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CHIRAL PERTURBATION THEORY CONFRONTED WITH EXPERIMENT

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Abstract

The general framework and the present status of the low energy theory of the standard model are briefly reviewed. Recent applications to a few topics of interest for the determinations of $|V_{ud}|$ and of $|V_{us}|$ are discussed.

1 Low energy theory of the standard model

At low energies, the standard model can be described in terms of an effective theory, involving only the lightest states as explicit degrees of freedom. In order that such an effective description becomes possible, two requirements need to be met. First, one must have a clear separation of scales (mass gap) between, on the one side, the light states, and, on the other side, the heavy states, which appear only indirectly in the effective theory, through their contribution to the infinite number of couplings, the low energy constants (LECs) describing the local interactions of the light states. The second requirement is that the masses of the light degrees of freedom are protected by some symmetry, in order that their lightness appears as natural, in the very precise sense defined by 't Hooft [1] some time ago. In practice, this means that light spin 0 states have to correspond to Goldstone bosons produced by the spontaneous breaking of some continuous global symmetry. The masses of light fermion will be protected by chiral symmetry, whereas gauge invariance will ensure that spin 1 gauge fields remain massless (or massive but light, in t he presence of a Higgs mechanism).

In the case of the standard model, the light degrees of freedom that can be identified in this way comprise: i) the pseudoscalar meson octet, π, K and η, which, in the limit of massless quarks, become the Goldstone bosons associated with the spontaneous breaking of the chiral symmetry of QCD, ii) the light leptons, $e^{\pm}$, $\mu^{\pm}$ and their neutrinos (in principle, one might add the τ neutrino to this list, although the τ lepton itself belongs to the heavy states in the context of the present discussion), and iii) the photon. The range of applicability of this effective theory is limited by the typical mass scale $\Lambda_H \sim 1$ GeV provided by the non Goldstone mesonic bound states. Notice that according to the criteria adopted above, other effective theories could be considered, for instance the one involving only the electron, the three neutrinos, and the photon, with the limiting mass scale set by $m_\mu \sim M_\pi$, etc.

Chiral perturbation theory [2, 3, 4] (ChPT) organizes the low energy effective theory in a systematic expansion in powers of momenta and of light masses. The most convenient tool to materialize this expansion is to construct

Table 1: *The low energy constants corresponding to some of the parts of $\mathcal{L}_{\mathrm{eff}}$ that have been constructed. They allow for a description of meson scattering amplitude and meson form factors up to two loops, anf for the inclusion of $\mathcal{O}(\alpha)$ radiative corrections up to one loop.*

	2 flavours	3 flavours
$\mathcal{O}(p^2)$	F, B	F_0, B_0
$\mathcal{O}(p^4)$	h_1,h_2, h_3, l_i, $i=1\dots7$ [3]	H_1,H_2, L_i, $i=1\dots10$ [4]
$\mathcal{O}(p^6)$	c_i, $i=1\dots57$ [5]	C_i, $i=1\dots94$ [5]
$\mathcal{O}(\alpha p^0)$	Z [6, 7]	Z [6]
$\mathcal{O}(\alpha p^2)$	k_i, $i=1\dots11$ [7]	K_i, $i=1\dots14$ [8], X_i, $i=1\dots8$ [11]

an effective lagrangian $\mathcal{L}_{\rm eff} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots$,where $\mathcal{L}_n$ contains all the terms of order δ^n, with $\delta \sim p/\Lambda_H \sim M_P/\Lambda_H \sim m_\ell/\Lambda_H \sim e$, for instance, modulated by LECs whose values depend on the dynamical properties of the heavy degrees of freedom that have been integrated out. At lowest order, one only needs to compute tree graphs generated by $\mathcal{L}_2$, whereas the NLO involves both tree graphs from $\mathcal{L}_4$ and one loop graphs, and so on. It is essential to include the loop graphs, with increasing number of loops at each new order, in order to correctly account for all the singularities (poles, cuts) coming from the light degrees of freedom. Computing higher orders in the effective theory potentially increases the theoretical precision. However, the number of LECs also increases, as shown in Table 1. Predictions can thus only be made if some knowledge about their values is available. How this problem can be adressed in practice will be illustrated in the case of the few examples discussed below.

2 Radiative corrections to $\pi_{\ell 2}$, $K_{\ell 2}$, and $K_{\ell 3}$ decay modes

As a first application, let us consider the $\mathcal{O}(\alpha)$ electromagnetic contributions to the semileptonic decays of the pion and the kaon. The general structure of the $\pi_{\ell 2}$ and $K_{\ell 2}$ decay rates with radiative corrections included is known [9)]

$$\Gamma_{P_{\ell 2(\gamma)}} = \frac{G_\mu^2}{8\pi} |V_{CKM}|^2 F_P^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 \times \left[1 + \frac{\alpha}{\pi} C_P + \mathcal{O}(\alpha^2)\right] \qquad (1)$$

with $(P, V_{CKM}) = (\pi, V_{ud})$ or (K, V_{us}). ChPT reproduces this structure, with $C_P = C_P^{(0)} + C_P^{(2)} + \ldots$. The expressions [10), 11)] for the $\mathcal{O}(p^0)$ contributions $C_{\pi,K}^{(0)}$ involve a (common) short distance logarithm [9)], chiral logarithms, and the low energy constants K_i and X_i, while $C_{\pi,K}^{(2)}$ and higher represent SU(3) breaking quark mass corrections. Interestingly, the contributions of the low energy constants drop out [11)] in the $\mathcal{O}(\alpha)$ correction to $\Gamma_{K_{\ell 2(\gamma)}}/\Gamma_{\pi_{\ell 2(\gamma)}}$,

$$C_\pi - C_K = \frac{Z}{4} \ln \frac{M_K^2}{M_\pi^2} + \mathcal{O}(M_K^2/\Lambda_H^2) = 0.50 \pm 0.15\,, \qquad (2)$$

with Z given by $M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2 F_\pi^2 Z$, and the error is a conservative estimate for SU(3) breaking corrections. This then leads to

$$\left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{F_K^2}{F_\pi^2} = (7558 \pm 23 \pm 3) \times 10^{-5}\,, \qquad (3)$$

where the first error comes from the experimental uncertainties on the decay rates, and the second error comes from Eq. (2).

Turning now to $K_{\ell 3}$, the general structure of the amplitudes reads

$$\mathcal{M}^{(0)}(K_{\ell 3}) = G_\mu V_{us}^* C_{CG} L^\mu \big[f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu \big] \, . \quad (4)$$

For the K_{e3} modes, only $f_+(t)$ needs to be considered, whereas for the $K_{\mu 3}$ modes $f_-(t)$ has to be included as well. The chiral expansions of these form factors read $f_+ = 1 + f_+^{(2)} + f_+^{(4)} + \ldots$ and $f_- = f_-^{(2)} + f_-^{(4)} + \ldots$ The one loop corrections $f_\pm^{(2)}(t)$ arising from mesonic intermediate states, including isospin breaking effects induced by $m_u \neq m_d$, are known [12, 13] for quite some time. Including $\mathcal{O}(\alpha)$ radiative corrections [14, 15] amounts to replacing $f_\pm(t)$ by

$$F_\pm(t,v) = \big[1 + \frac{\alpha}{\pi}\Gamma(v, m_\gamma)\big] \times \big(\widetilde{f}_\pm(t) + \widehat{f}_\pm(t)\big) \, . \quad (5)$$

In this expression, $\widetilde{f}_\pm(t)$ contains corrections from the loops and from $\pi^0 - \eta$ mixing, while $\widehat{f}_\pm(t)$ collects the remaining counterterm contributions. Finally, $\Gamma(v, m_\gamma)$, with $v = (p_K - p_\pi)^2$ for $K_{\ell 3}^\pm$, and $v = (p_K - p_\pi)^2$ for $K_{\ell 3}^0$, contains the long distance components of the loops with a virtual photon. The IR divergence, materialized by the dependence on the photon mass m_γ, is cancelled upon considering the differential rates with the emission of a real soft photon. Corrections at order $\mathcal{O}(\alpha p^2)$ were computed [14, 15] and the corresponding numerical estimates read

$$\widetilde{f}_\pm(0) = 1.0002 \pm 0.0022 \, , \; \widehat{f}_\pm(0) = 0.0032 \pm 0.0016 \quad [K^\pm] \quad (6)$$

$$\widetilde{f}_\pm(0) = 0.097699 \pm 0.00002 \, , \; \widehat{f}_\pm(0) = 0.0046 \pm 0.0008 \quad [K^0] \quad (7)$$

The expressions of the two loop corrections $f_\pm^{(4)}(t)$ were worked out [16] in the isospin limit, and will be discussed below.

3 The pion beta decay $\pi^+ \to \pi^0 e^+ \nu_e$ and $|V_{ud}|$

The beta decay of the charged pion ($\pi\beta$) in principle provides a determination of $|V_{ud}|$ which combines the advantages of the superallowed nuclear Fermi transitions (pure vector transition, no axial vector admixture), and of the neutron beta decay (no nuclear structure dependent radiative corrections). There is however a serious drawback, the tiny branching ratio, $\mathrm{Br}(\pi\beta) \sim 1 \times 10^{-8}$. In the absence of radiative corrections, the amplitude has the structure given in Eq. (4), with V_{us} replaced by V_{ud}, and $f_\pm(t)$ replaced by $f_\pm^{\pi\beta}(t)$. Contribution from $f_-^{\pi\beta}(t)$ are suppressed by m_e^2/M_π^2 and can be neglected. Furthermore, $f_+^{\pi\beta}(t) = 1 + f_{\pi\beta}^{(2)}(t) + \ldots$, where the one loop corrections [17] to the CVC result are small, $f_{\pi\beta}^{(2)}(0) = -7 \times 10^{-6}$. As a consequence, higher order corrections, $f_{\pi\beta}^{(4)}(0)$, etc., can be safely neglected. On the other hand, radiative corrections then become relevant. Including $\mathcal{O}(\alpha p^2)$ effects gives [17]

$$|V_{ud}|\cdot|f_+^{\pi\beta}(0)| = 9600.8\sqrt{\mathrm{Br}(\pi^+ \to \pi^0 e^+ \nu_e(\gamma))},\; f_+^{\pi\beta}(0) = 1.0046 \pm 0.0005. \quad (8)$$

Radiative corrections enhance the branching ratio by (3.34 ± 0.10) % . The (very small) uncertainties come from the counterterm contributions. It is thus possible to give a very accurate prediction for $|f_+^{\pi\beta}(0)|$ in ChPT. With the latest result [18] of the PIBETA experiment, the relative precision on $|V_{ud}|$ obtained this way is still limited by the experimental precision

$$\delta|V_{ud}|/|V_{ud}| = (\pm 3.2_{\,\mathrm{exp}} \pm 0.5_{\,\mathrm{th}}) \times 10^{-3}\,. \quad (9)$$

4 Two loop $K_{\ell 3}$ form factors and strategies to extract $|V_{us}|$

The situation is somewhat less ideal for the $K_{\ell 3}$ decays, since the corrections are larger, and the one loop result is not sufficient for an accurate determination [13] of $|V_{us}|$. The NNLO expressions for the $K_{\ell 3}$ form factors $f_\pm(t)$ decompose into a two loop part, which depends only on the masses and on F_π, a one loop part involving the L_i's, and a tree level contribution depending on some of the $\mathcal{O}(p^6)$ LECs C_i. It should be stressed that the estimate of $f_+^{(4)}(0)$ given in Ref. [13] is neither a two loop calculation, nor an estimate of the LECs that enter the two loop expression. While the LECs giving the $\mathcal{O}(t)$ and the $\mathcal{O}(t^2)$ terms of $f_+(t)$ can in principle be obtained from the experimental measurements of the slope λ_+ and the curvature c_+, there remain two unknown LECs in $f_+(0)$, C_{12} and C_{34}. The important observation [16] here is that these same two LECs also appear in a combination of the scalar form factor $f_0(t)$ and of F_K/F_π. For instance,

$$\lambda_0 = 8\frac{M_\pi^2(M_K^2 + M_\pi^2)}{F_\pi^4}(2C_{12} + C_{34}) + \frac{M_\pi^2}{M_K^2 - M_\pi^2}\left(\frac{F_K}{F_\pi} - 1\right) + \Delta'(0)\,, \quad (10)$$

$$c_0 = -8\frac{M_\pi^4}{F_\pi^4}C_{12} + \Delta''(0)/2\,. \tag{11}$$

In the kinematical region of interest, the known function $\Delta(t)$ is well approximated by a polynomial [16], $\Delta(t) = \alpha t + \beta t^2 + \gamma t^3$. Thus, one may extract C_{12} from the knowledge of the curvature c_0 of $f_0(t)$, and then get C_{34} from its slope λ_0 *provided* F_K/F_π *is known.* The reason for the emphasis [19] here comes from the fact that the value usually quoted, $F_K/F_\pi = 1.22 \pm 0.01$, actually results from the analysis of Ref. [13], and thus cannot be used *a priori.* The effect of a variation in F_K/F_π on $f_+(0)$ reads,

$$\delta f_+(0)|_{F_K/F_\pi} = \frac{M_K^2 - M_\pi^2}{M_K^2 + M_\pi^2}\,\delta\left(\frac{F_K}{F_\pi}\right)\,, \tag{12}$$

and even a variation of F_K/F_π as small as a few percents directly affects the value of $f_+(0)$, and thus the determination of $|V_{us}|$, by about the same relative amount. This assumes that all the dependence on F_K/F_π is explicitly shown in Eqs. (10) and (11). The situation is however more complicated, since the values of the coefficients α, β, γ depend on the values of the L_i's, which are obtained from a fit [20] to various input observables, *including the fixed value* $F_K/F_\pi = 1.22{\pm}0.01$. A more accurate description of the dependence on F_K/F_π therefore requires to perform this fit for different values of this ratio, in the range, say, from 1.17 to 1.27, expressing, for instance, the numerical coefficients α, β, γ in the form $\alpha = \alpha_0 + \alpha_1(F_K/F_\pi - 1.22) + \alpha_2(F_K/F_\pi - 1.22)^2 + \ldots$, etc. The situation is thus similar to the one encountered previously in a different, but not unrelated, context [19], and the strategies to extract $|V_{us}|$ discussed there may be easily adapted. From Eq. (3), one can obtain F_K/F_π in terms of $|V_{us}/V_{ud}|$, thus expressing $f_+(0)$ as $1 + \mathcal{F}(\lambda_0, c_0, |V_{ud}|, |V_{us}|)$. Given a value of $|V_{ud}|$ and sufficiently accurate experimental determinations of λ_0 and of c_0 from the $K_{\mu 3}$ data (see the discussion in Ref. [16]

for the accuracy that is required), this would then allow to extract $|V_{us}|$ from the values of the $K_{\ell 3}$ branching ratios, and then to obtain F_K/F_π from Eq. (3). Independent information on F_K/F_π can of course modify the situation. For instance, there exists now a rather accurate determination of F_K/F_π from partially quenched lattice data with staggered fermions [21]. Using this imput allows to extract $|V_{us}|$ directly from Eq. (3) [22], given a value of $|V_{ud}|$. On the other hand, there exists also a direct, although quenched, lattice calculation [23] of $f_+(0)$. These new developments offer possibilities for cross checks.

In particular, one would like to have a determination of both F_K/F_π and $f_+(0)$ from the same lattice simulation with dynamical (domain wall ?) fermions, in order to check whether they satisfy the correlation implied by the above analysis of the two loop ChPT expression. As far as the latter is concerned, the inclusion of isospin breaking corrections would be welcome.

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