Rare Decays as Window to New Physics

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DAPHNE 04

1. Ones and Zeros of the Standard Model
2. Rare K- Decays
3. Miscellaneous Comments

1. Ones and Zeros of SM

- Most important numbers of a theory are its "ones" and "zeros", i.e. its intensity rules and selection rules.

- In the case of the SM, these "ones" and "zeros" follow from the symmetry properties of the theory, and are most succinctly expressed as properties of the CKM matrix:

$$J_\mu^{cc} = \overline{\left(\bar{u}\ \bar{c}\ \bar{t}\right)}\ \gamma_\mu \frac{1-\gamma_5}{2} V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V V^\dagger = V^\dagger V = \mathbb{1}$$

$$\rightarrow\ V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Physical Meaning of 'One':

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

is an expression of Lepton-Hadron Universality (generalization of the old idea of Cabibbo and Gell-Mann)

Present Status:

$|V_{ud}| = 0.9740 \pm 0.0005$ (superallowed pure Fermi decays)

$$|V_{us}| = \begin{cases} 0.2250 \pm 0.0027 & \text{(hyperon decays)} \\ 0.2196 \pm .0023 & \text{(K-decay)} \end{cases}$$

$|V_{ub}| = 0.0036 \pm 0.0007$

Deviation from universality:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta$$

$\Delta = 0.0007 \pm 0.0014$ [Hyperon]

$ 0.0032 \pm 0.0014$ [Kaon]

c.f. Cabibbo, Swallow, Winston
hep-ph/0307214

- Interpretation of Zeros:

A. $V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0,$

$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0,$

etc.

6 Unitarity Triangles (CP Violation)

Can have diverse shapes:

UT for $s \to d$

UT for $b \to d$

$(\lambda \approx 0.2)$

Unexplained diversity of V_{ij}

Unexplained structure of V_{ij}:

$$[V_{ij}] \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Properties of UT's:

(i) Unity in diversity: All UT's have same area (universal measure of CP violation):

$$|J_{CP}| = 2A_\Delta = \lambda\left(1-\frac{\lambda^2}{2}\right)|\mathrm{Im}\,\lambda_t|$$

$$\lambda_t = V_{ts}V_{td}^*$$

(ii) Unification of CP-conserving and CP-violating observables: Sides of UT determined by moduli $|V_{ij}|$, measurable in CP-conserving processes. Knowledge of sides fixes angles of triangle, which are measures of CP-violation.

N.B. Features (i) and (ii) are specific to world with three generations.

B. Second consequence of "Zeros":
Absence of FCNC (GIM cancellation)

$$\overbrace{\underbrace{d\ s\ b}}\ \gamma_\mu \frac{1-\gamma_5}{2} V^\dagger V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= \bar{d}\gamma_\mu \frac{1-\gamma_5}{2} d + \bar{s}\gamma_\mu \frac{1-\gamma_5}{2} s + \bar{b}\gamma_\mu \frac{1-\gamma_5}{2} b$$

as a consequence of $V^\dagger V = 1$.

C. Symmetries leading to FCNC zeros are broken $\Rightarrow$ FCNC amplitudes not exactly zero.

Source of symmetry breaking:

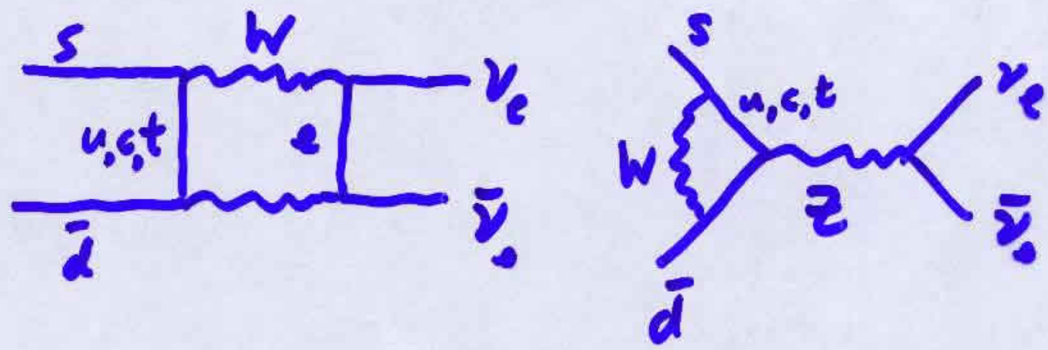
Yukawa couplings of scalar fields to fermions; strength proportional to fermion mass:

$$y_f = \sqrt{2}\, m_f / v$$

E.g. $\underbrace{y_e = 2.9 \times 10^{-6}}_{\approx 0 \text{ (Unexplained)}}$, $\underbrace{y_t \approx 1.006}_{\text{Unexplained '1'}}$

D. Effective Hamiltonian for FCNC Processes:

Derived from Box and Penguin diagrams. Example:



Typical FCNC amplitude has the structure

$$A_{FCNC} = G_F \, [0] + G_F \, \alpha \, f(m_f, V_{ij})$$

GIM Zero

sensitive to fermion mass spectrum, hence to Yukawa couplings.

Thus: study of FCNC decays tests the Yukawa (chiral symmetry breaking) sector of the standard model.

2. Rare K Decays

2.1 Golden Modes: $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$

- $$H_{eff}^{SM} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \left[V_{cs}^* V_{cd} X_{NL} + V_{ts}^* V_{td} X(x_t) \right]$$

$$x_t = m_t^2 / m_W^2$$

$$X(x_t) = \frac{x_t}{8} \left[-\frac{2+x_t}{1-x_t} + \frac{3x_t - 6}{(1-x_t)^2} \ln x_t \right]$$

X_{NL} : contribution of charm quarks

[Inami-Lim ; Buras, Buchalla ....]

- Theoretical Significance

$$X(x_t) \sim a + b x_t$$

a : represents gauge interaction

$$H_{eff} \sim G_F \frac{\alpha}{2\pi \sin^2\theta_W} a$$

$b x_t$: represents Yukawa interaction

$$H_{eff} \sim G_F \underbrace{\frac{\alpha}{2\pi \sin^2\theta_W} m_t^2 / m_W^2}_{= \frac{1}{4\pi^2} y_t^2 \,!}$$ (indep. of α)

- Branching Ratios can be accurately predicted (long-distance effects negligible), in terms of ρ, η (c.f. Buras, Schwab, Uhlig, hep-ph/0405132)

Estimated accuracy :

$\sim 10\%$ for $K^+ \to \pi^+ \nu \bar{\nu}$

$\sim 5\%$ for $K_L \to \pi^0 \nu \bar{\nu}$

Consistency Check of SM

$$(\sin 2\beta)_{\pi\nu\nu} = (\sin 2\beta)_{\psi K_S}$$

Current Estimate

$$Br(K^+ \to \pi^+ \nu \bar{\nu}) = (7.8 \pm 1.2) \cdot 10^{-11}$$

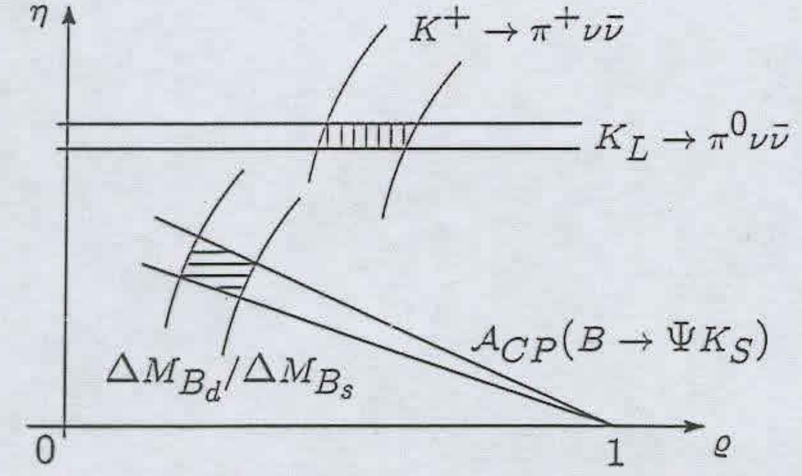
Data: $(14.7 {}^{+13}_{-8.9}) \cdot 10^{-11}$

E949 + E787

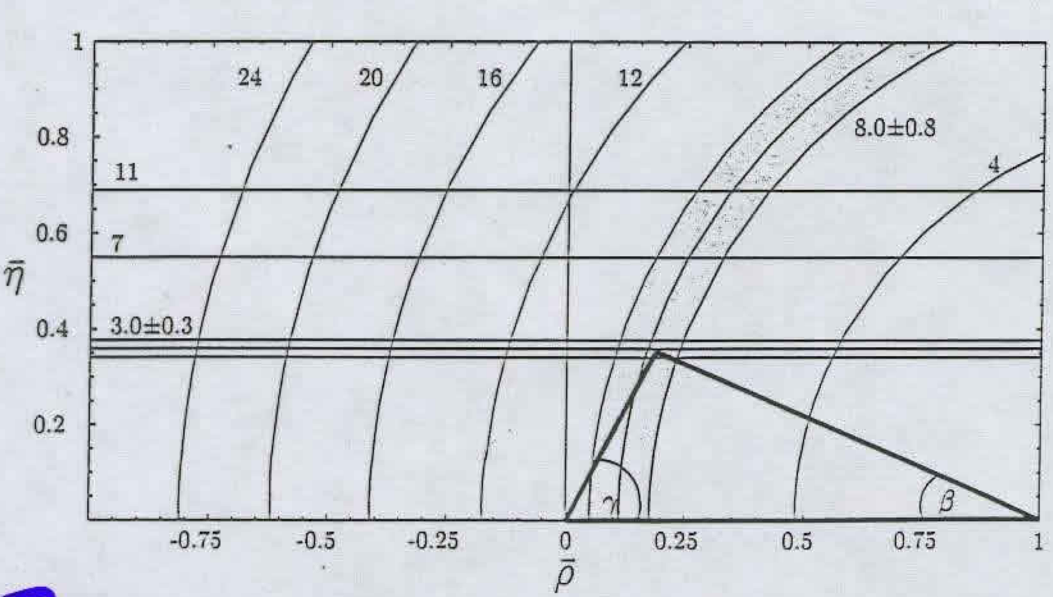
$$Br(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.6) \cdot 10^{-11}$$

Conclude : $K \to \pi \nu \bar{\nu}$ has high priority, both as precision test of (ρ, η), and as probe of New Physics.

Future Scenarios



I. Deviation from SM

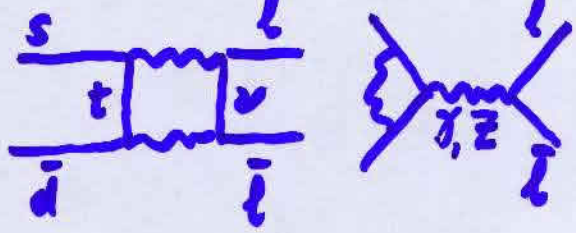


II. Precision Test of SM

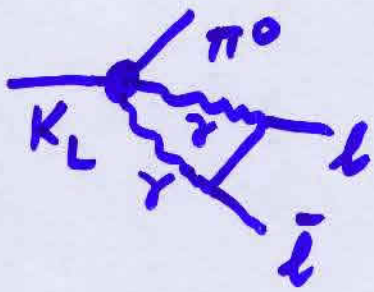
2.2. Decay Modes $K_L \to \pi^0 \ell^+ \ell^-$

Amplitude has three components

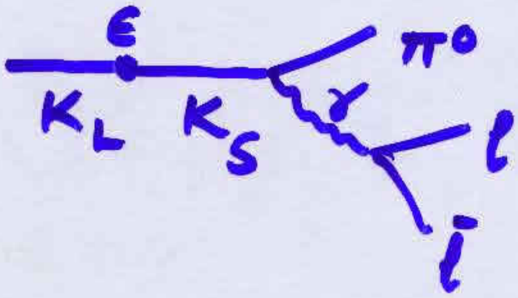
$A = A_{\substack{\text{short}\\ \text{dist}}}$



$+ \; A_{2\gamma}$



$+ \; A_{\text{indirect}}$



$$\therefore \; A = \underbrace{\alpha^2 A_{2\gamma}}_{\text{CP-cons.}} + \underbrace{\alpha \, \varepsilon \, A_{1\gamma}}_{\text{Indirect } \not{CP}} + \underbrace{\eta \lambda^4 A_{sd}}_{\text{Direct } \not{CP}}$$

Can be estimated from data on $K_L \to \pi^0 \gamma\gamma$ (rate and spectrum)

(NA48, KTeV)

determined (up to sign) by recent measurement of $K_S \to \pi^0 e^+ e^-$ & $K_S \to \pi^0 \mu^+ \mu^-$

NA 48/1

calculable from short-dist. Hamitonian

Dib, Dunietz, Gilman; Buchalla .....

Recent analysis:

Buchalla, D'Ambrosio, Isidori (hep-ph/0308008)

Isidori, C. Smith, Unterdorfer (hep-ph/0404127)

Essential Results:

- $$Br(K_L \to \pi^0 \ell^+ \ell^-) = \left[C^\ell_{ind} \pm C^\ell_{interf} \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right) + C^\ell_{dir} \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right)^2 + C^\ell_{CPC} \right]$$

- Constructive Interf. preferred (Buchalla et al, Isidori et al, Friot et al)

- 2γ contrib. small for e^+e^-, significant for $\mu^+\mu^-$ (Heiliger & LMS)

- SM prediction for branching ratio:

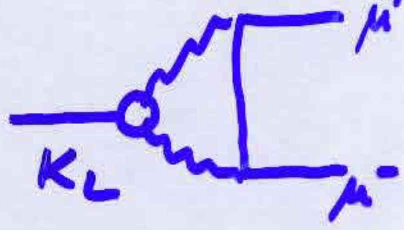
$$Br(K_L \to \pi^0 e^+ e^-) = (3.7 \pm 1.0) \cdot 10^{-11}$$

$$Br(K_L \to \pi^0 \mu^+ \mu^-) = (1.5 \pm 0.3) \cdot 10^{-11}$$

- e : μ ratio could be affected by New Physics [Example: modification of electroweak penguin (Buras et al hep-ph/0402112)]

2.3 Decay $K_L \to \mu^+\mu^-$

- Unitarity Bound
associated with 2γ intermediate state:



$$R^K = \frac{\Gamma(K_L \to \mu\mu)}{\Gamma(K_L \to \gamma\gamma)} \geq \frac{\alpha^2}{2\beta} \frac{m_\mu^2}{m_K^2} \left(\ln \frac{1+\beta}{1-\beta} \right)^2$$

$$\beta = \sqrt{1 - 4 m_\mu^2 / m_K^2}$$

- Measured Ratio

$$R^K_{exp} = (1.238 \pm 0.024) . 10^{-5}$$

is only 4% above the unitarity bound!

- $$Br(K_L \to \mu\bar{\mu}) - Br(K_L \to \mu\mu)\big|_{unitarity} = \left| A_{disp}(2\gamma) + A_{short\text{-}dist.} \right|^2$$

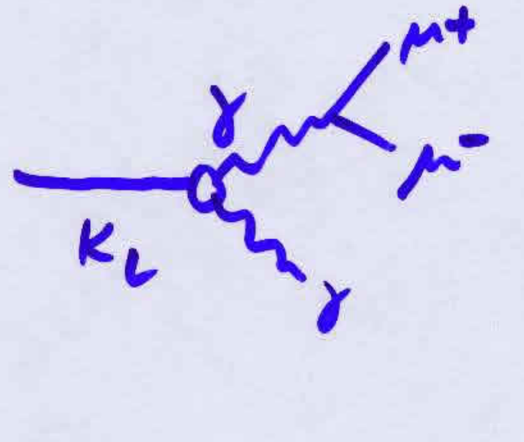
- A_{disp} requires knowledge of $K_L \to \gamma^* \gamma^*$ form factor. [Isidori, Unterdorfer, hep-ph/0311084]
- Alternatively: compare $R^{(K)}$ with $R^{(\eta)} = (1.45 \pm 0.20) . 10^{-5}$

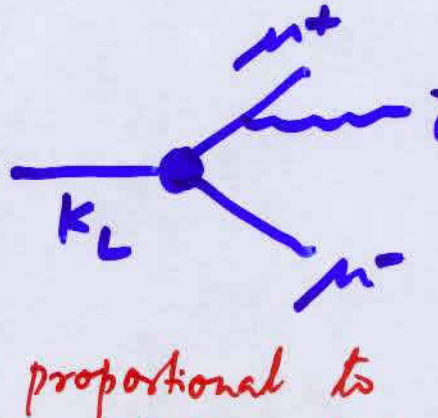
- Conclusion :

$$-0.5 < \rho < 2.1$$

- Direct Probe of Real Part of $K_L \to \mu^+\mu^-$ Amplitude :

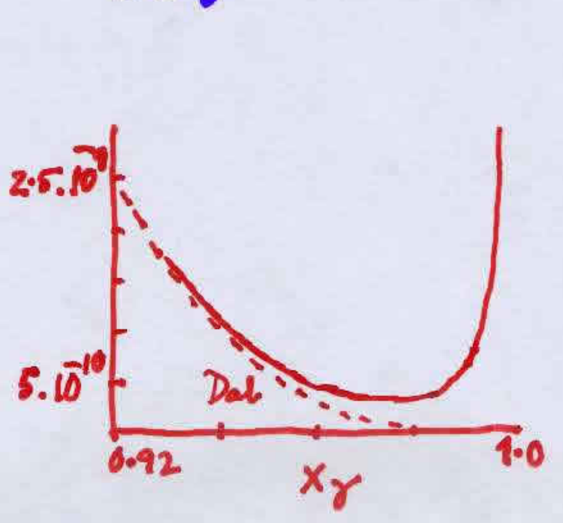
Study $K_L \to \mu^+\mu^-\gamma$ at large $\mu^+\mu^-$ mass.
Interference of Conversion and Bremsstrahlung amplitudes :





proportional to
$A(K_L \to \mu^+\mu^-)$

$$\frac{d\Gamma}{dx_\gamma} = \text{Dalitz} + \text{Interf} + \text{Bremsst.}$$

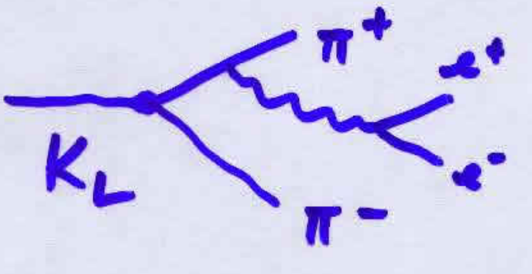


Position and height of minimum depends on interf. term; can probe sign of Re $A_{\mu\mu}$

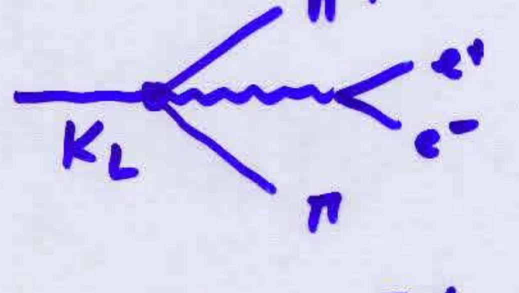
[Poulose and Sehgal].

2.4. Decay $K_L \to \pi^+\pi^- e^+e^-$

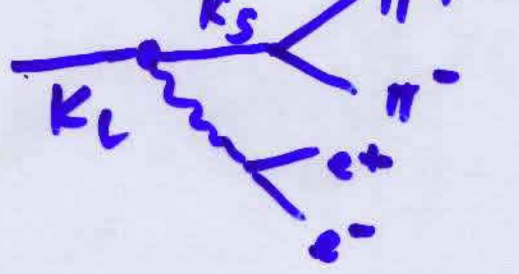
Amplitude has three components



Bremsstrahlung ($\not{CP}$)
$\propto \epsilon$



Direct M1 (CPC)



$(\pi\pi)_{S\text{-wave}} \sim K^0$ Charge Radius

Theoretical Prediction

$$\frac{d\Gamma}{d\phi} = \Gamma_1 \cos^2\phi + \Gamma_2 \sin^2\phi + \Gamma_3 \underbrace{\sin\phi \cdot \cos\phi}_{\not{CP},\ T\text{-odd}}$$

$\phi = \measuredangle$ angle between $\pi^+\pi^-$ and e^+e^- planes

Asymmetry $A_\phi = \dfrac{\int_{I+III} d\Gamma/d\phi - \int_{II+IV} d\Gamma/d\phi}{\int_{I+II+III+IV} d\Gamma/d\phi}$

Predicted Result (Sehgal & Wanninger '92, Heiliger & Sehgal '93)

$$A_\phi = 15\% \sin(\phi_{+-} + \delta_0 - \delta_1)$$

$$\approx 14\%$$

$$\langle R^2 \rangle_{K^0} = \frac{1}{2}\left[\frac{1}{M_\phi^2} - \frac{1}{M_\rho^2}\right] = -0.07\ fm^2$$

(VMD)

Data

$$A_\phi = 13.7 \pm 1.4 \pm 1.5\% \quad (KTeV)$$

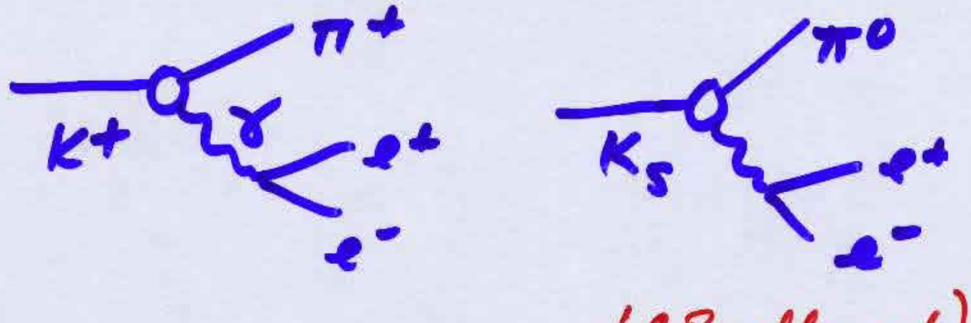
$$14.2 \pm 3.6\% \quad (NA48)$$

$$\langle R^2 \rangle_{K^0} = -0.077 \pm 0.014\ fm^2 \quad (KTeV)$$

$$-0.09 \pm 0.02 \quad (NA48)$$

2.5. Decays $K^+ \to \pi^+ e^+ e^-$ and $K_S \to \pi^0 e^+ e^-$

Examples of K-decays mediated by a single photon



(CP allowed)

Probe the effective interaction

$$\mathcal{L}_{eff}\,(\pi, K, \gamma)$$

Matrix Element:

$$A(K^+ \to \pi^0 e^+ \nu) = \frac{G_F}{\sqrt{2}} \frac{f_+}{\sqrt{2}} \sin\theta_c (k+p)_\alpha \bar{\nu} \gamma^\alpha (1+\gamma_5) e$$

$$A(K^+ \to \pi^+ e^+ e^-) = a_+ \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} f_+ \sin\theta_c (k+p)_\alpha \bar{e} \gamma^\alpha e$$

$$A(K_S \to \pi^0 e^+ e^-) = a_S \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} f_+ \sin\theta_c (k+p)_\alpha \bar{e} \gamma^\alpha e$$

Vainshtein et al (1976):

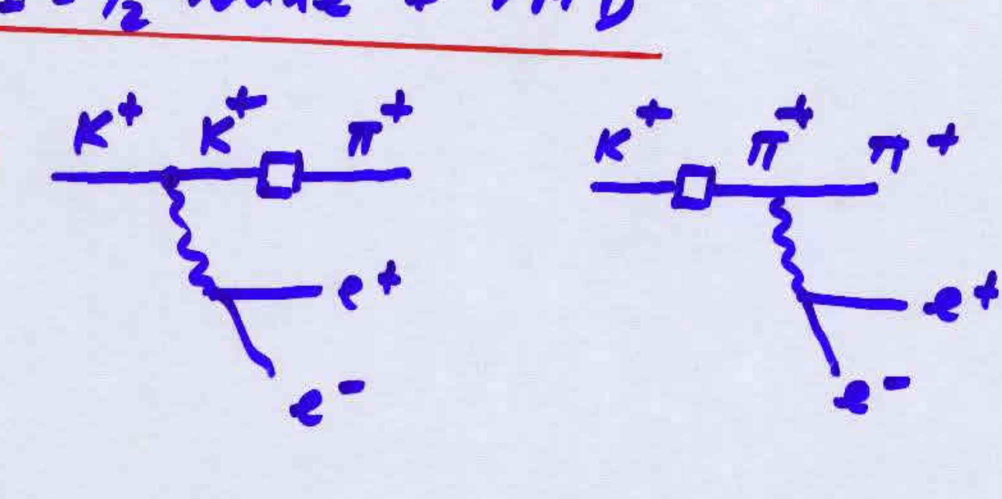
$$a_+ = -0.7\,, \quad a_S = 2.4$$

Chiral Pert. theory: $a_+/a_S < 0$

Phenomenological Model (Burkhardt et al hep-ph/0011345)

$\Delta I = 1/2$ rule + VMD

(a)



$$\mathcal{M}(K^+ \to \pi^+ e^+ e^-) = A(q^2)\,(p_K + p_\pi)^\mu\, \bar{e}\gamma_\mu e$$

$$A = e^2 \frac{\langle \pi^+ | H_w | K^+ \rangle}{m_{K^+}^2 - m_{\pi^+}^2} \left[\frac{F_\pi(q^2) - F_K(q^2)}{q^2} \right]$$

$$|A^{(0)}| \approx e^2 \left| \frac{\langle \pi^+ | H_w | K^+ \rangle}{m_K^2 - m_\pi^2} \right| \cdot \frac{1}{3} \left(\frac{1}{m_\rho^2} - \frac{1}{m_\phi^2} \right)$$

Comparison with measured branching ratio [E865 collaboration]

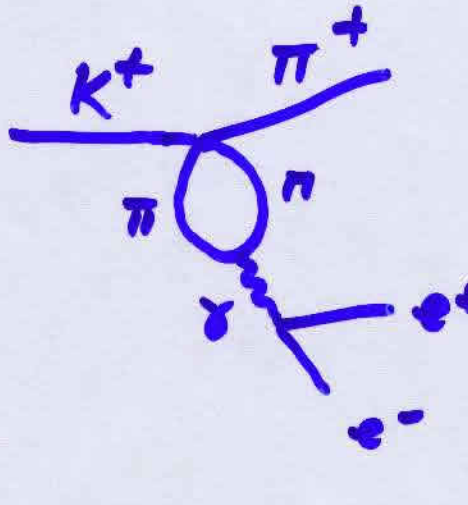
$$Br(K^+ \to \pi^+ e^+ e^-) = (2.99 \pm 0.06) . 10^{-7}$$

$$\Rightarrow |A(0)| = (4.0 \pm 0.2)\, 10^{-9}\ \mathrm{GeV}^{-2}$$

in good agreement with CA-PCAC estimate of $\langle \pi^+ | H_w | K^+ \rangle$, which yields

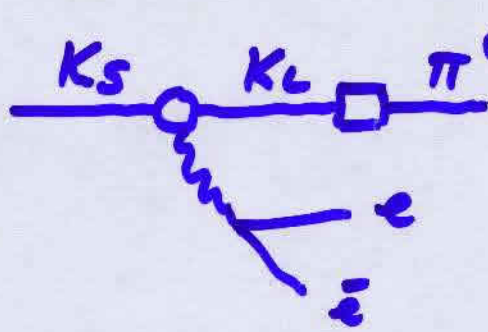
$$|A(0)|_{\text{theory}} = 3.9 \times 10^{-9}\ \mathrm{GeV}^{-2}$$

Form Factor $A(q^2)$ in VMD model is <u>slower</u> than observed. Possible resolution: <u>Pion loop correction</u> given by ChPT:



(b) <u>Similar Model for $K_S \to \pi^0 e^+ e^-$</u>

(Sehgal, 1970)



$$\mathcal{M}(K_S \to \pi^0 e^+ e^-) = \frac{1}{3} e^2 \langle R^2 \rangle_{K^0} \frac{\langle \pi^0 | H_w | K_L \rangle}{m_\pi^2 - m_K^2} \bar{u} \not{p}_\pi v$$

$$\text{CA-PCAC} \Rightarrow \langle \pi^0 | H_w | K_L \rangle = - \langle \pi^+ | H_w | K^+ \rangle$$

$$\Rightarrow Br(K_S \to \pi^0 e^+ e^-) = 5.5 \times 10^{-9} \quad \text{(model)}$$

NA48 : $(5.8 {}^{+2.8}_{-2.3} \pm 0.8) \cdot 10^{-9}$ (expt.)

Note: Model predicts $a_S / a_+ < 0 \Rightarrow$ Constructive Interf. in $K_L \to \pi^0 e^+ e^-$

3. Miscellaneous Remarks

3.1 Standard Model has two types of couplings:

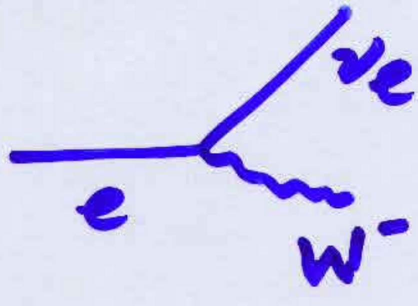
$\{g, g'\}$ gauge couplings (chirality-conserving)

$\{y_f\}$ Yukawa couplings (proportional to m_f; violate chirality)

Question I: What happens to the electron when its gauge couplings are switched off (keeping $v = (\sqrt{2}\, G_F)^{-1/2}$ fixed?)
This is the "gaugeless" limit studied by Bjorken.
In this limit, fermions interact purely via scalar (Yukawa) couplings.

Remarkable Consequence: Electron is unstable!

$$\Gamma(e^- \to \nu_e W^-) = \frac{\sqrt{2}\, G_F\, m_e^3}{16\pi}$$



$$= \overset{\to\, y_e^2}{\left(\frac{m_e}{v}\right)^2} m_e \Big/ 16\pi$$

$$= (10.3\ \text{ns})^{-1}$$

[For fixed v (fixed G_F) does not decouple from longitudinal massless W^-: this is just the Goldstone boson s^-:

$$e^- \to \nu + s^-\ ]$$

Question II : What happens when the Yukawa coupling of the electron is switched off ($y_e \to 0$ or $m_e \to 0$, v fixed)? Is chirality of electron conserved ?

Answer : No !

Example 1 : QED with $m_e \to 0$

- Compton scattering with helicity-flip :

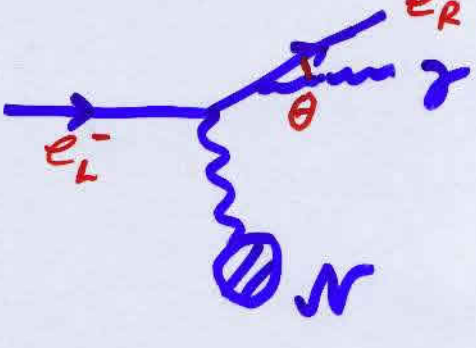
$$\gamma + e_L^- \to \gamma + e_R^-$$

$$\lim_{m \to 0} \sigma_{hf}(s) = 2\pi \frac{\alpha^2}{s} \quad (\neq 0)$$

(see, for example, Peskin & Schroeder)

- Consider bremsstrahlung with helicity flip

$$e^-_L + N \to e^-_R + \gamma + N$$



$$d\sigma_{hf} \sim \alpha \left(\frac{m}{E}\right)^2 \frac{d\theta^2}{\left(\theta^2 + \frac{m^2}{E^2}\right)^2}$$

$$\Rightarrow \sigma_{hf} \xrightarrow[m \to 0]{} \alpha \,(\text{finite}) \qquad (\neq 0)$$

(Lee and Nauenberg)

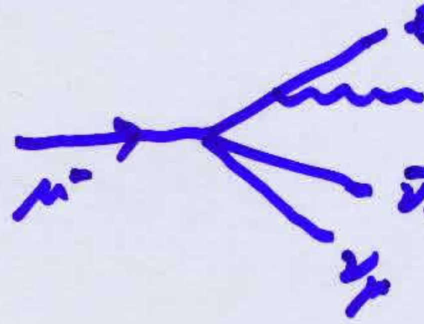
- Equivalent Particle Formula for Helicity-Flip : $e_L \to e_R$ $\gamma(z)$

$$D_{hf}(z) = \frac{\alpha}{2\pi} z \qquad \text{(Falk and Sehgal)}$$

N.B. This is the probability (fragm. function) for an electron to emit a photon of fractional mom. z, in a process where electron helicity is flipped. (Universal function, valid for $m_e \to 0$.)

Example 2 : Muon Decay, $m_e \to 0$

Consider radiative μ-decay



V-A theory

[Sehgal hep-ph/0306166, Schulz and Sehgal hep-ph/0404023]

Q : What is the helicity of the electron in the limit $m_e \to 0$?

Conventional Wisdom : $P_{long} = -1$.

True Answer :

Electron has significant probability of being right-handed !

Polarization P_{long} deviates from -1, depending on energy of photon.

Contribution of right-handed electrons to muon decay width is

$$\Gamma_R = \frac{\alpha}{4\pi}\left(\frac{G_F^2 m_\mu^5}{192\pi^3}\right) \; !$$

Radiative Muon Decay

$\mu^- \to e^-_{L,R}\,\bar{\nu}_e\,\nu_\mu\,\gamma$

Photon Energy Spectrum

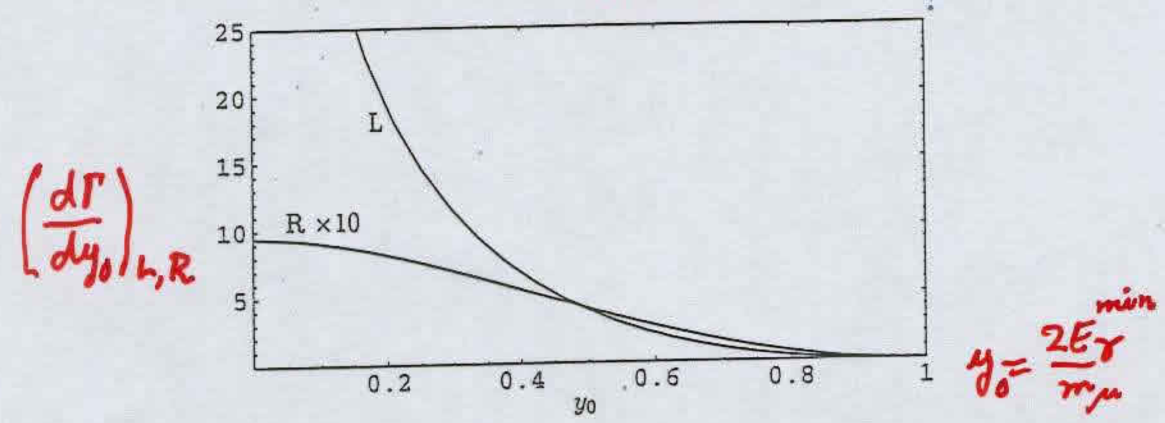


Figure 4: Integrated rates for R- and L-electrons $\Gamma_{R,L}(y_0)$ in units of $\Gamma_0 \frac{\alpha}{4\pi}$ as function of minimum photon energy y_0.

Electron Polarization

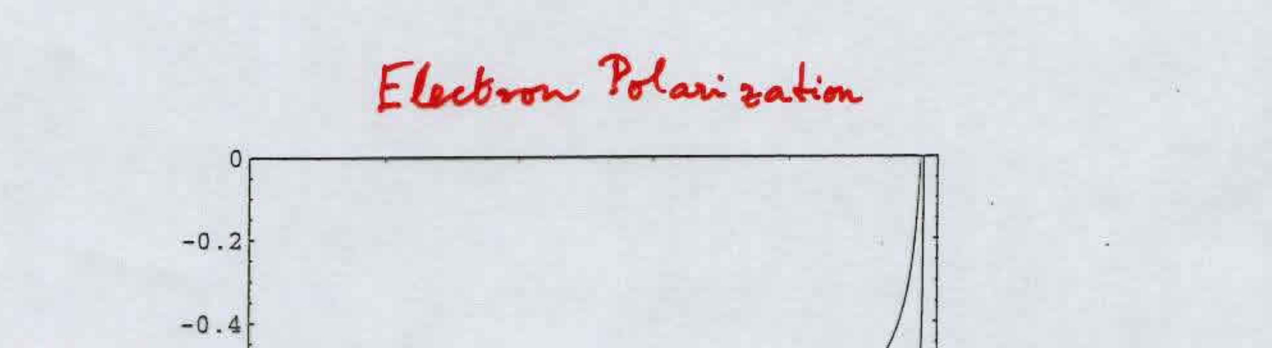


Figure 5: Longitudinal polarization of electron P_L as function of minimum photon energy y_0: a) $\mu \to e$ decay, b) $\tau \to e$ decay.

$\theta_{e\gamma}$ distribution

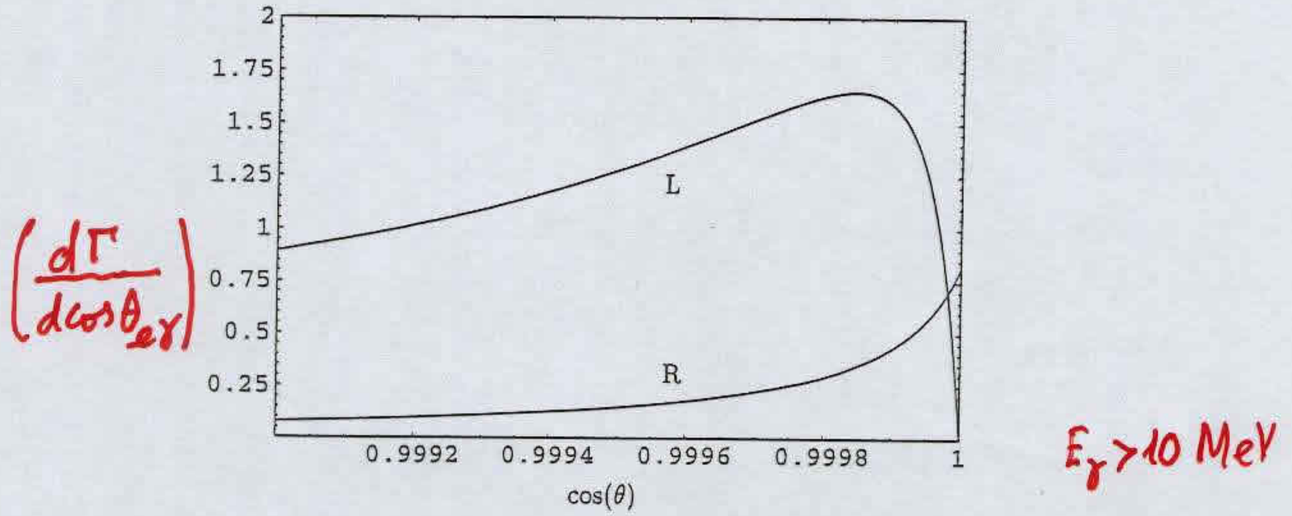
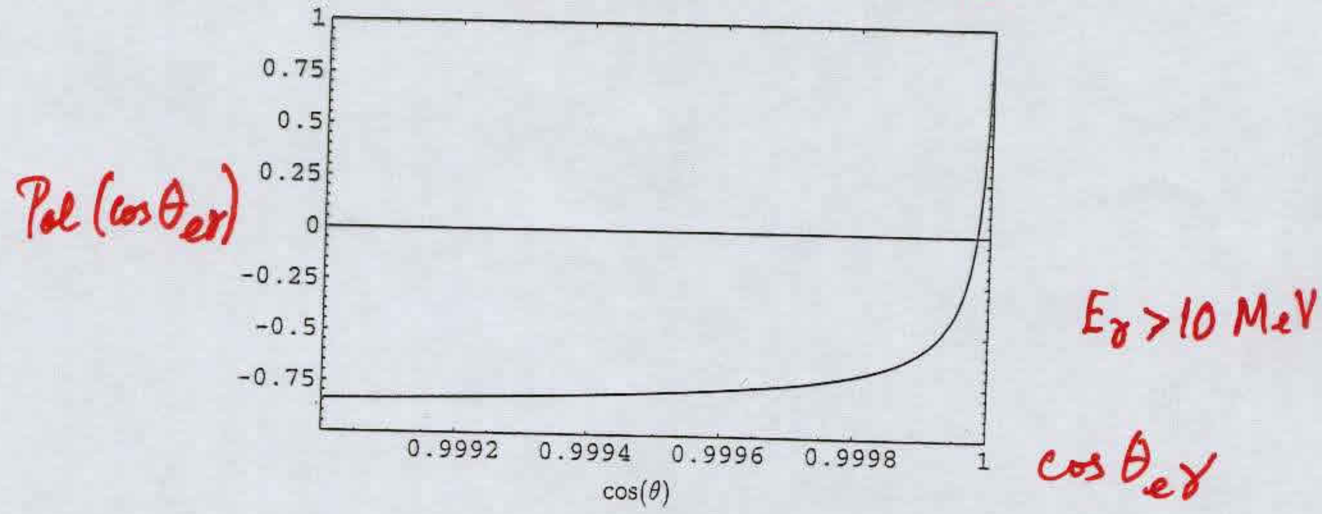


Figure 6: $\cos(\theta)$ spectrum $\left(\frac{d\Gamma}{d\cos(\theta)}\right)_{L,R}$ for $\lambda = 1/207$ and $y_0 = 0.189$ in units of Γ_0.

Electron Polarization vs. $\theta_{e\gamma}$



3.2. Final Comments

(i) We are still trying to understand the Standard Model.

(ii) Scalar sector essential for symmetry-breaking, responsible for fermion masses and CKM parameters: not yet directly observed.

(iii) Scalar fields are present in loop diagrams responsible for rare K- and B-decays.

(iv) Attempts to modify or extend the SM usually disturb its ONES and ZEROS.

(v) Pursuit of FCNC, CP-violation, as well as search for exotic processes ($K_L \to \mu e$ etc) is of the utmost importance.