Kaonic Atoms

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1. Introduction
2. πK atoms
3. Kaonic hydrogen
4. More complicated systems
5. Conclusions

1. Introduction

- Kaonic atoms are of the type

$$\boxed{\bar{K}X}$$

$$X = \pi, K; \text{p; d; } {}^3\text{He; } {}^4\text{He} \ldots$$

- These are by definition bound by electromagnetic interactions, so a more precise title of my talk would be

Kaonic Atoms in QCD + QED

- Deeply bound kaonic nuclear states are of a different variety - predicted to exist already in the framework of QCD [1], Maxwell is not needed to bind

I do not consider these systems here (nor $K\bar{K}$ bound states [2])

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[1] Akaishi, this conference; Outa, this conference

[2] Krewald 2003

Basic idea:

- System bound by electromagnetic interactions
- Hadronic interactions
  - change spectrum
  - let atoms decay

Energy shift, lifetime

$\Updownarrow$ [1]

$T-$ matrix at threshold

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[1] Deser, Goldberger, Baumann, Thirring 1954; Trueman 1961

Classic examples of this information:

Pionic hydrogen	PSI	$T_{\pi N}$ [1]
Pionium	DIRAC	$T_{\pi\pi}$ [2]
Kaonic Hydrogen	DEAR	$T_{\bar{K}N}$ [3]

These experiments have the potential to replace low energy experiments on

$\pi N \to \pi N$

$\pi\pi \to \pi\pi$

$\bar{K}N \to \bar{K}N$

that are difficult (or impossible) to perform

Why do we want to know?

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[1] Trasinelli, this conference

[2] Tauscher, this conference

[3] Jensen, Zmeskal, this conference

We wish to confront high precision, low energy QCD predictions with data

Example: $\pi\pi$ scattering lengths

Theory [1]

$$a_0 = 0.220 \pm 0.005$$

$$a_0 - a_2 = 0.265 \pm 0.004$$

Experiment ?

K_{e4} E865 [2]

$$a_0 = 0.216 \pm 0.013 \pm 0.002 \pm 0.002$$

Pionium DIRAC [3]

$K_{e4}, K \rightarrow 3\pi$ NA48, in progress [4]

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[1] Colangelo, Gasser, Leutwyler 2000; [2] E865, Brookhaven, 2002,2003; [3] Tauscher, this afternoon; [4] NA48 Workshop 2004; Cabibbo, this conference

Procedure consists of 2 steps:

$\boxed{1}$

Relate properties of atomic spectra to QCD scattering amplitudes at threshold

$\boxed{2}$

Predict QCD amplitudes using effective field theories or lattice calculations, and compare with data from atomic spectra, using $\boxed{1}$

Situation for kaonic atoms?

	experiment	*theory*
$\bar{K}\pi$	LoI [1]	*** [2]
$\bar{K}p$	DEAR ** [3]	** [4]
$\bar{K}d$	SIDDHARTA [5]	
$\bar{K}X$	FINUDA [6]	

X=^{6}Li,^{7}Li,^{12}C,^{27}Al,^{51}V, . . .

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[1] DIRAC collaboration 2004. [2] Schweizer 2003, 2004; Sazdjian, work in progress. [3] Zmeskal, this conference. [4] A. Ivanov et al., 2003; Meißner, Rusetsky and Raha 2004. [5] Iliescu, this conference. [6] Outa, this conference

2. $K\pi$ atoms

$K\pi$ atoms are interesting, because change in spectrum is related to $SU(3) \times SU(3)$ ChPT

1. How is this relation worked out?

momenta of

- constituents
- outgoing mesons

are small, order of MeV

$$\Downarrow$$

May use non relativistic framework [1]

Has many technical advantages

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[1] Caswell, Lepage 1985

2. Why is the procedure valid?

$\boxed{1}$

Bohr radius $\simeq 250$ fm

Lifetime $\tau \simeq 4 \cdot 10^{-15}$ sec

$\tau \cdot E_B \simeq 1.4 \cdot 10^4$ orbits before decaying

$\boxed{2}$

binding energy $E_B \simeq$ 2.9 keV

strong energy shift $\simeq$ - 9 eV

Conclude:

perturbation of spectrum due to hadronic interactions is small

$$M_{K^+} + M_{\pi^+} = M_{K^0} + M_{\pi^0} + 0.44 \text{ MeV}$$

$$\Downarrow$$

$$A_{K^+\pi^-} \to K^0\pi^0, K^0 + n\gamma, \cdots$$

Expand Γ in powers of $\delta \simeq \alpha \simeq m_d - m_u$.

Ground state: leading and next-to-leading terms are due to decay into $K^0\pi^0$:

$$\Gamma_G = \underbrace{a\,\delta^{7/2} + b\,\delta^{9/2}}_{K^0\pi^0} + \underbrace{\mathcal{O}(\delta^5)}_{K^0\pi^0\text{+other channels}}$$

$$\Gamma_G = \Gamma_{K^0\pi^0} + \mathcal{O}(\delta^5)$$

3. Results

Decay of ground state:

$$\Gamma_G = 8\alpha^3\mu_c^2 p^*[a_0^-]^2(1+\epsilon) + \mathcal{O}(\delta^5)$$

p^* relative 3-momentum of $\pi^0 K^0$ pair

ϵ: correction due to isospin breaking, known at order δ [1]

Therefore:

$\Gamma_G \quad \rightarrow \quad a_0^- \quad \leftrightarrow$ low energy QCD

Main point: a_0^- is scattering length in pure QCD, purified from electromagnetic corrections, evaluated at $m_u = m_d$

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[1] J. Schweizer 2004; H. Sazdjian, in progress

Roy-Steiner equations [1], ChPT [2]

$$\to a_0^- \to$$

$$\tau_G = (3.7 \pm 0.4) \cdot 10^{-15}\text{s}$$

Main open problem: Experiment?

Notice [3]:

$\tau_{n=2,l=0} = (29.4 \pm 3.3) \cdot 10^{-15}\text{s}$

$\tau_{n=2,l=1} \simeq 7 \cdot 10^{-12}$ s

$2P$-state: leading decay into $1S$ state via photon emission

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[1] Buettiker, Descotes-Genon, Moussallam 2003

[2] Bijnens, Talavera 2004

[3] Schweizer 2004

Similarly for the energy levels

$$\triangle E^h = -2\alpha^3 \mu_r^2 (a_0^+ + a_0^-)(1 + \epsilon')$$

ϵ' is known at order δ [1]

Provided energy levels and width can be measured accurately, one measures $a_0^\pm$

$$\begin{array}{lcl} a_0^- & \leftrightarrow & L_5^r \\ a_0^+ & \leftrightarrow & 2L_6^r + L_8^r, \ldots \end{array}$$

$2L_6^r + L_8^r \leftrightarrow$ vacuum properties of QCD

Main conclusion: Theory of πK atom very well understood. Experiment missing

Measurability of L_i^r remains to be clarified

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[1] Schweizer 2004; Sazdjian in progress

3. Kaonic hydrogen

1. Preliminaries

$\boxed{1}$

Bohr radius $\simeq 80$ fm

Lifetime $\simeq 2.6 \cdot 10^{-18}$ sec [1]

$\tau \cdot E_B \simeq 35 \ (!)$ orbits before decaying

$\boxed{2}$

binding energy $\simeq$ 9 keV

strong energy shift $\simeq$ .5 keV [1]

perturbation of electromagnetic spectrum due to hadronic interactions is small enough to treat it as a perturbation

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[1] See talk by Zmeskal, and below

2. Theory

Electromagnetic interactions:

$$E_{1s}^{em} = -\frac{\alpha^2}{2}\mu_r + O(\alpha^4)$$

$A_{\bar{K}p}$ is not stable:

$$A_{\bar{K}p} \to \pi\Sigma, \Sigma\pi\gamma, \Sigma\pi e^+e^-, \Sigma\gamma, \ldots$$

$$E_{1s} = E_{1s}^{em} + \epsilon_{1s}$$

In the absence of isospin breaking corrections:

$$\epsilon_{1s} = -2\alpha^3\mu_r^2 \, \mathsf{T}^{\mathsf{QCD}} \qquad (1)$$

where $\mathsf{T}^{\mathsf{QCD}}$ is the QCD threshold amplitude

$$\bar{K}p \to \bar{K}p$$

in the isospin symmetry limit

.

T$^{\mathsf{QCD}}$ is the object of our desire:

$$\mathsf{T}^{\mathsf{QCD}} = (a_0 + a_1)/2$$

a_0, a_1 are $\bar{K}N$ scattering lengths in pure QCD, at $m_u = m_d$

Unfortunately, formula (1) is not correct: ϵ_{1s} contains information on the decay channels listed above → contributions from $\alpha, m_u \neq m_d$, further isospin breaking effects are present on the right hand side

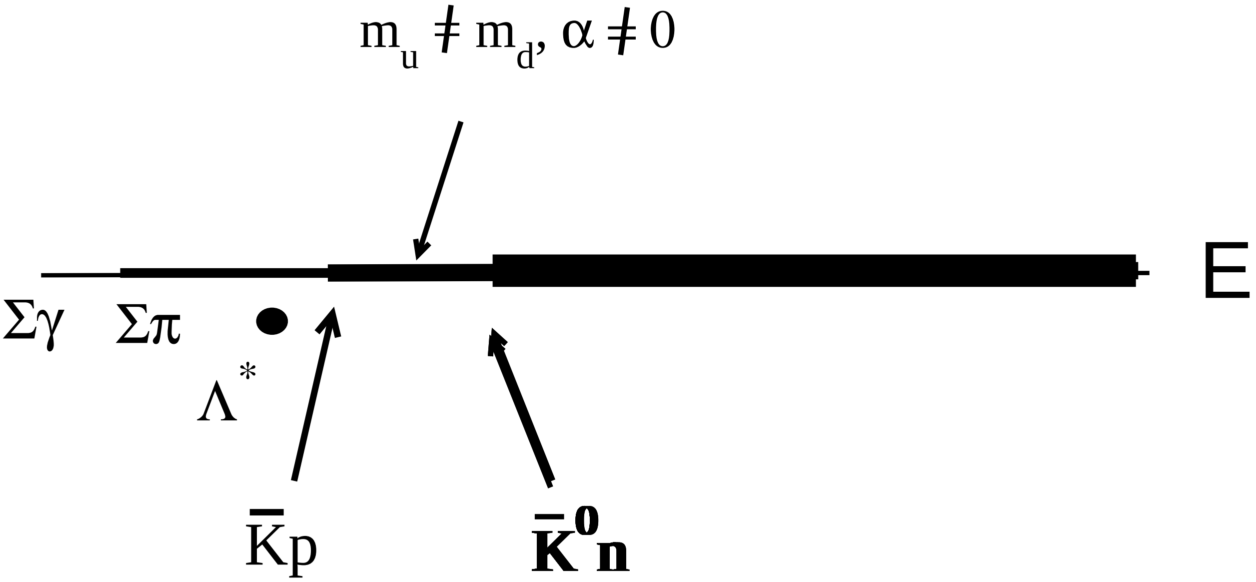
How to proceed?

Consider $\bar{K}p \to \bar{K}p$ in the real world:

amplitude T^{P}

Complicated object. Analytic properties:

Physical amplitude T^P



$m_u \neq m_d, \alpha \neq 0$

E

$\Sigma\gamma$ $\Sigma\pi$

Λ^*

$\bar{K}p$

$\mathbf{\bar{K}^0 n}$

T^P occurs in the formula for the energy shift and width:

1^{st} step: $\epsilon_{1s} \leftrightarrow T^P$

2^{nd} step: $T^P \leftrightarrow T^{QCD}$

These 2 steps have been performed recently by Meißner, Rusetsky and Raha [1]. Very clear presentation, all effects at NL order identified.

See also [2]

Main observation: Large isospin breaking effects (as large as uncertainty in present DEAR data) [3]

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[1] Meißner, Rusetsky and Raha 2004

[2] A. Ivanov et al. 2003

[3] Dalitz and Tuan 1960, Deloff and Law 1979

Result for strong shift [1,2]

$$\begin{aligned} \triangle E_n^s - \frac{i}{2}\Gamma_n &= -2\alpha^3\mu_c^2\,\mathsf{T}^{\mathsf{QCD}}\,(1+X)\,, \\ X &= \ldots \end{aligned}$$

This is step $\boxed{1}$ on p. 7

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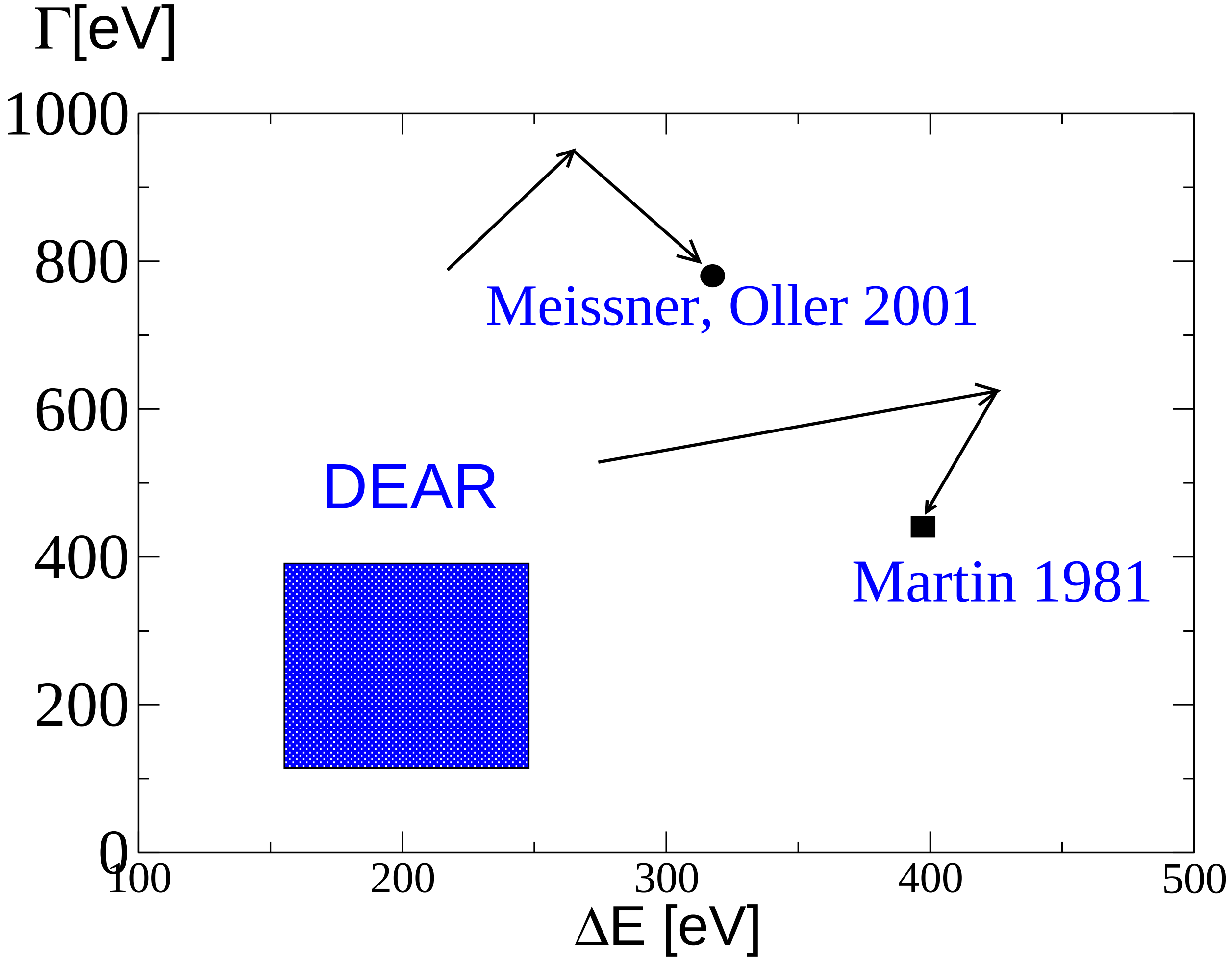
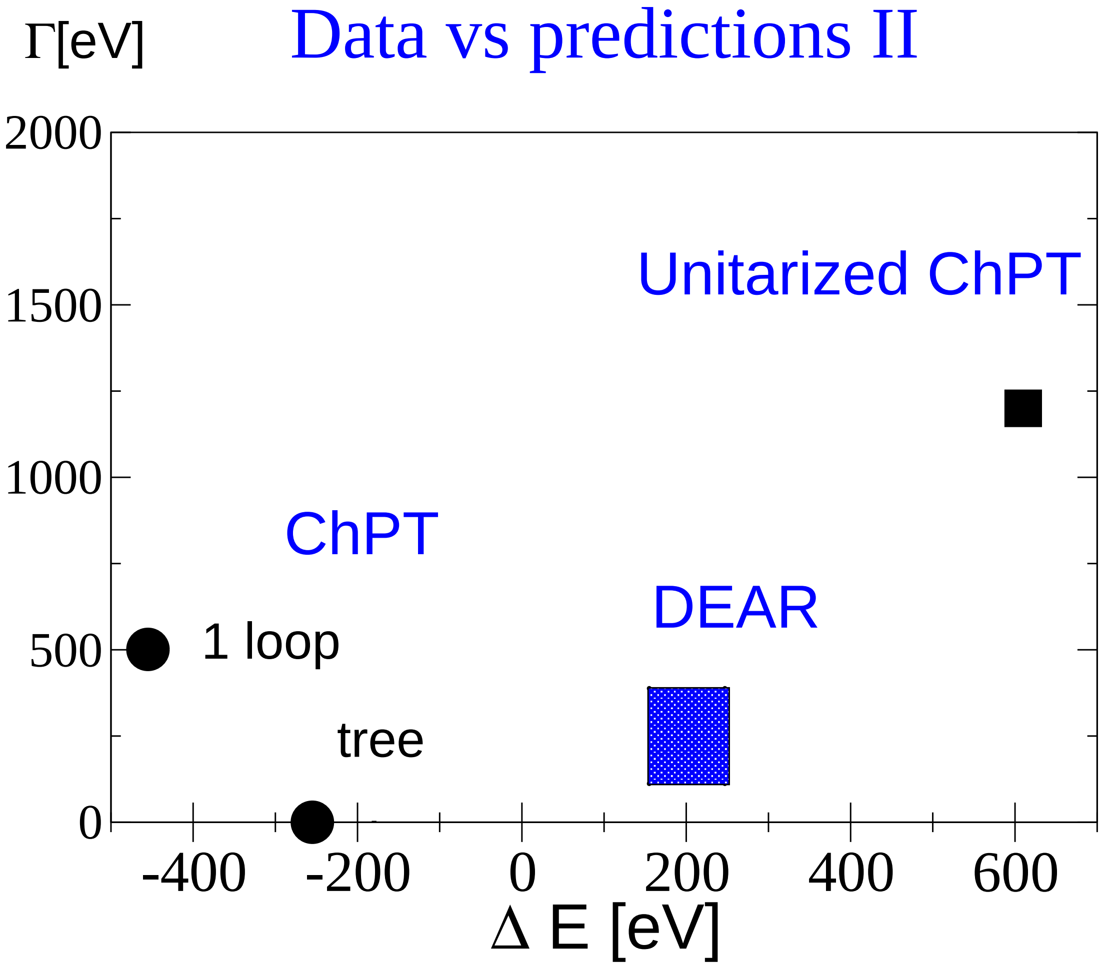
[1] Meißner, Rusetsky and Raha 2004

[2] A. Ivanov et al. 2003

3. Comparison with data (step 2)



Data vs predictions II



ChPT: Kaiser 2001

Reason for failure: Λ^*

Unitarized ChPT: Oset, Ramos 1998

Reason for failure: cutoff introduced violates chiral symmetry [1]

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[1] Meißner, Rusetsky and Raha 2004

4.Conclusion

ChPT needs improvement. Pioneered in Ref. [1]

Last word not yet available: e.g., no error bars on theoretical calculation in [2]

If data precise at eV level: It will be even more interesting to confront theoretical predictions with data

Incidentally:

$$a_0, a_1 \Leftrightarrow \sigma_{KN}$$

???

terrific entreprise (theoretically)

See more on this in the Proceedings

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[1] Kaiser, Siegel, Weise 1995

[2] Meißner and Oller 2001

4. More complicated systems

Kaonic deuterium

SIDDHARTA see following talks by Iliescu, Jensen

Can provide information on

$\bar{K}p, \bar{K}n$ scattering amplitude at threshold

Needed theoretically: The relevant Deser formula

Compare πd: Irgaziev, Fayzullaev 2004; Ruestsky et al., work in progress

two steps:

$\boxed{1}$ Scattering on deuterium as elementary particle

$\boxed{2}$ Relate deuterium scattering lengths to $\pi p, \pi n$ scattering lengths

$\bar{K}d$ considerably more complicated, but doable

5. Conclusions

1. πK atom DIRAC

   clean case, atom fully understood

   connection with

   - ChPT
   - vacuum properties of QCD

   Theory of $K\pi$ scattering not yet fully worked out [1]. Data lacking

2. Kaonic hydrogen DEAR

   $$\boxed{\Lambda^*}$$

   data available, atom understood. Error analysis in calculation of scattering lengths is missing

   More precise data will reveal whether theory is able to describe the complex situation properly

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[1] Bijnens and Talavera 2004

3. Kaonic deuterium SIDDHARTA
   Data and calculation missing

How long does $\bar{K}$d live?

How many orbits before decaying?

Can one extract $\bar{K}p$, $\bar{K}n$ scattering lengths?