Dalitz plot analysis in

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Physics at meson factories

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The high statistics and excellent quality of charm data now available allow for unprecedented sensitivity & sophisticated studies:

- lifetime measurements @ better than 1%
- CPV, mixing and rare&forbidden decays
- investigation of 3-body decay dynamics: **Dalitz plot analysis**
  - Phases and Quantum Mechanics interference: FSI
  - CP violation probe Focus $D^+ \to K^+K^-\pi^+$ (ICHEP 2002), Cleo $D^0 \to K_s\pi^+\pi^-$

but decay amplitude parametrization problems arise

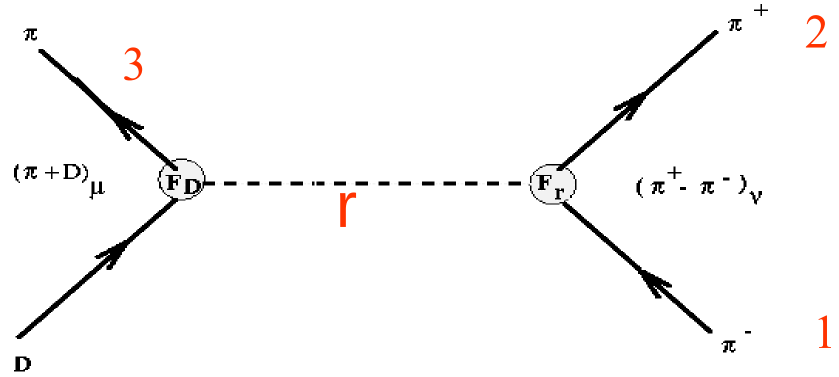
Complication for charm **Dalitz** plot **analysis**

Focus had to **face** the **problem** of dealing with **light scalar particles** populating charm meson hadronic decays, such as **$D\to\pi\pi\pi$**, **$D\to K\pi\pi$** including **σ(600)** and **κ(900)**, (i.e, $\pi\pi$ and $K\pi$ states produced close to threshold), whose **existence** and **nature** is still **controversial**

Amplitude parametrization

$D \longrightarrow r + 3$

$\quad \hookrightarrow 1 + 2$



The problem is to write the propagator for the resonance r

For a well-defined wave with specific isospin and spin *(IJ)* characterized by narrow and well-isolated resonances, we know how:

the propagator is of the simple BW type

$$A = F_D F_r \times \left| \vec{p}_1 \right|^J \left| \vec{p}_3 \right|^J P_J(\cos \vartheta_{13}^r) \times \frac{1}{m_r^2 - m_{12}{}^2 - i m_r \Gamma_r}$$

The isobar model

$$A = F_D F_r \times \left|\vec{p}_1\right|^J \left|\vec{p}_3\right|^J P_J(\cos\vartheta_{13}^r) \times BW(m_{12}^2)$$

Where

$$F = 1$$
$$F = (1 + R^2 p^2)^{-1/2}$$
$$F = (9 + 3R^2 p^2 + 3R^4 p^4)^{-1/2}$$

Spin 0: $P_J = 1$

Spin 1: $P_J = (-2\vec{p}_3 \cdot \vec{p}_1)$

Spin 2: $P_J = 2(p_3 p_1)^2 (3\cos^2\vartheta_{13} - 1)$

and

$$BW(12\,|\,r) = \frac{1}{M_r^2 - m_{12}^2 - i\Gamma M_r}$$

$$\Gamma = \Gamma_r \left[\frac{p}{p_0}\right]^{2j+1} \frac{M_r}{m_{12}} \frac{F_r^2(p)}{F_r^2(p_0)}$$

Dalitz total amplitude

$$\mathcal{M} = \sum_j a_j e^{i\delta_j} A_j$$

fit parameters (a_j, δ_j)

fit fraction

$$f_r = \frac{\int \left|a_r e^{i\delta_r} A_r\right|^2 dm_{12}^2 dm_{13}^2}{\int \left|\sum_j a_j e^{i\delta_j} A_j\right|^2 dm_{12}^2 dm_{13}^2}$$

traditionally applied to charm decays

In contrast

when the specific *IJ*–wave is characterized by large and heavily overlapping resonances (just as the scalars!), the problem is not that simple.

Indeed, it is very easy to realize that the propagation is no longer dominated by a single resonance but is the result of a complicated interplay among resonances.

In this case, it can be demonstrated on very general grounds that the propagator may be written in the context of the K-matrix approach as

$$(I - iK \cdot \rho)^{-1}$$

where K is the matrix for the scattering of particles 1 and 2.

⇩

i.e., to write down the propagator we need the scattering matrix

K-matrix formalism

E.P.Wigner,
Phys. Rev. 70 (1946) 15

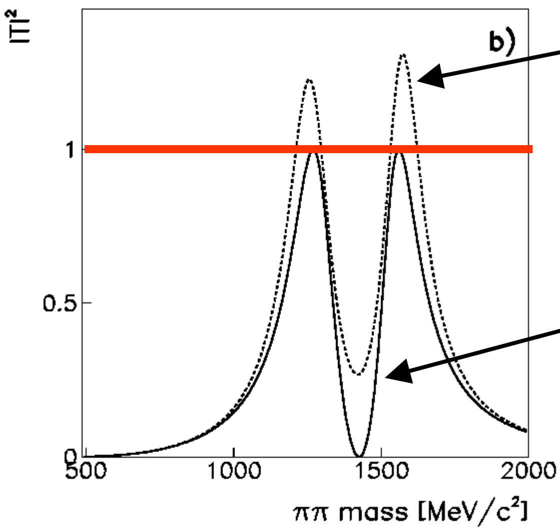
$$S = I + 2iT$$

T transition matrix

K-matrix is defined as: $K^{-1} = T^{-1} + i\rho$ i. e. $T = \left(I - iK\rho\right)^{-1} K$

real & symmetric

ρ = phase space diagonal matrix

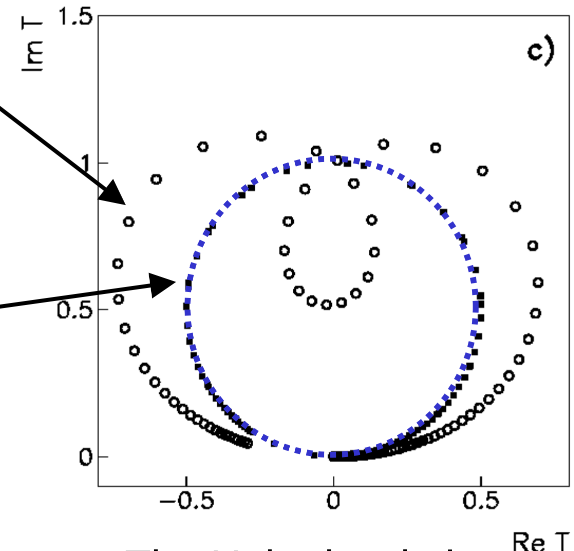


Add two BW ala Isobar model

Adding BW violates unitarity

Add two K matrices

Adding K matrices respects unitarity



The Unitarity circle

pioneering work by Focus

Dalitz plot analysis of D^+ and $D^+_s \rightarrow \pi^+\pi^-\pi^+$

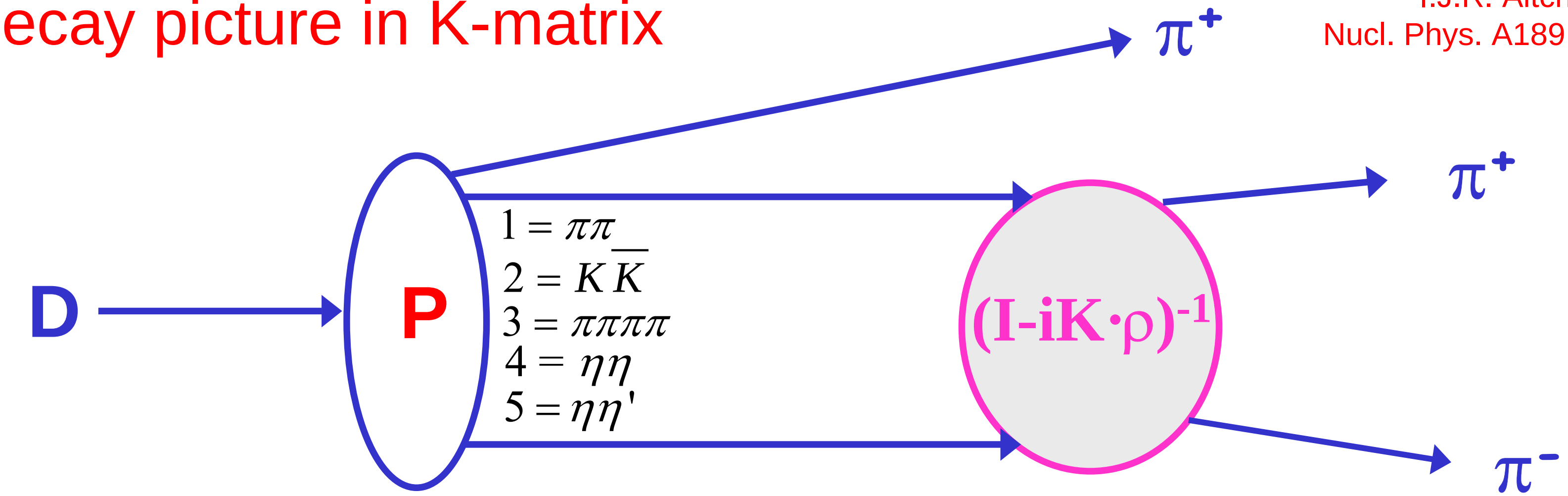
Phys. Lett. B 585 (2004) 200

first attempt to fit charm data

with the K-matrix formalism

D decay picture in K-matrix

I.J.R. Aitchison
Nucl. Phys. A189 (1972) 514



"K-matrix analysis of the 00^{++}-wave in the mass region below 1900 MeV"

V.V Anisovich and A.V.Sarantsev Eur. Phys.J.A16 (2003) 229

$$F = \frac{P}{I - iK\cdot\rho}$$

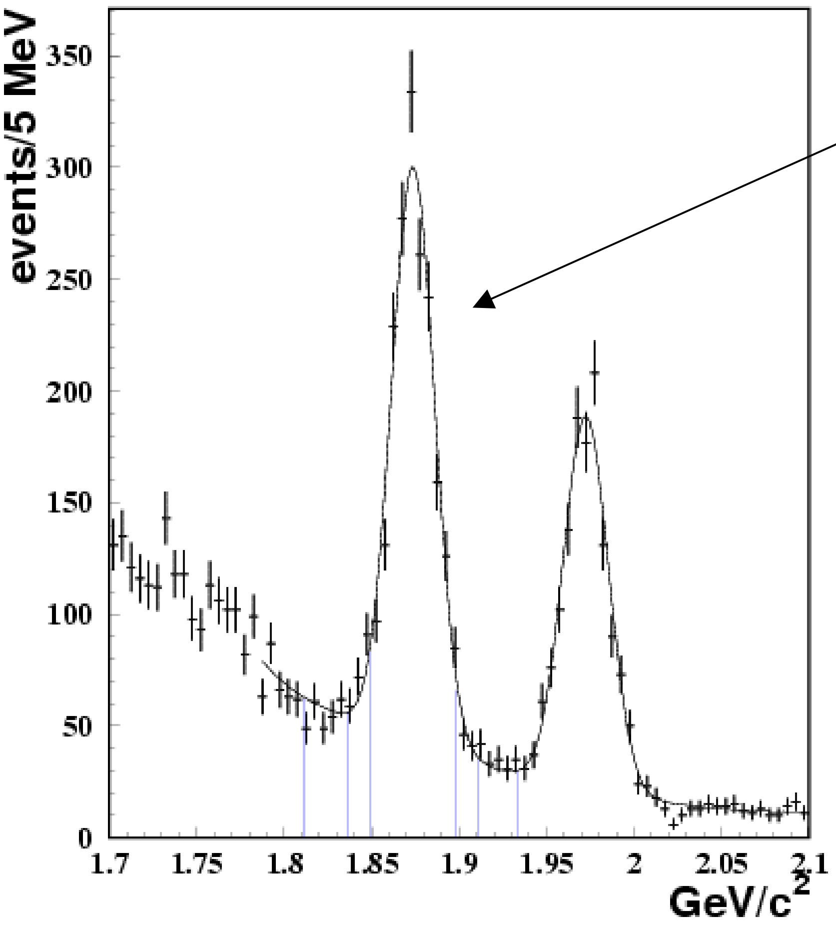
fixed to

BNL	$\pi^- p \to K\bar{K} n$
CERN-Münich	$\pi^+\pi^- \to \pi^+\pi^-$
Crystal Barrel	$p\bar{p} \to \pi^0\pi^0\pi^0,\ \pi^0\pi^0\eta$
Crystal Barrel	$p\bar{p} \to \pi^+\pi^-\pi^0,\ K^+K^-\pi^0,\ K_sK_s\pi^0,\ K^+K_s\pi^-$
etc...	

$$P_i = \sum_\alpha \frac{\beta_\alpha \gamma_{i\alpha} m_\alpha \Gamma_\alpha}{m_\alpha^2 - m^2} + d_i(m^2)$$

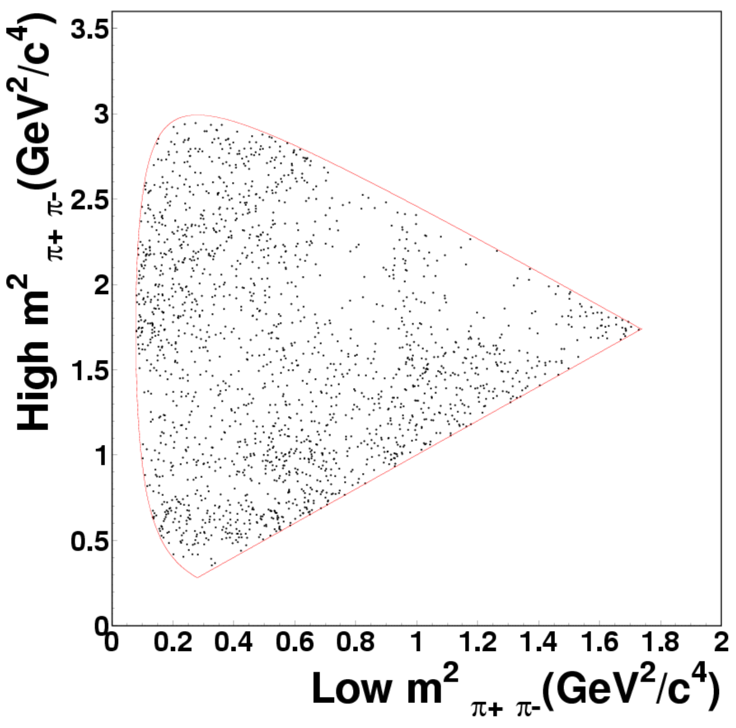
carries the production information
COMPLEX

$D^+ \to \pi^+ \pi^- \pi^+$

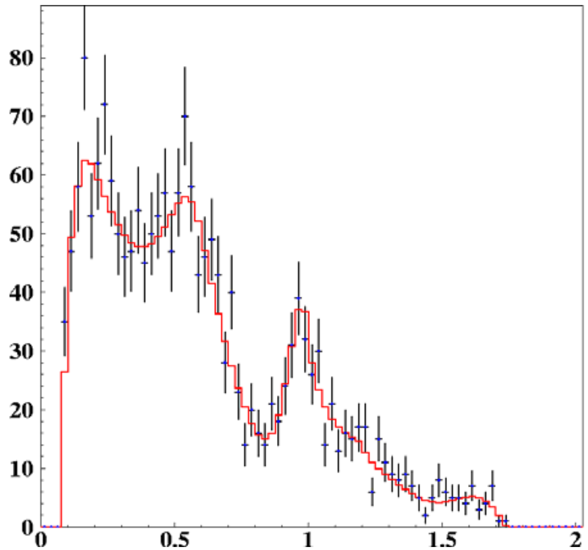


Yield $D^+ = 1527 \pm 51$

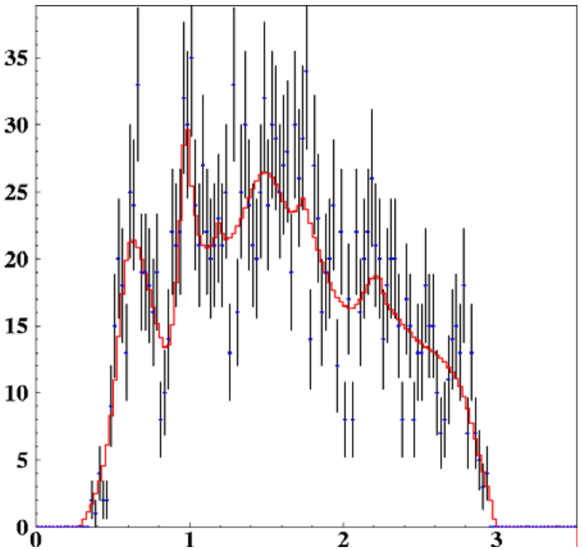
S/N $D^+ = 3.64$



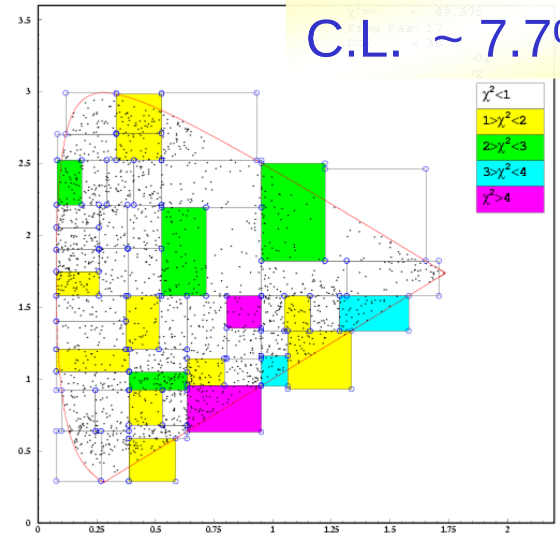
K-matrix fit results



Low $m^2_{\pi\pi}$ mass projection



High $m^2_{\pi\pi}$ mass projection



C.L. ~ 7.7%

	Decay fractions	Phases
(S-wave) π^+	$(56.00 \pm 3.24 \pm 2.08)$ %	0 (fixed)
$f_2(1275)\ \pi^+$	$(11.74 \pm 1.90 \pm 0.23)$ %	$(-47.5 \pm 18.7 \pm 11.7)$ °
$\rho(770)\ \pi^+$	$(30.82 \pm 3.14 \pm 2.29)$ %	$(-139.4 \pm 16.5 \pm 9.9)$ °

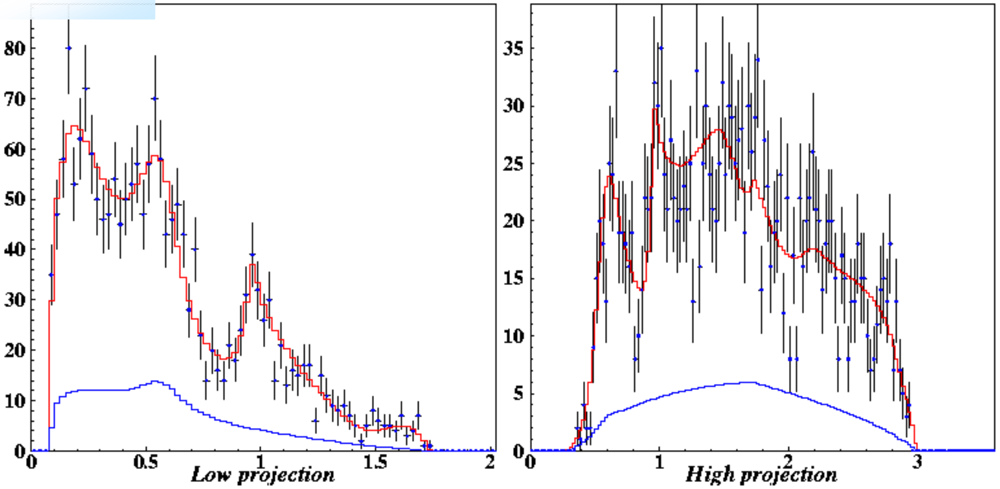
No new ingredient (resonance) required not present in the scattering!

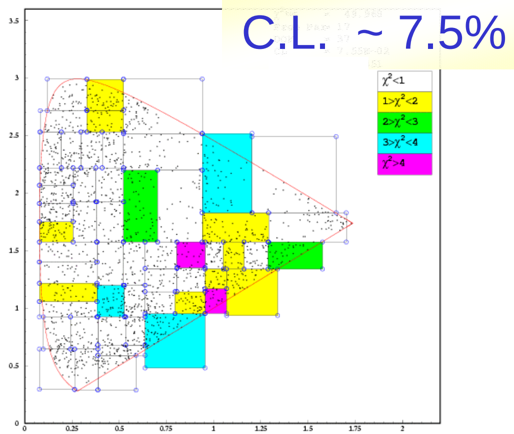
Isobar analysis of $D^+ \to \pi^+ \pi^+ \pi^-$ would instead require an ad hoc scalar meson: σ(600)

preliminary

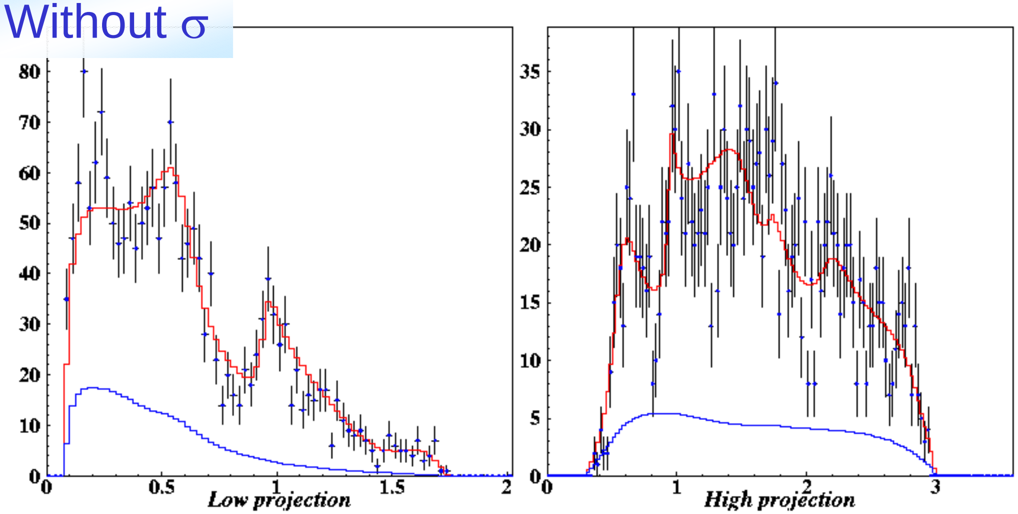
$m = 442.6 \pm 27.0$ MeV/c²
$\Gamma = 340.4 \pm 65.5$ MeV/c²

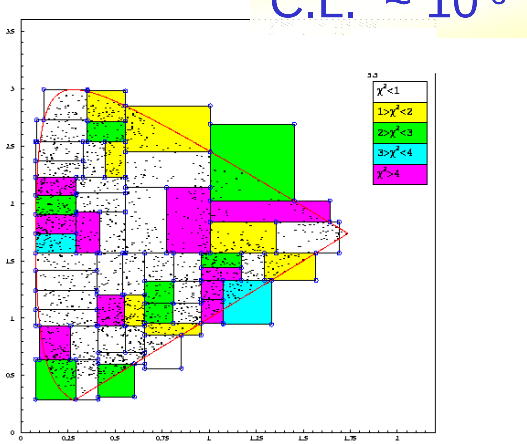
With σ





Without σ

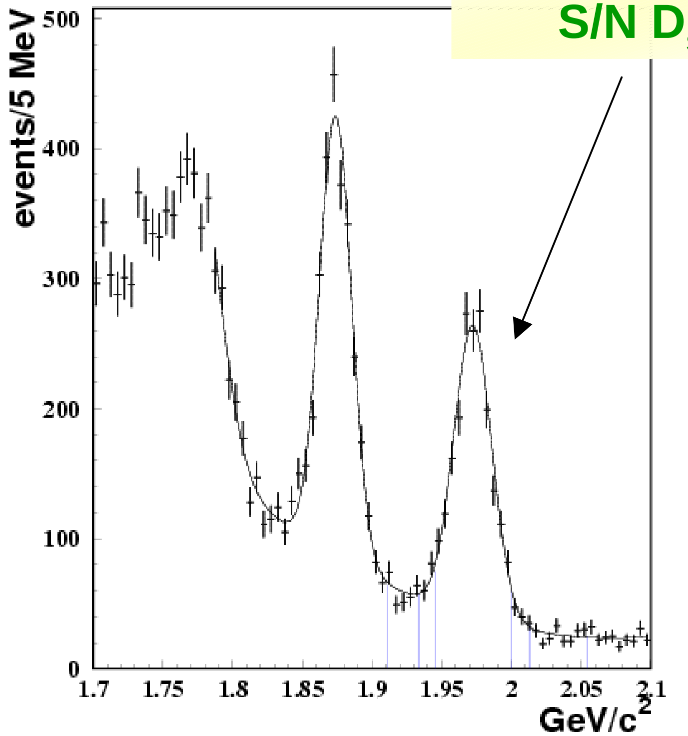


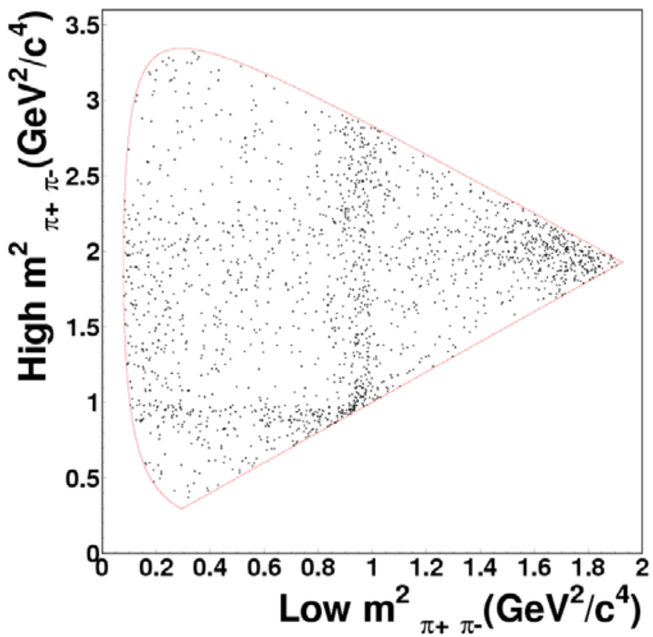


$D_s^+ \to \pi^+ \pi^- \pi^+$

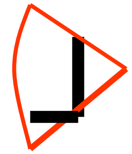
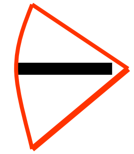
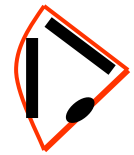
Yield D_s^+ = 1475 ± 50

S/N D_s^+ = 3.41

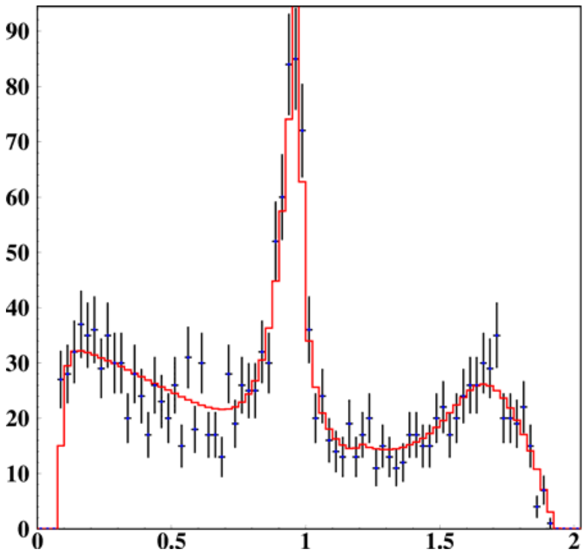




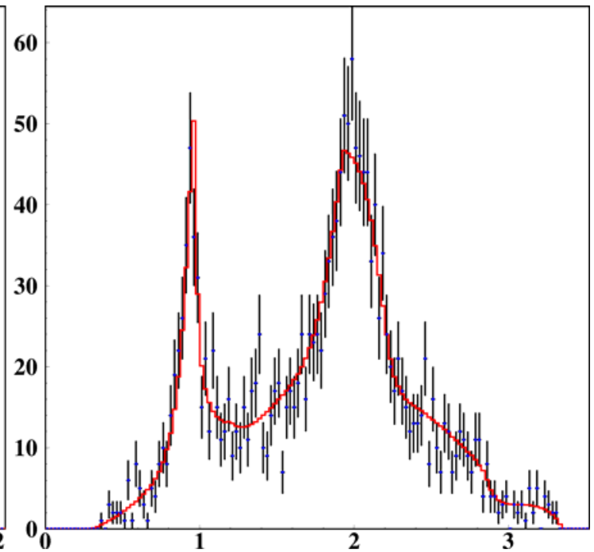
Observe:

- $f_0(980)$ 
- $f_0(1500)$ 
- $f_2(1270)$ 

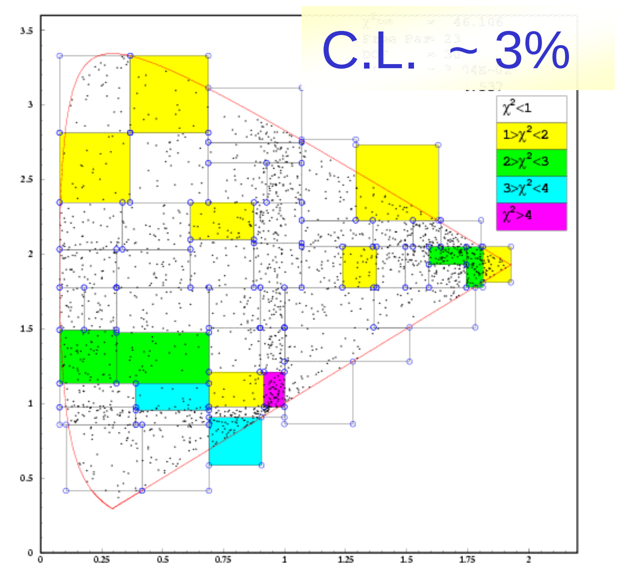
K-matrix fit results



Low $m^2_{\pi\pi}$ mass projection



High $m^2_{\pi\pi}$ mass projection



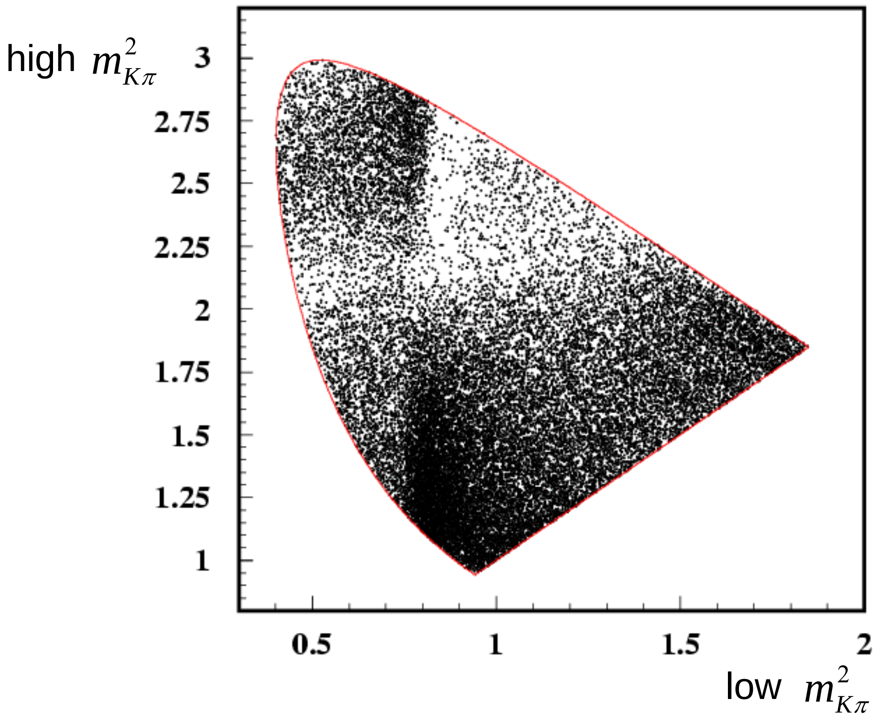
	Decay fractions	Phases
(S-wave) π^+	(87.04 ± 5.60 ± 4.17) %	0 (fixed)
$f_2(1275)$ π^+	(9.74 ± 4.49 ± 2.63) %	(168.0 ± 18.7 ± 2.5) °
$\rho(1450)$ π^+	(6.56 ± 3.43 ± 3.31) %	(234.9 ± 19.5 ± 13.3) °

from $D^+ \to \pi^+\pi^-\pi^+$ to $D^+ \to K^-\pi^+\pi^+$

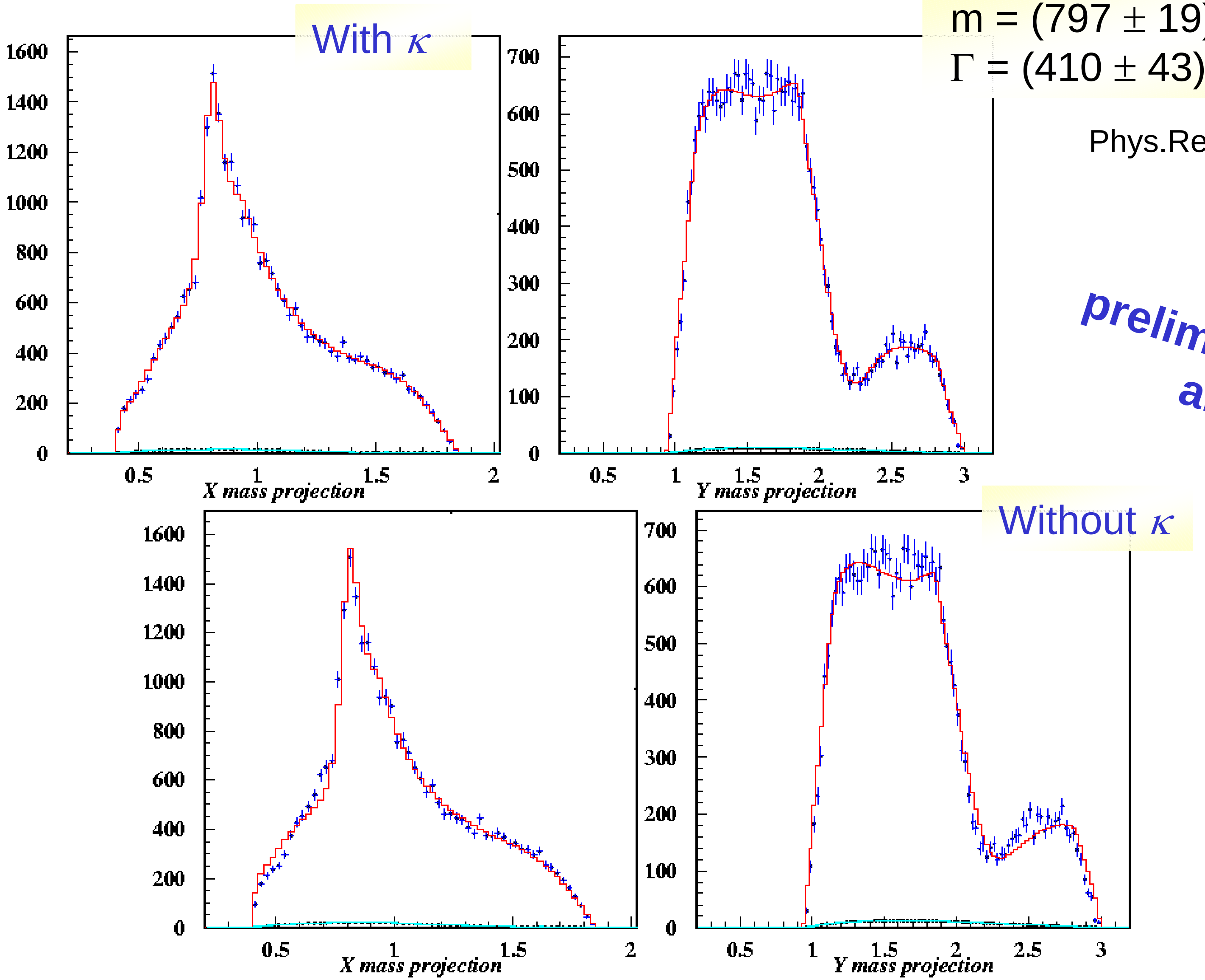
from $\pi\pi$ wave to $K\pi$ wave

from $\sigma(600)$ to $\kappa(900)$

from 1500 events to more than 50000!!!



Isobar analysis of $D^+ \to K^- \pi^+ \pi^+$ would require an ad hoc scalar meson: $\kappa(900)$



$m = (797 \pm 19)$ MeV/c^2

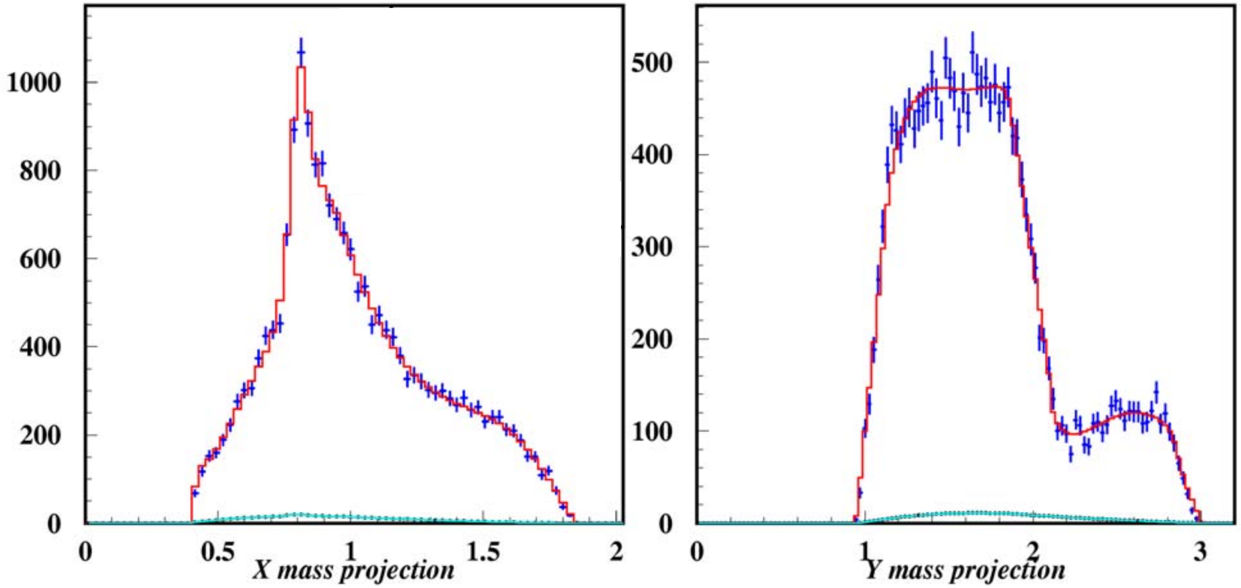
$\Gamma = (410 \pm 43)$ MeV/c^2

E791

Phys.Rev.Lett.89:121801,2002

preliminary FOCUS analysis

First attempt to fit the $D^+ \to K^- \pi^+ \pi^+$ in the K-matrix approach



very preliminary

$K\pi$ scattering data available from LASS experiment

a lot of work to be performed!!

a "real" test of the method (high statistics)…

in progress...

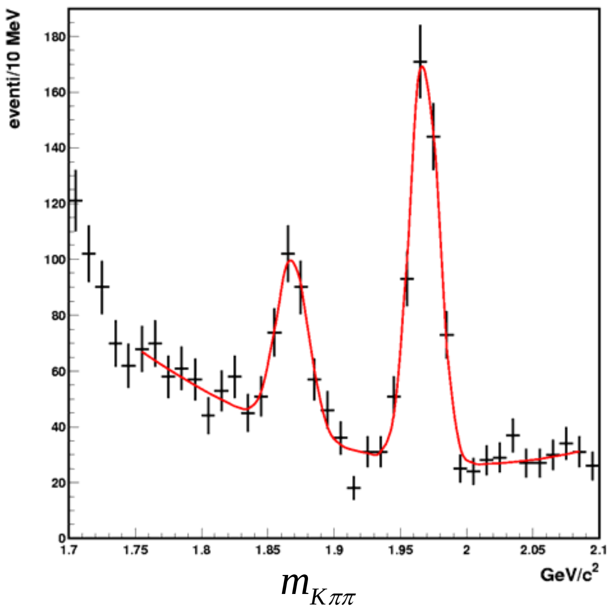
The excellent statistics allow for investigation of suppressed and even heavily suppressed modes

Doubly Cabibbo Suppressed

$D^+ \to K^+\pi^+\pi^-$

Yield $D^+ = 189 \pm 24$

S/N $D^+ = 1.0$



Singly Cabibbo Suppressed

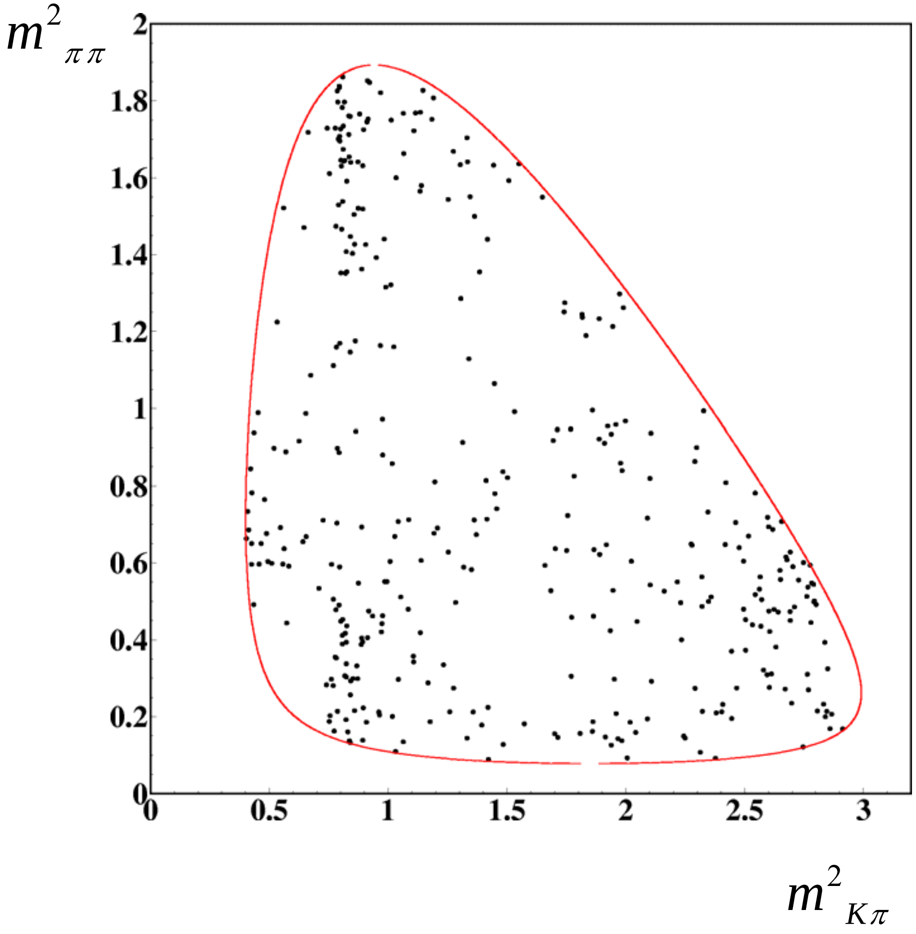
$D_s^+ \to K^+\pi^+\pi^-$

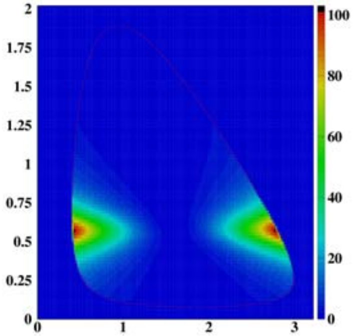
Yield $D_s^+ = 567 \pm 31$

S/N $D_s^+ = 2.4$

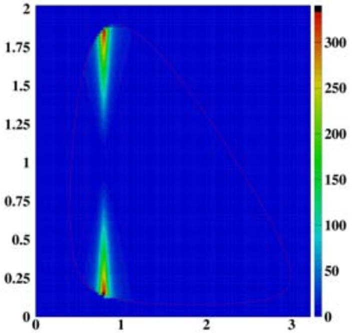
$\pi\pi$ & $K\pi$ s-waves are necessary...

$D^+ \to K^+ \pi^- \pi^+$





ρ(770)



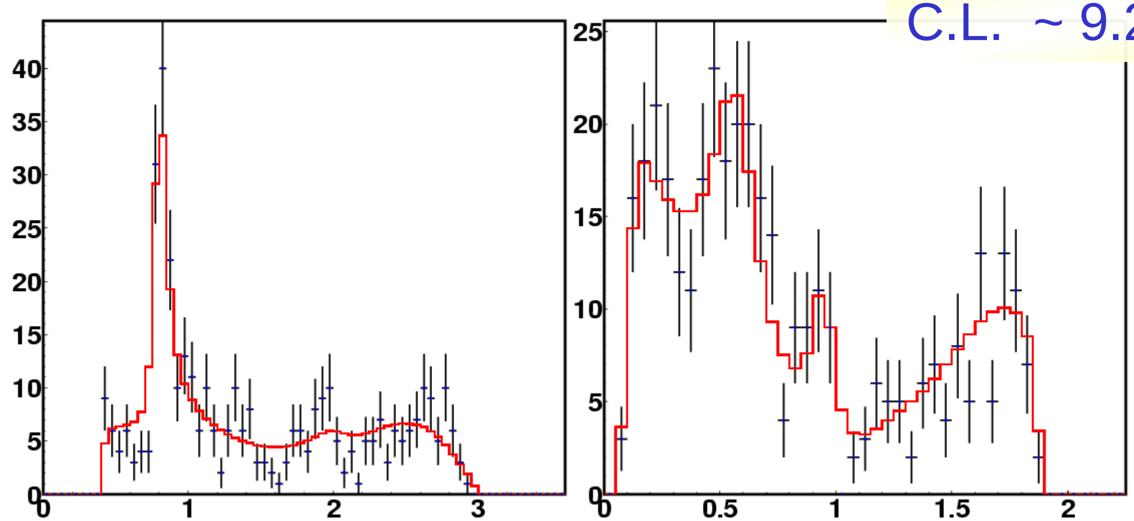
K*(892)

$D^+ \to K^+ \pi^- \pi^+$

isobar *effective* model

Doubly Cabibbo Suppressed Decay

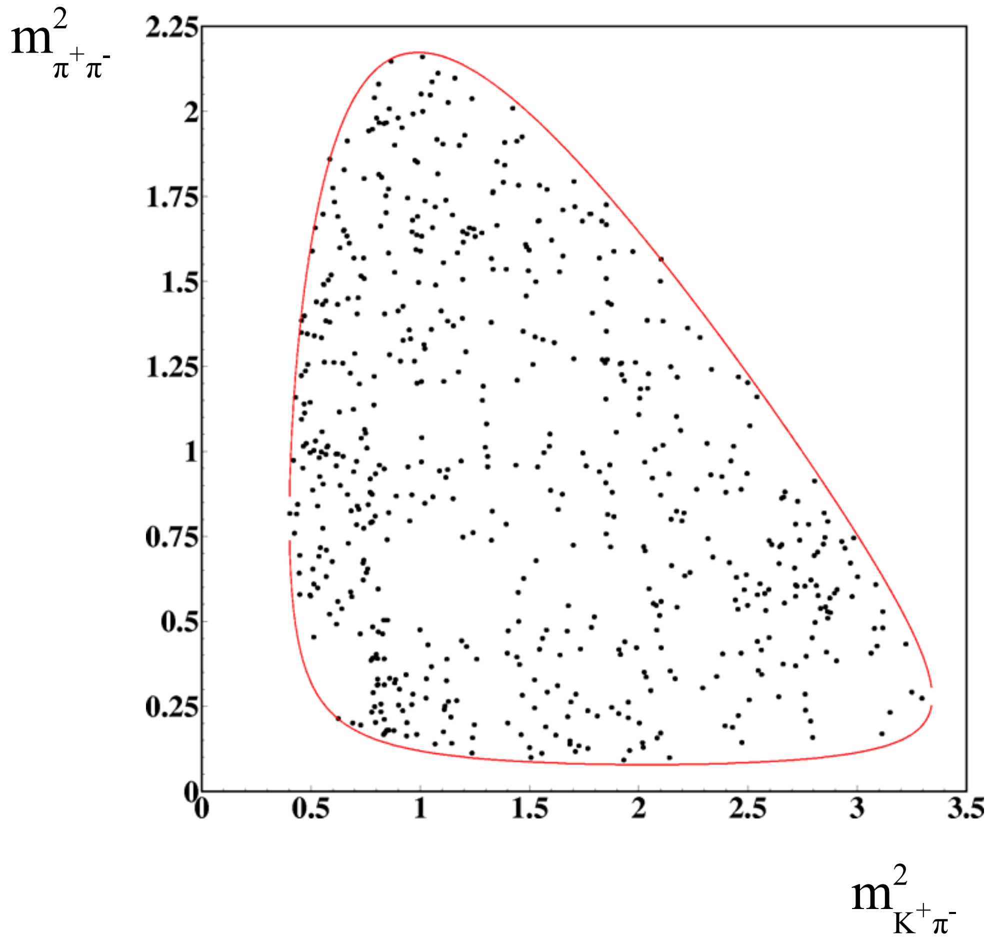
C.L. ~ 9.2%



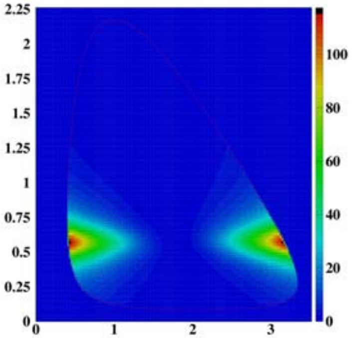
$K^+ \pi^-$ projection $\pi^+ \pi^-$ projection

Decay fractions	**Coefficients**	**Phases**
K*(892) = (52.2 ± 6.8 ± 6.4) %	1.15 ± 0.17 ± 0.16	(-167 ± 14 ± 23) °
ρ (770) = (39.4 ± 7.9 ± 8.2) %	1 fixed	(0 fixed)
K_2(1430) = (8.0 ± 3.7 ± 3.9) %	0.45 ± 0.13 ± 0.13	(54 ± 38 ± 21) °
f_0(980) = (8.9 ± 3.3 ± 4.1) %	0.48 ± 0.11 ± 0.14	(-135 ± 31 ± 42) °

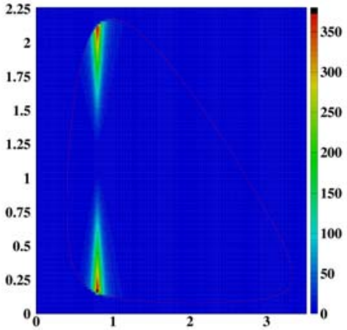
$D_s^+ \to K^+ \pi^- \pi^+$



visible contributions:
$\rho(770)$, $K^*(892)$



$\rho(770)$



$K^*(892)$

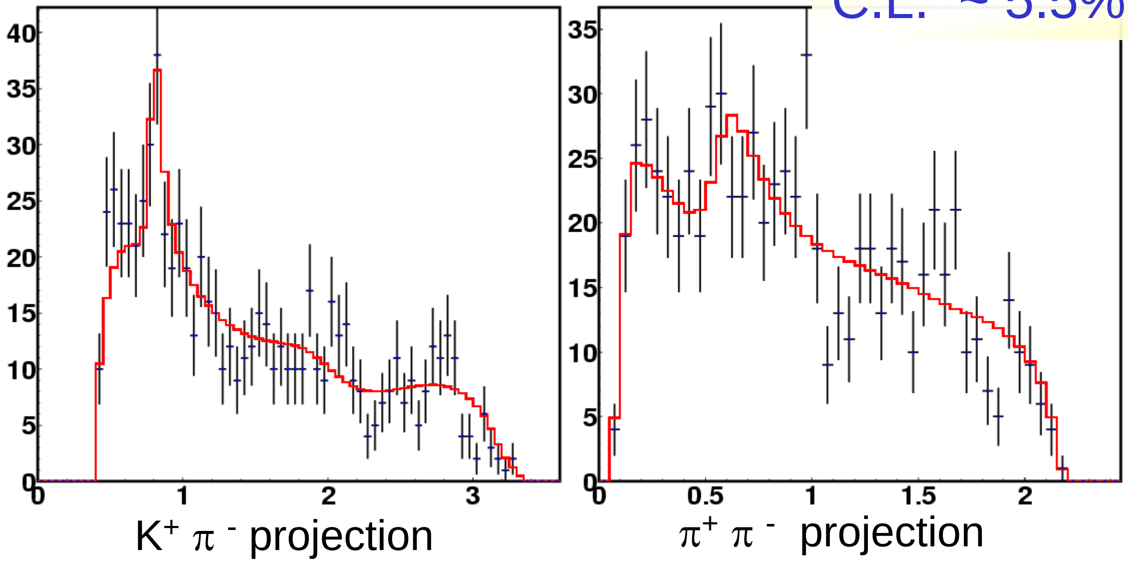
$D_s^+ \to K^+ \pi^- \pi^+$

isobar *effective* model

Singly Cabibbo Suppressed Decay

First Dalitz analysis

C.L. ~ 5.5%



Decay fractions	coefficients	Phases
ρ (770) = (38.8 ± 5.3 ± 2.6) %	1 fixed	(0 fixed)
K*(892) = (21.6 ± 3.2 ± 1.1) %	0.75 ± 0.08 ± 0.03	(162 ± 9 ± 2) °
NR = (15.9 ± 4.9 ± 1.5) %	0.64 ± 0.12 ± 0.03	(43 ± 10 ± 4) °
K*(1410) = (18.8 ± 4.0 ± 1.2) %	0.70 ± 0.10 ± 0.03	(-35 ± 12 ± 4) °
K^*_0(1430) = (7.7 ± 5.0 ± 1.7) %	0.44 ± 0.14 ± 0.06	(59 ± 20 ± 13) °
ρ (1450) = (10.6 ± 3.5 ± 1.0) %	0.52 ± 0.09 ± 0.02	(-152 ± 11 ± 4) °

Conclusions

- Dalitz analysis ⇨ interesting and promising results

- Focus has carried out a pioneering work!

  The K-matrix approach has been applied to charm decay for the first time

  The results are extremaly encouraging since the same parametrization of two-body $\pi\pi$ resonances coming from light-quark experiments works for charm decays too

- Cabibbo suppressed channels started to be analyzed

  now easy (isobar model), complications for the future ($\pi\pi$ and $K\pi$ waves)

- What we have just learnt will be crucial at higher charm statistics and for future beauty studies, such as $B \to \rho\pi$

slides for possible questions...

K-matrix formalism

Resonances are associated with poles of the S-matrix

$$S = I + 2iT$$

T transition matrix

K-matrix is defined as: $K^{-1} = T^{-1} + i\rho$ i. e. $T = (I - iK\rho)^{-1} K$

real & symmetric

ρ = phase space diagonal matrix

$$K_{ij} = \sum_{\alpha} \frac{\gamma_{i\alpha}\gamma_{j\alpha} m_\alpha \Gamma_\alpha}{m_\alpha^2 - m^2} + c_{ij}(m^2)$$

decay channels (ij)

sum over all poles (α)

γ_{ia} = coupling constant to channel i

m_a = K-matrix mass

Γ_a = K-matrix width

from scattering to production (from T to F):

carries the production information COMPLEX (β_α)

$$P_i = \sum_{\alpha} \frac{\beta_\alpha \gamma_{i\alpha} m_\alpha \Gamma_\alpha}{m_\alpha^2 - m^2} + d_i(m^2)$$

$$F = (I - iK\rho)^{-1} P$$

production vector (P_i)

from scattering to production (from T to F):

I.J.R. Aitchison
Nucl. Phys. A189 (1972) 514

$$T = \left(I - iK\rho\right)^{-1} K$$

⇩

$$F = \left(I - iK\rho\right)^{-1} P$$

$$P_i = \sum_{\alpha} \frac{\beta_\alpha \gamma_{i\alpha} m_\alpha \Gamma_\alpha}{m_\alpha^2 - m^2} + d_i(m^2)$$

production vector

carries the production information
COMPLEX

Dalitz
total
amplitude

$$\mathcal{M} = a_0 e^{i\vartheta_0} + F + \sum_{j} a_j e^{i\vartheta_j} BW$$

vector and
tensor
contributions

Only in a few cases the description through a simple BW is satisfactory.

- If $m_0 = m_a = m_b$

$$K = \frac{\gamma_a^2 m_a \Gamma_a}{m_a^2 - m^2} + \frac{\gamma_b^2 m_b \Gamma_b}{m_b^2 - m^2} \Rightarrow T = \frac{m_0 \left[\Gamma_a(m) + \Gamma_b(m)\right]}{m_0^2 - m^2 - i m_0 \left[\Gamma_a(m) + \Gamma_b(m)\right]}$$

The results is a single BW form where $\Gamma = \Gamma_a + \Gamma_b$ ⇒ The observed width is the sum of the two individual widths

- If m_a and m_b are far apart relative to the widths (no overlapping)

$$T \simeq \left[\frac{m_a \Gamma_a^0}{m_a^2 - m^2 - i m_a \Gamma_a(m)}\right]\left[\left(\frac{m_a}{m}\right)\left(\frac{q}{q_a}\right)\right] + \left[\frac{m_b \Gamma_b^0}{m_b^2 - m^2 - i m_b \Gamma_b(m)}\right]\left[\left(\frac{m_b}{m}\right)\left(\frac{q}{q_b}\right)\right]$$

The transition amplitude is given merely by the sum of 2 BW

The K-matrix formalism gives us the correct tool to deal with the nearby resonances

e.g. 2 poles ($f_0(1370)$ - $f_0(1500)$) coupled to 2 channels ($\pi\pi$ and KK)

$$K_{ij} = \frac{\gamma_{ai}\gamma_{aj} m_a \Gamma_a}{m_a^2 - m^2} + \frac{\gamma_{bi}\gamma_{bj} m_b \Gamma_b}{m_b^2 - m^2} \qquad P_i = \frac{\beta_a \gamma_{ai} m_a \Gamma_a}{m_a^2 - m^2} + \frac{\beta_b \gamma_{bi} m_b \Gamma_b}{m_b^2 - m^2}$$

total amplitude

$$F_i = \frac{\beta_a m_a \Gamma_a \gamma_{a1}\left(m_b^2 - m^2\right) + \beta_b m_b \Gamma_b \gamma_{b1}\left(m_a^2 - m^2\right) - i m_a \Gamma_a m_b \Gamma_b \rho_2 \left(\gamma_{a2}\beta_b - \beta_a \gamma_{b2}\right)\left(\gamma_{a2}\gamma_{b1} - \gamma_{a1}\gamma_{b2}\right)}{\left(m_a^2 - m^2\right)\left(m_b^2 - m^2\right) - i m_a \Gamma_a \left(\gamma_{a1}^2 \rho_1 + \gamma_{a2}^2 \rho_2\right)\left(m_b^2 - m^2\right) - i m_b \Gamma_b \left(\gamma_{b1}^2 \rho_1 + \gamma_{b2}^2 \rho_2\right)\left(m_a^2 - m^2\right) - m_a \Gamma_a m_b \Gamma_b \rho_1 \rho_2 \left(\gamma_{a2}\gamma_{b1} - \gamma_{a1}\gamma_{b2}\right)^2}$$

if you treat the 2 f_0 scalars as **2 independent BW**:

$$F_i = \frac{\beta_a \gamma_{ai} m_a \Gamma_a}{m_a^2 - m^2 - i m_a \Gamma_a \left(\gamma_{a1}^2 \rho_1 + \gamma_{a2}^2 \rho_2\right)} + \frac{\beta_b \gamma_{bi} m_b \Gamma_b}{m_b^2 - m^2 - i m_b \Gamma_b \left(\gamma_{b1}^2 \rho_1 + \gamma_{b2}^2 \rho_2\right)}$$

no ‘mixing’ terms!

the unitarity is not respected!

$IJ^{PC} = 00^{++}$ wave has been reconstructed on the basis of a complete available data set

K^{IJ}_{ab} is a 5x5 matrix (a,b = 1,2,3,4,5)

1 = ππ 2 = K$\bar{K}$

3 = ηη 4 = ηη'

5 = multimeson states (4π)

Scattering amplitude:

$$K_{ij}^{00}(s) = \left(\sum_\alpha \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{M^2_\alpha - s} + f_{ij}^{scatt} \frac{1 GeV^2 - s_0^{scatt}}{s - s_0^{scatt}} \right) \frac{s - s_A m_\pi^2 / 2}{(s - s_{A0})(1 - s_{A0})}$$

$g_i^{(\alpha)}$ is the coupling constant of the bare state α to the meson channel $g_i^{(\alpha)}(m) = \sqrt{m_\alpha \Gamma_i^{(\alpha)}(m)}$

f_{ij}^{scatt} and s_0^{scatt} describe a smooth part of the K-matrix elements

$(s - s_A m_\pi^2 / 2) / (s - s_{A0})(1 - s_{A0})$ suppresses false kinematical singularity at s=0 near ππ threshold

Production of resonances:

$$P_j = \left(\sum_\alpha \frac{\beta_\alpha g_j^{(\alpha)}}{M^2_\alpha - s} + f_{bck} \frac{1 GeV^2 - s_0^{prod}}{s - s_0^{prod}} \right) \frac{s - s_A m_\pi^2 / 2}{(s - s_{A0})(1 - s_{A0})}$$

fit parameters

A description of the scattering ...

A global fit to all the available data has been performed!

"K-matrix analysis of the 00++-wave in the mass region below 1900 MeV''

V.V Anisovich and A.V.Sarantsev Eur.Phys.J.A16 (2003) 229

* **GAMS**	$\pi p \to \pi^0\pi^0 n, \eta\eta n, \eta\eta' n$, $\lvert t\rvert < 0.2$ (GeV/c²)	
* **GAMS**	$\pi p \to \pi^0\pi^0 n$, $0.30 < \lvert t\rvert < 1.0$ (GeV/c²)	
* **BNL**	$\pi p^- \to K\bar{K}n$	
* **CERN-Munich**	$\pi^+\pi^- \to \pi^+\pi^-$	
* **Crystal Barrel**	$p\bar{p} \to \pi^0\pi^0\pi^0, \pi^0\pi^0\eta, \pi^0\eta\eta$	At rest, from liquid H_2
* **Crystal Barrel**	$p\bar{p} \to \pi^0\pi^0\pi^0, \pi^0\pi^0\eta$	At rest, from gaseous H_2
* **Crystal Barrel**	$p\bar{p} \to \pi^+\pi^-\pi^0, K^+K^-\pi^0, K_sK_s\pi^0, K^+K_s\pi^-$	At rest, from liquid H_2
* **Crystal Barrel**	$n\bar{p} \to \pi^0\pi^0\pi^-, \pi^-\pi^-\pi^+, K_sK^-\pi^0, K_sK_s\pi^-$	At rest, from liquid D_2
* **E852**	$\pi^- p \to \pi^0\pi^0 n$, $0 < \lvert t\rvert < 1.5$ (GeV/c²)	

A&S K-matrix poles, couplings etc.

$Poles$	$g_{\pi\pi}$	g_{KK}	$g_{4\pi}$	$g_{\eta\eta}$	$g_{\eta\eta'}$
0.65100	0.24844	–0.52523	0	–0.38878	–0.36397
1.20720	0.91779	0.55427	0	0.38705	0.29448
1.56122	0.37024	0.23591	0.62605	0.18409	0.18923
1.21257	0.34501	0.39642	0.97644	0.19746	0.00357
1.81746	0.15770	–0.17915	–0.90100	–0.00931	0.20689
s_0^{scatt}	f_{11}^{scatt}	f_{12}^{scatt}	f_{13}^{scatt}	f_{14}^{scatt}	f_{15}^{scatt}
–3.30564	0.26681	0.16583	–0.19840	0.32808	0.31193
s_A	s_{A0}				
1.0	–0.2				

A&S T-matrix poles and couplings

$(m, \Gamma/2)$	$g_{\pi\pi}$	g_{KK}	$g_{4\pi}$	$g_{\eta\eta}$	$g_{\eta\eta'}$
(1.019, 0.038)	$0.415 e^{i\,13.1}$	$0.580 e^{i\,96.5}$	$0.1482\, e^{i\,80.9}$	$0.484\, e^{i\,98.6}$	$0.401\, e^{i\,102.1}$
(1.306, 0.167)	$0.406\, e^{i\,116.8}$	$0.105\, e^{i\,100.2}$	$0.8912\, e^{-i\,61.9}$	$0.142\, e^{i\,140.0}$	$0.225\, e^{i\,133.0}$
(1.470, 0.960)	$0.758\, e^{i\,97.8}$	$0.844 e^{i\,97.4}$	$1.681\, e^{i\,91.1}$	$0.431\, e^{i\,115.5}$	$0.175\, e^{i\,152.4}$
(1.489, 0.058)	$0.246\, e^{i\,151.5}$	$0.134\, e^{i\,149.6}$	$0.4867\, e^{-i\,123.3}$	$0.100\, e^{-i\,170.6}$	$0.115\, e^{-i\,133.9}$
(1.749, 0.165)	$0.536\, e^{i\,101.6}$	$0.072\, e^{i\,134.2}$	$0.7334\, e^{-i\,123.6}$	$0.160\, e^{i\,126.7}$	$0.313\, e^{i\,101.1}$

A&S fit does not need a σ as measured in the isobar fit

D_s production coupling constants

f_0(980)	(1.019,0.038)	1 e^{i 0} (fixed)
f_0(1300)	(1.306,0.170)	(0.43 \pm 0.04) e^{i(-163.8 \pm 4.9)}
f_0(1200-1600)	(1.470,0.960)	(4.90 \pm 0.08) e^{i(80.9 \pm1.06)}
f_0(1500)	(1.488,0.058)	(0.51 \pm 0.02) e^{i(83.1 \pm 3.03)}
f_0(1750)	(1.746,0.160)	(0.82 \pm0.02) e^{i(-127.9 \pm 2.25)}

D^+ production coupling constants

f_0(980)	(1.019,0.038)	1 e^{i0} (fixed)
f_0(1300)	(1.306,0.170)	(0.67 \pm 0.03) e^{i(-67.9 \pm 3.0)}
f_0(1200-1600)	(1.470,0.960)	(1.70 \pm 0.17) e^{i(-125.5\pm 1.7)}
f_0(1500)	(1.489,0.058)	(0.63 \pm 0.02) e^{i(-142.2\pm 2.2)}
f_0(1750)	(1.746,0.160)	(0.36 \pm 0.02) e^{i(-135.0 \pm 2.9)}

The Q-vector approach

- We can view the decay as consisting of an initial production of the five virtual states ππ, $K\overline{K}$, ηη, ηη' and 4π, which then scatter via the physical T-matrix into the final state.

$$F = (I - iK\rho)^{-1} P = (I - iK\rho)^{-1} K K^{-1} P = T K^{-1} P = TQ$$

The Q-vector contains the production amplitude of each virtual channel in the decay

The resulting picture

- The S-wave decay amplitude primarily arises from a $s\bar{s}$ contribution.
- For the D^+ the $s\bar{s}$ contribution competes with a $d\bar{d}$ contribution.
- Rather than coupling to an S-wave dipion, the $d\bar{d}$ piece prefers to couple to a vector state like $\rho(770)$, that alone accounts for about 30 % of the D^+ decay.
- This interpretation also bears on the role of the annihilation diagram in the $D_s^+ \to \pi^+\pi^-\pi^+$ decay:
  - the S-wave annihilation contribution is negligible over much of the dipion mass spectrum. It might be interesting to search for annihilation contributions in higher spin channels, such as $\rho^0(1450)\pi$ and $f_2(1270)\pi$.

CP violation on the Dalitz plot

- For a two-body decay

$$\mathbf{A_{tot}} = g_1 M_1 e^{i\delta_1} + g_2 M_2 e^{i\delta_2}$$

CP conjugate

$$\overline{\mathbf{A}}_{\mathbf{tot}} = g_1^* M_1 e^{i\delta_1} + g_2^* M_2 e^{i\delta_2}$$

δ_i = strong phase

CP asymmetry:

$$a_{CP} = \frac{|A_{tot}|^2 - |\overline{A}_{tot}|^2}{|A_{tot}|^2 + |\overline{A}_{tot}|^2} = \frac{2\,\mathrm{Im}(g_2\, g_1^*)\,\sin(\delta_1 - \delta_2) M_1 M_2}{|g_1|^2 M_1^2 + |g_2|^2 M_2^2 + 2\,\mathrm{Re}(g_2\, g_1^*)\cos(\delta_1 - \delta_2) M_1 M_2}$$

2 different amplitudes

strong phase-shift

CP violation & Dalitz analysis

Dalitz plot = FULL OBSERVATION of the decay

COEFFICIENTS and **PHASES** for each amplitude

Measured phase: $\theta = \delta + \phi$

CP conserving: $\bar{\delta} = \delta$

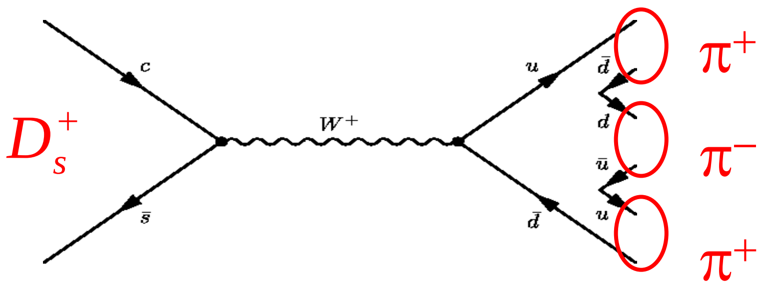
CP violating: $\bar{\phi} = -\phi$

CP conjugate → $\bar{\theta} = \delta - \phi$

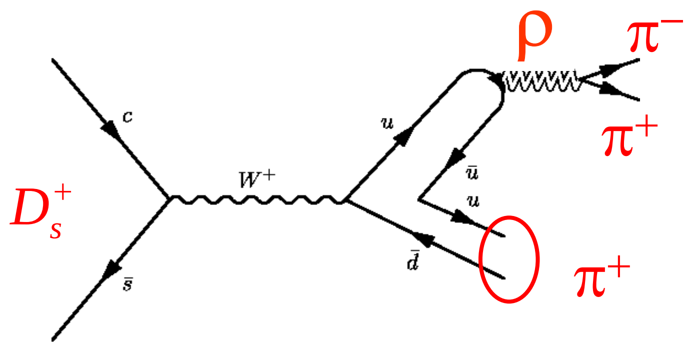
E831 → Measure of direct CP violation: asymmetrys in decay rates of $D^{\pm} \to K^{\mp} K \pi^{\pm}$

$a_{CP} = 0.006 \pm 0.011 \pm 0.005$

•**No significant direct three-body-decay component**



•**No significant ρ(770) π contribution**



⬇

Marginal role of annihilation in charm hadronic decays

But need more data!