A Different Approach to X-ray Stress Measurement

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The General Electric X-ray Camera with a Molybdenum X-ray tube used by H. Lester and R. Aborn in 1925.

Drawing of the X-ray pattern from the unstressed and stressed specimen.
A spherical element becomes an Ellipsoid Under stress

Assume that the specimen is homogeneous and isotropic
The lattice parameter ellipsoid with axes $a_x$, $a_y$, and $a_z$. 
The lattice parameter ellipsoid

$L_x$ and $L_y$ are the laboratory coordinates.

$a_x$ and $a_y$, and $a_z$ are the principle axes of the ellipsoid and the directions of the principle strains/stresses.
Using the following equation for the Ellipsoid,

\[ Ax^2 + Bxy + Cy^2 + Dz^2 + Exz + Fyz = 1 \quad (1) \]

The coefficients A, B, C, …F are determined by least squares from the data, \( \phi \), \( \psi \), and a or d.

Using the general rotation of axes equations we obtain the following equation for the ellipse in the xy plane;

\[ A'(x')^2 + B'x'y' + C'(y')^2 = 1 \quad (2) \]

Where \( A' \), \( B' \), and \( C' \) related to \( A,B, \) and \( C \) by;

\[ A' = A \cos^2 \alpha + B \cos \alpha \sin \alpha + C \sin^2 \alpha \]
\[ B' = B (\cos^2 \alpha - \sin^2 \alpha) + 2 (C - A) \sin \alpha \cos \alpha = 0 \]
\[ C' = A \sin^2 \alpha - B \sin \alpha \cos \alpha + C \cos^2 \alpha \; ; \; D' = D \]

The angle between the laboratory and the specimen coordinates, \( \alpha \), is obtained by setting \( B' = 0 \);

\[ \tan(2\alpha) = B / (A - C) \quad (3) \]

\[ a_x = 1 / (A')^{\frac{1}{2}} \quad a_y = 1 / (C')^{\frac{1}{2}} \quad a_z = 1 / (D')^{\frac{1}{2}} \]
Using Hooke’s Law to determine the unstressed $a_0$ value

$$
\varepsilon_x = \left(\frac{1}{E}\right) \left[ \sigma_x - \nu (\sigma_y + \sigma_z) \right]
$$

$$
\varepsilon_y = \left(\frac{1}{E}\right) \left[ \sigma_y - \nu (\sigma_x + \sigma_z) \right]
$$

$$
\varepsilon_z = \left(\frac{1}{E}\right) \left[ \sigma_z - \nu (\sigma_x + \sigma_y) \right]
$$

Assume $\sigma_z = 0$

$$
\varepsilon_x = \left(\frac{1}{E}\right) \left[ \sigma_x - \nu \sigma_y \right] = \frac{a_x - a_0}{a_0}
$$

$$
\varepsilon_y = \left(\frac{1}{E}\right) \left[ \sigma_y - \nu \sigma_x \right] = \frac{a_y - a_0}{a_0}
$$

$$
\varepsilon_z = \left(\frac{1}{E}\right) \left[ -\nu (\sigma_x + \sigma_y) \right] = \frac{a_z - a_0}{a_0}
$$

$$
a_x = \left(\frac{a_0}{E}\right) \left[ \sigma_x - \nu \sigma_y \right] + a_0
$$

$$
a_y = \left(\frac{a_0}{E}\right) \left[ \sigma_y - \nu \sigma_x \right] + a_0
$$

$$
a_z = \left(\frac{a_0}{E}\right) \left[ -\nu (\sigma_x + \sigma_y) \right] + a_0
$$

$$
a_0 = \frac{M_x}{a_0 \sigma_x / E} = \frac{M_y}{a_0 \sigma_y / E}
$$
From the values of Young’s modulus, $M_x$, $M_y$, and $a_o$, the magnitude of the principle stresses are determined.

$$\sigma_x = E \frac{M_x}{a_o}$$

$$\sigma_y = E \frac{M_y}{a_o}$$

The stress in the x-y plane as a function $\phi'$ can be evaluated from the following equation;

$$\sigma_{\phi'} = \frac{1}{2} (\sigma_x + \sigma_y) + \frac{1}{2} (\sigma_x - \sigma_y) \cos(2\phi')$$
Rigaku MSF-3M X-ray stress analyzer
Results of the analysis on a specimen of 1018 Carbon steel, Which employed the 4 point bending device.

\[ A = 1.215357 \times 10^{-1} \quad B = 6.061435 \times 10^{-5} \quad C = 1.218008 \times 10^{-1} \]
\[ D = 1.217557 \times 10^{-1} \quad E = 8.770656 \times 10^{-6} \quad F = 3.705822 \times 10^{-7} \]

\[ \alpha = -6.44^\circ \]

\[ A' = 1.215323 \times 10^{-1} \quad B' = 0.000000 \times 10^{-0} \quad C' = 1.218043 \times 10^{-1} \]
\[ D' = 1.217557 \times 10^{-1} \quad E' = 1.346428 \times 10^{-1} \quad F' = 1.073322 \times 10^{-1} \]

\[ a_x = 2.868495 \quad a_y = 2.865291 \quad a_z = 2.865862 \quad a_o = 2.866313 \]

\[ \sigma_x = 158.02 \text{ MPa} \quad \sigma_y = -34.29 \text{ MPa} \]

\[ \% \varepsilon_x = 0.0761 \quad \% \varepsilon_y = -0.0357 \quad \% \varepsilon_z = -0.0157 \]

Residual Standard Deviation
\[ \text{RSD} = 7.02 \times 10^{-5} \]
Plots of Two-Theta vs $\sin^2(\psi)$ for values of $\phi$ from 0 to 180°

$\sigma_\phi \text{ (MPa)} = -318.04 \times \text{Slope}$
The solid line was obtained from the following equation;

\[ \sigma_{\phi'} = \frac{1}{2} (\sigma_x + \sigma_y) + \frac{1}{2} (\sigma_x - \sigma_y) \cos(2\phi') \]

- From \( \sin^2(\psi) \) plots (laboratory coordinates)
  - Corrected to the specimen coordinates.
Results of the analysis on a specimen of Inconel 600
In which the (420) and the (331) were employed.

\[ A = 7.895888 \times 10^{-2} \quad B = 3.532094 \times 10^{-5} \quad C = 7.919767 \times 10^{-2} \]
\[ D = 7.913487 \times 10^{-2} \quad E = -9.481498 \times 10^{-6} \quad F = 9.772829 \times 10^{-6} \]

\[ \text{Alpha} = -4.21^\circ \]

\[ A' = 7.895758 \times 10^{-2} \quad B' = 0.000000 \times 10^0 \quad C' = 7.919897 \times 10^{-2} \]
\[ D' = 7.913487 \times 10^{-2} \quad E' = 8.472686 \times 10^{-2} \quad F' = 7.311644 \times 10^{-2} \]

\[ a_x = 3.558796 \quad a_y = 3.553368 \quad a_z = 3.554807 \]
\[ a_o = 3.55538 \]

\[ \sigma_x = 179.89 \text{ MPa} \quad \sigma_y = -64.89 \text{ MPa} \]
\[ \% \varepsilon_x = 0.0961 \quad \% \varepsilon_y = -0.0566 \quad \% \varepsilon_z = -0.0161 \]

Residual Standard Deviation
\[ \text{RSD} = 1.21 \times 10^{-4} \]
$2\theta$ vs $\sin^2(\Psi)$ for Inconel 600 (420)

$\sigma$(MPa) = -358.9 * slope
Inconel 600, * (420) and ● (331) data.
The solid line was obtained from the following equation;

\[ \sigma_{\phi'} = \frac{1}{2} (\sigma_x + \sigma_y) + \frac{1}{2} (\sigma_x - \sigma_y) \cos(2\phi') \]

Using the combined data from the (420) and (331).
$E(100) = 123.3 \text{ GPa}$  
$E(110) = 223.0 \text{ GPa}$  
$E(111) = 305.3 \text{ GPa}$  
$E(420) = 172.8 \text{ GPa}$  
$E(331) = 249.3 \text{ GPa}$  
$E(\text{bulk}) = 206.8 \text{ GPa}$
I would like to take this opportunity to thank Sugiyama-san for his constant help during this study and arranging for the x-ray data to be taken at Rigaku in Japan and in particular I want to thank Kaminago-san for his diligent work in obtaining the necessary data for this paper.