x) - I^{\perp}(x)] / [I^{\parallel}(x) + I^{\perp}(x)] \\ &= \tanh[\frac{1}{2}x(\Sigma^{\parallel} - \Sigma^{\perp})]. \end{aligned} \quad (2)$$





Nicola 1963

# 70's

- In late 60's first evidence for a pointlike hadron internal structure from DIS at Slac. Formulated in terms of the parton model (Feynman, Bjorken, ..).
- In 1970 Adone data give evidence for a large multihadron production and new resonances.
- Cabibbo, Parisi, Testa, L.N.C. 4, 35, 1970.



## Hadron Production in $e^+e^-$ Collisions (\*).

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*Istituto di Fisica dell'Università - Roma*

(ricevuto il 30 Maggio 1970)

1. — The simple properties of deep inelastic electron-proton scattering has suggested models where these processes arise as interactions of virtual photons with an « elementary » component of the proton. These as yet unspecified elementary components of the proton have been given the name of « partons » by FEYNMAN <sup>(1)</sup>. The model has been studied by BJORKEN and PASCHOS <sup>(2)</sup> and successively by DRELL, LEVY and TUNG MOW YAN <sup>(3)</sup> who gave a field-theoretical treatment of the parton model, and were able to recover some of the experimentally observed properties of this process. In this letter we wish to extend the method of ref. <sup>(3)</sup> to the study of the total cross-section of electron-positron annihilation into hadrons.

This treatment leads to an asymptotic (very high cross-section c.m. energy,  $2E$ ) of the form

$$(1) \quad \sigma \rightarrow \frac{\pi\alpha^2}{12E^2} \left[ \sum_{\text{spin } 0} (Q_i)^2 + 4 \sum_{\text{spin } \frac{1}{2}} (Q_i)^2 \right],$$

where  $Q_i$  is the charge of the  $i$ -th parton in units of  $e$ . This is simply the sum of the contributions of the single partons considered as pointlike <sup>(4)</sup>. Each parton contributes a different kind of events to the total cross-section. The typical high-energy event should consist in the production of a pair of virtual partons, each of which develops into a jet of physical hadrons.

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(\*) This research has been sponsored in part by the Air Force Office of Scientific Research, through the European Office of Aerospace Research OAR, United States Air Force, under contract f 61 05267 C 0084.

<sup>(1)</sup> R. P. FEYNMAN: unpublished.

<sup>(2)</sup> J. D. BJORKEN and E. A. PASCHOS: *Phys. Rev.*, **185**, 1976 (1969).

<sup>(3)</sup> S. D. DRELL, D. J. LEVY and TUNG MOW YAN: *Phys. Rev. Lett.*, **22**, 744 (1969); *Phys. Rev.*, **187**, 2159 (1969).

<sup>(4)</sup> Equation (1) extends the well-known result obtained by J. D. BJORKEN: *Phys. Rev.*, **148**, 1467 (1966) in the case of spin- $\frac{1}{2}$  partons.

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- In 1970 Adone data give evidence for a large multihadron production and new resonances.
- Cabibbo, Parisi, Testa, L.N.C. 4, 35, 1970.  
Cabibbo, Parisi, Testa, Verganelakis, L.N.C. 4, 569, 1970.
- Workshop at Frascati, 1973, with N.C. and M. Veltman.
- Nov. 1974, J/Psi discovery, big excitement but wrong suggestion: Altarelli, Cabibbo, Maiani, Parisi, Petronzio, INFN-ROME-592, Nov 1974. "Is the 3104-MeV Vector Meson the psi - Charm or the W0?"
- 1975: SuperAdone.  
QG-plasma (N.C., Parisi, Phys.Lett. B59)

# 80's

1983: Nicola President of INFN.

- LNGS: general structure of the lab. and support to expts. ICARUS, LVD, MACRO, Gallex. Role of LNF.
- LNF: Adone as a source of synchr. radiation. Need for new programs for acceler. and particle phys. to keep Frascati tradition and give locally a physics goal to young people.

Study groups in the lab. since 1987 leading to a project for a Phi-factory. Final proposal in 1990: Giorgi, Greco, Piccolo - Bassetti, Vignola (LNF-90/031).

- Nicola very enthusiastic and asks for independent th advice.

Barbieri, Maiani, Martinelli, Paoluzzi, Paver, Petronzio, Remiddi: “Report from the Phi factory working group”, LNF-90/041.

- Daphne and KLOE have led to very important progress in K-physics.

- Very precise measurement of Cabibbo angle.

## Test of CKM unitarity

Determine  $|V_{us}|$  and  $|V_{ud}|$  from a **fit to the results**:

$$|V_{us}f_+(0)|=0.2163(5), \quad f_+(0)=0.959(5);$$

$$|V_{us}|/|V_{ud}|f_K/f_\pi=0.2758(5), \quad f_K/f_\pi=1.193(6)$$

$$|V_{us}| = 0.2254(13) \quad [K_{\ell 3} \text{ only}],$$

$$|V_{us}/V_{ud}| = 0.2312(13) \quad [K_{\ell 2} \text{ only}]$$

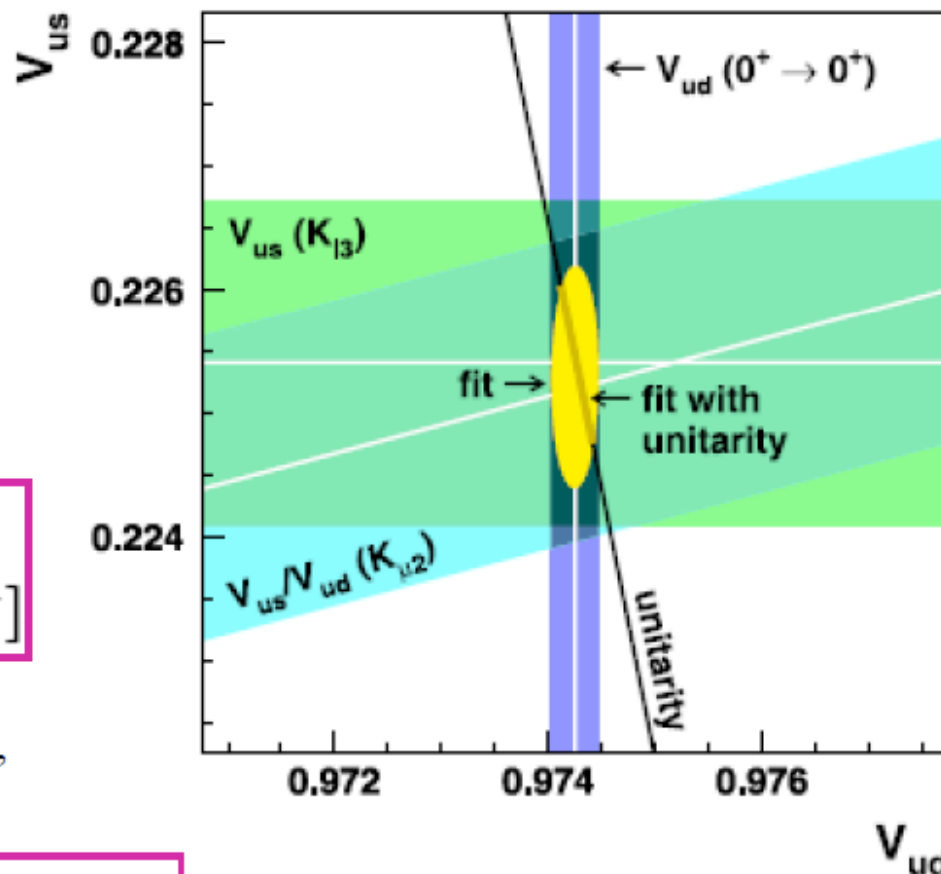
Adding  $|V_{ud}|=0.97425(22)$ , obtains  
( $\chi^2/\text{ndf}=0.014/1$ ,  $P=91\%$ , negligible  
correlation between  $V_{us}$  and  $V_{ud}$ ):

$$|V_{ud}| = 0.97425(22),$$

$$|V_{us}| = 0.2253(9) \quad [K_{\ell 3}, K_{\ell 2}, 0^+ \rightarrow 0^+]$$

Including in the fit the unitarity constraint,  
obtains ( $\chi^2/\text{ndf}=0.024/2$ ,  $P=99\%$ ):

$$|V_{us}| = \sin \theta_C = \lambda = 0.2254(6) \quad [\text{with unitarity}]$$

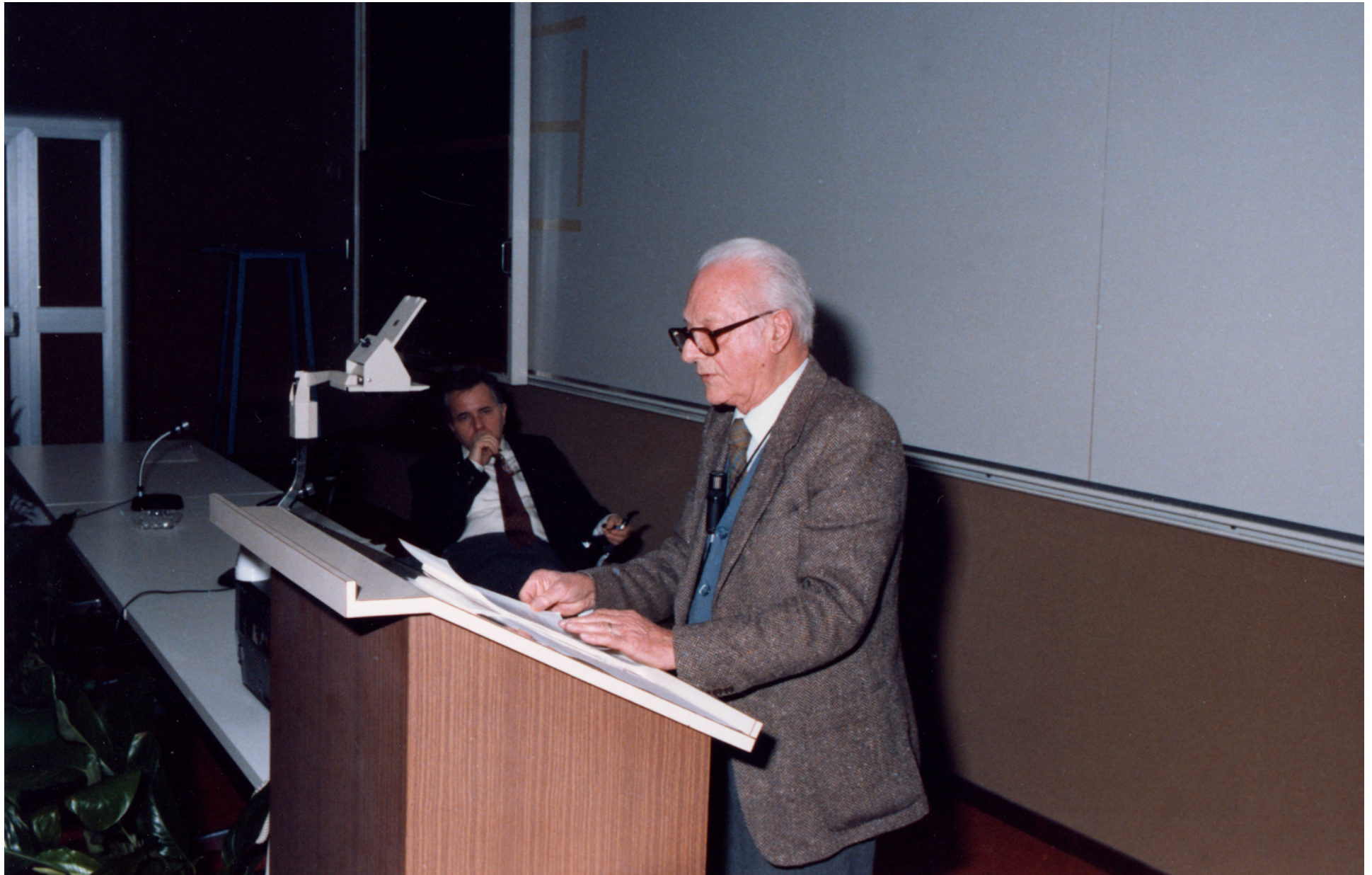






BTML 1987





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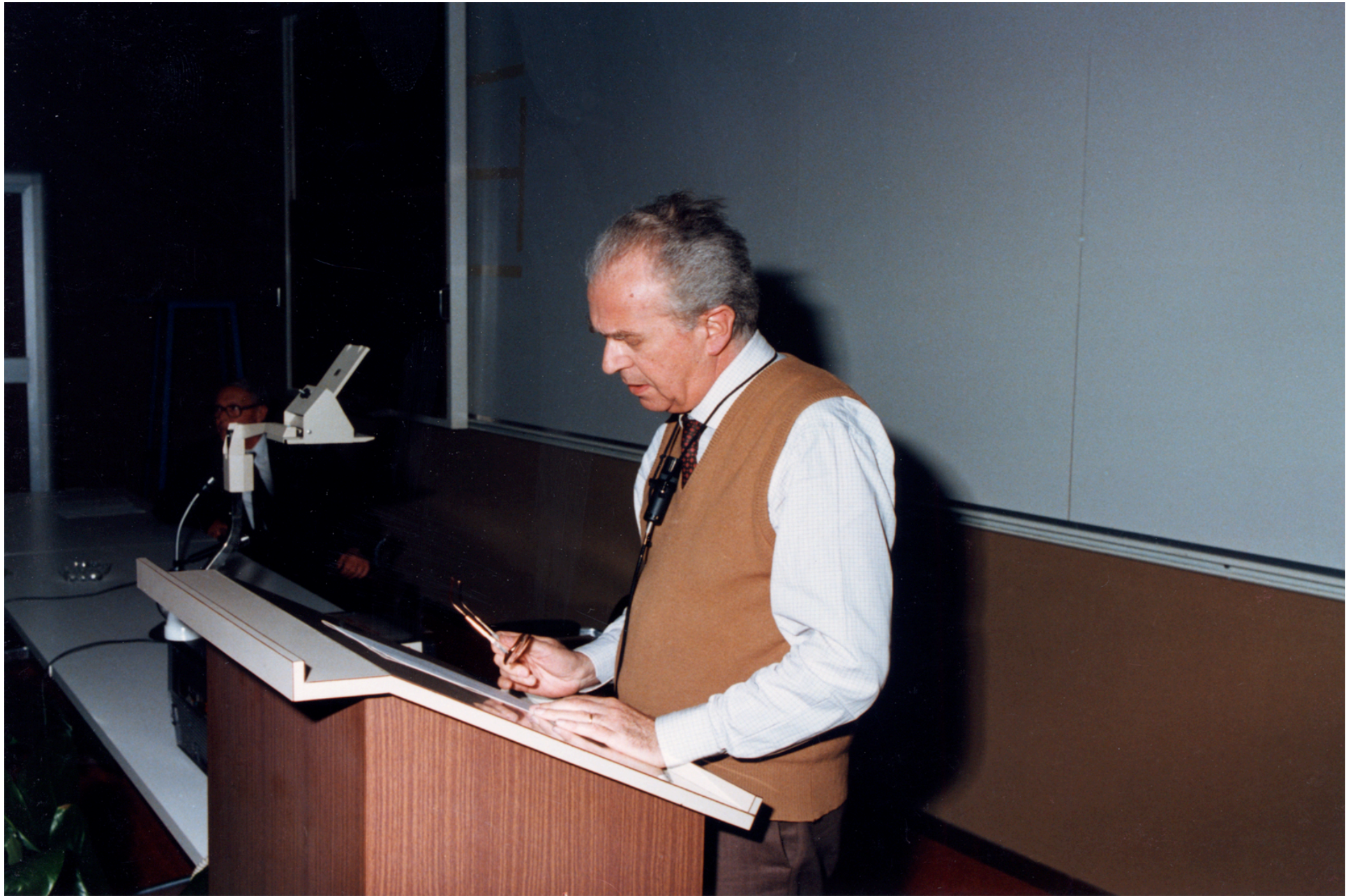
BTML 1987





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Liceo Touschek, January 2009