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$(g_\mu - 2)/2$ AND HADRONIC e^+e^- CROSS SECTIONS

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Abstract

Recent measurements of the cross section of the process $e^+e^- \to hadrons$ are reviewed. Their implications for the calculation of the muon anomalous magnetic moment are discussed. e^+e^- based calculations are compared to those using τ data.

1 Introduction

Recently the E821 Collaboration at BNL published the final results on $a_\mu = (g-2)_\mu/2$, the muon anomalous magnetic moment (MAMM) [1]. The relative accuracy achieved in this experiment is $5 \cdot 10^{-7}$: $a_\mu = (11659208 \pm 6) \cdot 10^{-10}$. Although $a_{\rm e}$ is measured with a $4 \cdot 10^{-9}$ accuracy, a_μ is much more sensitive to new physics effects: the gain is usually $\sim (m_\mu/m_{\rm e})^2 \approx 4.3 \cdot 10^4$. Any significant difference of $a_\mu^{\rm exp}$ from $a_\mu^{\rm th}$ indicates new physics beyond the Standard Model (SM). It is conventional to express the theoretical prediction as

$$a_\mu^{\rm th} = a_\mu^{\rm SM} + a_\mu^{\rm non-SM}, \qquad a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}. \tag{1}$$

For the quantum electrodynamics (QED) term the analytical calculation of the α^3 terms, the numerical calculation of the α^4 terms and estimation of some of the α^5 terms gives after recent improvements [2, 3]: $a_\mu^{\rm QED} = (11658471.9 \pm 0.2) \cdot 10^{-10}$. For the electroweak (EW) term the most recent estimation including all one- and two-loop terms as well as part of three-loop terms gives [4]: $a_\mu^{\rm EW} = (15.4 \pm 0.1 \pm 0.2) \cdot 10^{-10}$. The hadronic contribution can also be written as a sum of three terms:

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}. \tag{2}$$

The dominant contribution comes from the first, leading order term.

Although this term can't be calculated from first principles, one obtains from dispersion relations:

$$a_\mu^{\rm had,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{R(s)\, \hat{K}(s)}{s^2}, \tag{3}$$

where $R(s) = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$, and the kernel $\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \to \infty$. The factor $1/s^2$ emphasizes the role of low energies, particularly important is the reaction $e^+e^- \to \pi^+\pi^-$ with a large cross section below 1 GeV. As a result, 73% of the central value comes from this channel and more than 92% comes from the energy region below 2 GeV.

Numerous attempts to estimate the leading order hadronic term exist in literature (see Ref. [5] and references therein), but most of them are model-dependent and do not take into account systematic uncertainties of the data. The authors of Ref. [5] were the first to perform a model-independent estimate based only on the data up to 40 GeV. Their result used the whole bulk of the experimental information existing before 1995 and gave: $a_\mu^{\rm had,LO} = (702 \pm 6 \pm 14) \cdot 10^{-10}$. Today, almost 10 years later, there are a lot of new precise measurements, mostly due to the experiments at the VEPP-2M collider in Novosibirsk and the BEPC collider in Beijing. We describe below a new evaluation based on new e^+e^- and τ lepton data.

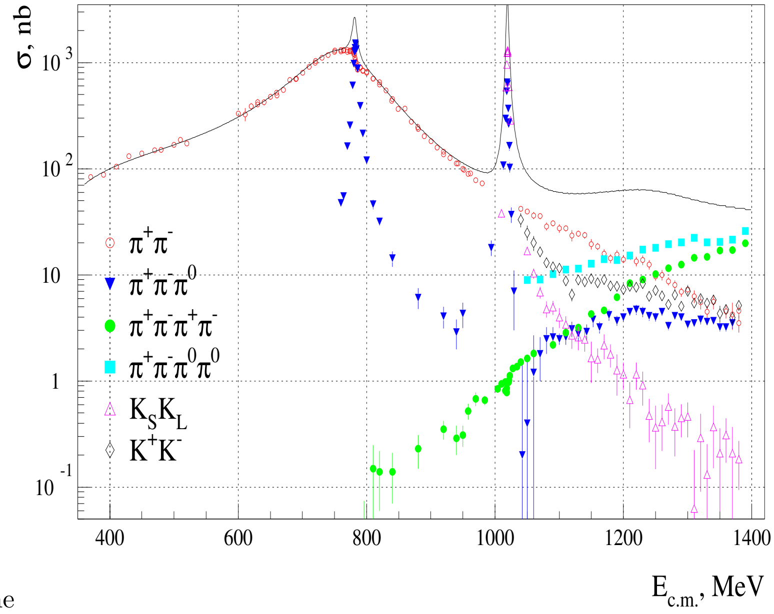

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Figure 1: *Hadronic cross sections at CMD-2.*

2 New e^+e^- Based Evaluation of $a_\mu^{\rm had,LO}$

Two Novosibirsk groups (CMD-2 [6]) and SND [7]) studied a lot of exclusive modes at $\sqrt{s} < 1.4$ GeV $(\pi^+\pi^-, \pi^+\pi^-\pi^0, \ldots, K\bar{K})$ including the channels with a very small cross section $(\pi\pi\gamma, \pi^0\gamma, \eta\gamma, \ldots)$ whereas BES studied the energy range from 2 to 5 GeV and measured the total cross section (R value) [8]).

The VEPP-2M e^+e^- collider in Novosibirsk was running for experiment since 1974 and until 2000. It covered the c.m. energy range $0.36 < \sqrt{s} < 1.40$ GeV with the peak luminosity of $3 \cdot 10^{30}$ cm^{-2} s^{-1}. The integrated luminosity of ≈ 100 pb^{-1} was collected in Novosibirsk below 1.4 GeV compared to ≈ 6 pb^{-1} in Orsay and Frascati at $1.4 < \sqrt{s} < 3.0$ GeV!

As mentioned above, the contribution of the process $e^+e^- \to \pi^+\pi^-$ dominates $a_\mu^{\rm had,LO}$ ($\sim 73\%$). Therefore, its low systematics study is of paramount importance. CMD-2 reached the 0.6% systematic error in the crucial ρ meson region from 600 to 960 MeV using only a part of the full data sample [6]). Final analysis of the whole statistics will soon be completed.

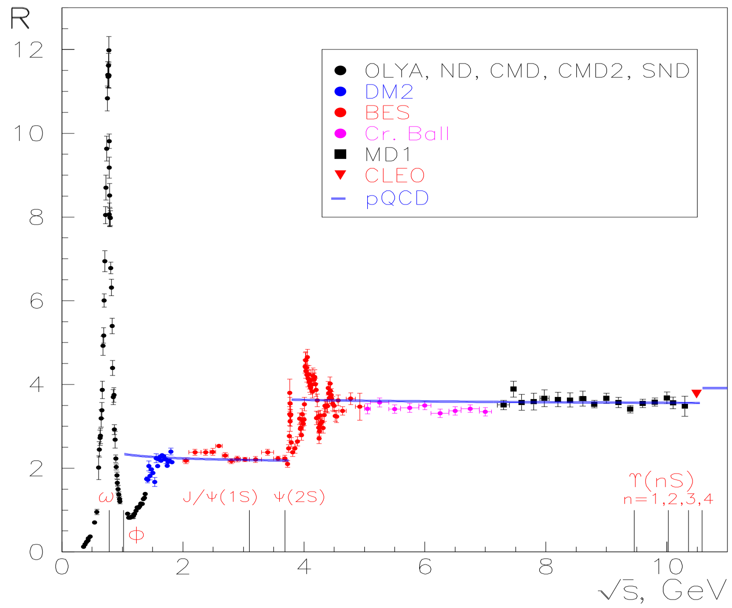

Figure 2: *R values below 10 GeV.*

Above 1 GeV another process - production of four pions dominates. Both possible final states have been studied: $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$, and the cross sections are known with good accuracy below 1.4 GeV while a large data scatter is observed from 1.4 to 2 GeV.

In Fig. 1 we show various hadronic cross sections studied at CMD-2. One can see that the values of the cross section vary by three–four orders of magnitude, from more than one thousand nb for the processes $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ at the peak of the ρ and ω mesons, respectively, to about 100 pb only for the process $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$.

Another important feature of these studies is that for the first time cross sections of various purely neutral final states $\pi^0\gamma$, $\eta\gamma$, $\pi^0\pi^0\gamma$ and $\eta\pi^0\gamma$ were studied in a broad energy range with both detectors [9, 10], not only at the resonance peaks, but also at the energies between the resonances. The results of CMD-2 and SND are consistent with each other and show that the contributions of the ρ, ω and ϕ mesons dominate the cross sections. From the upper limits on nonresonant cross sections CMD-2 set a 90% CL upper limit for the non-resonant contributions of such processes to the MAMM: $a_\mu^{\rm rad,LO} < 0.7 \cdot 10^{-10}$.

As we have already mentioned, measurements at 1.4 GeV $< \sqrt{s} <$ 2 GeV were performed long ago in Orsay and Frascati and their results have much worse accuracy. A detailed study of the energy range would be important not only for the MAMM problem, but also for the resonance spectroscopy. For example, even the basic properties of the five resonances in these energy range (2 ρ' mesons, 2 ω' mesons and the ϕ' meson) are rather badly known. There are indications for additional structures, e.g., the narrow one at 1.9 GeV in the 6π system [11]. There are also speculations that the spectroscopy of these states is even more complicated involving mixing with exotic states, e.g., hybrids [12].

A very important measurement of the total cross section $e^+e^- \to hadrons$ recently performed at the BES detector improved upon the accuracy of R in the energy range between 2 and 5 GeV at least by a factor of two [8].

In Fig. 2 we show the value of R obtained in the most precise experiments below 10 GeV. Good agreement is observed between the data and the theoretical predictions based on perturbative QCD. This justifies a possibility to use theory above 5 GeV for the calculations, particularly taking into account that this energy range gives less than a 1.5% contribution to the leading order hadronic term [13, 14]. As a result, our calculation gives $a_\mu^{\rm had,LO} = 696.3 \pm 6.2 \pm 3.6$. Note that a higher accuracy of e^+e^- data makes the error of $a_\mu^{\rm had,LO}$ two times smaller.

We should also mention the status of the higher order terms. The higher order hadronic contribution $a_\mu^{\rm had,HO}$ can also be calculated in terms of the $\int R(s)G(s)ds/s^{2(3)}$, where $G(s)$ is a smooth function of s, so that the low energy range again dominates the integral. Several calculations agree. The accepted value is [15]: $a_\mu^{\rm had,HO} = (-10.0 \pm 0.6) \cdot 10^{-10}$. Until recently the accepted value for a controversial light-by-light scattering term was $a_\mu^{\rm LBL} = (8.6 \pm 3.5) \cdot 10^{-10}$ based on the Refs. [16, 17]. In Ref. [18] this quantity was reevaluated to be $a_\mu^{\rm LBL} = (13.6 \pm 2.5) \cdot 10^{-10}$. Adding together all the contributions described above, we obtain for the theoretical prediction $a_\mu^{\rm th} = (11659180.9 \pm 8.0) \cdot 10^{-10}$ and $a_\mu^{\rm exp} - a_\mu^{\rm th} = (27.1 \pm 10.0) \cdot 10^{-10}$ or 2.7 standard deviations. If recent theory progress is taken into account, the difference becomes smaller, 2.1σ only.

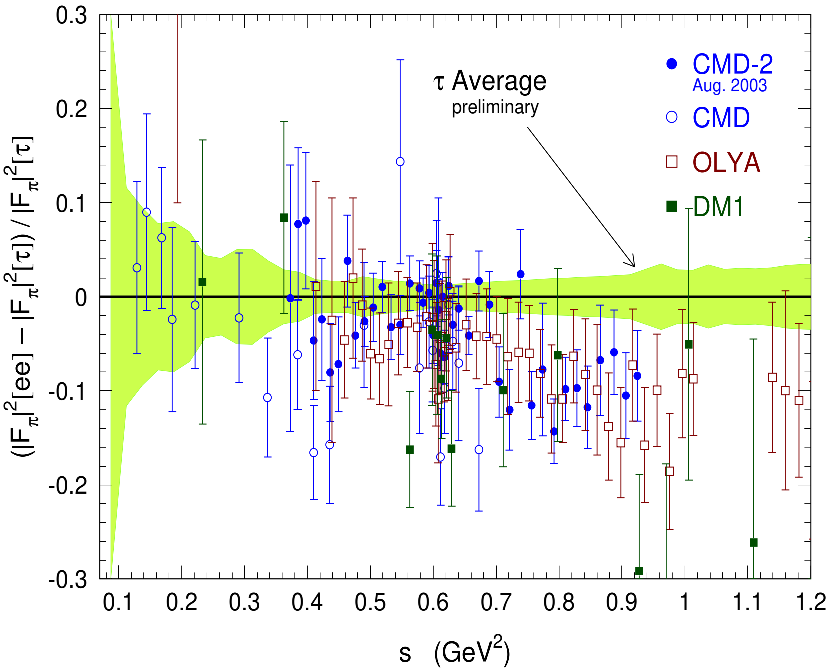

Figure 3: *Comparison of* 2π *spectral functions.*

3 Confronting e^+e^- and τ Lepton Based Evaluations

To improve on a theoretical error, it is very tempting to use the data on τ lepton decays into vector states 2π and 4π. It is known that from the conservation of vector current (CVC) and SU(2) symmetry the corresponding hadronic spectra in τ decays are related to the values of the e^+e^- cross sections [19]. Various CVC tests performed in 80-ies and the beginning of 90-ies showed that this relation was valid within the accuracy of the measurements available at that time [20]. Therefore, in Ref. [15] the authors assumed the CVC validity to use an independent τ lepton data set to perform a new τ lepton based estimate of $a_\mu^{\rm had,LO}$. Averaging their result with the e^+e^- based one they improved on the accuracy of $a_\mu^{\rm had,LO}$ by a factor of 1.5. With the increasing accuracy both in e^+e^- and τ sectors indications for some discrepancy appeared: the spectral functions in τ decays were higher than those in e^+e^- annihilation. A recent

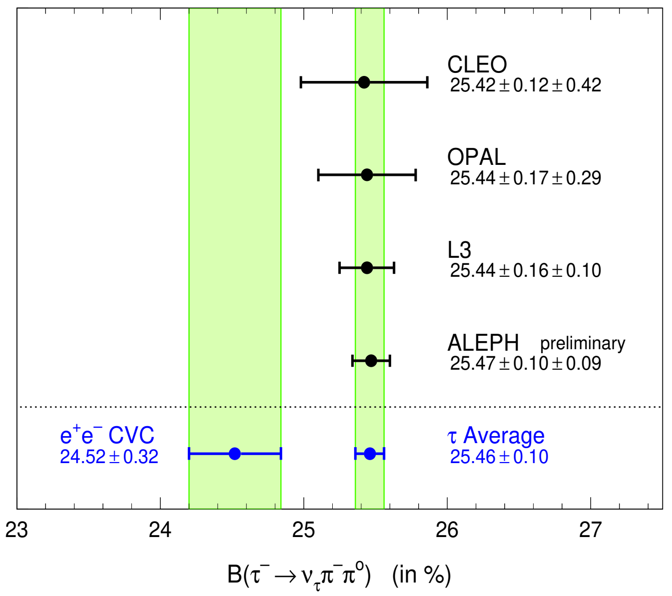

Figure 4: *$Br(\tau^- \to \pi^-\pi^0\nu_\tau)$ from CVC and τ decays.*

analysis, which used the latest data from both e^+e^- and τ sectors as well as a large set of corrections for SU(2) breaking [21, 22], confirmed the existence of such a discrepancy. It is illustrated for the 2π channel in Fig. 3, from which it is clear that above the ρ meson peak the τ spectral function is significantly higher than that in e^+e^- annihilation. Integrating the spectral function from e^+e^- one can obtain the prediction for the branching fraction $Br(\tau^- \to \pi^-\pi^0\nu_\tau)$ confronted in Fig. 4 to that measured in τ decays by different groups. It is clear that the value of the branching fraction from all groups is systematically higher than the CVC prediction. As a result, a possibility to use τ lepton data for MAMM becomes problematic. Indeed, the τ lepton based estimate of $a_\mu^{\mathrm{had,LO}}$ is $(711.0 \pm 5.8) \cdot 10^{-10}$, i.e., substantially higher than $(696.3 \pm 7.2) \cdot 10^{-10}$ from e^+e^- data [14]. Averaging of the two estimates becomes meaningless. The reasons of this discrepancy are not yet clear [23]. The current situation is summarized in Fig. 5.

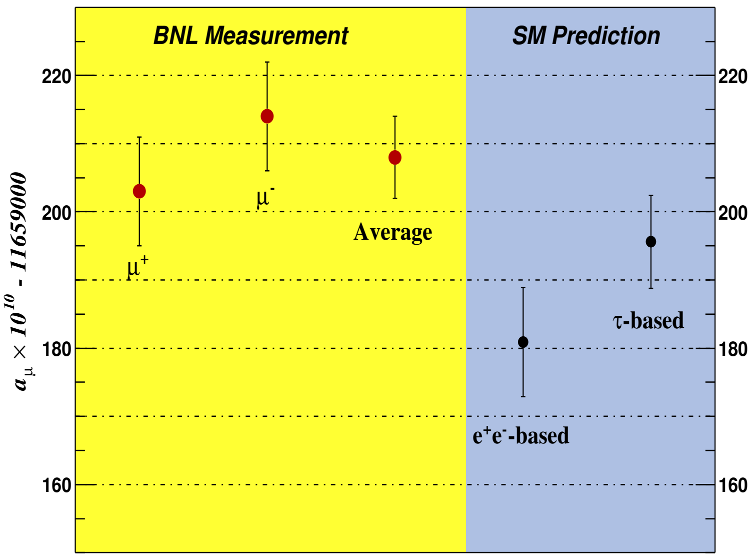

Figure 5: *Current status of* a_μ.

4 Prospects and Conclusions

We can conclude that the increased accuracy of e^+e^- data (VEPP-2M and BEPC) decreased an error of $a_\mu^{\rm had,LO}$ by a factor of 2, but the experimental accuracy of a_μ is still better. τ data could further improve upon the accuracy by a factor of 1.5 but e^+e^- and τ data differ. The experimental value of a_μ is higher than the theoretical prediction by (2.1–2.7) σ.

Future progress will be possible after experiments planned at the new machine VEPP-2000 (VEPP-2M upgrade) with 2 detectors (CMD-3 and SND) up to $\sqrt{s}$=2 GeV with $L_{\rm max} = 10^{32}$ cm^{-2}s^{-1} as well as other experiments at the CESRc and $(c-\tau)$ factory.

Also attractive is the idea to use for more precise R measurements the method of radiative return recently successfully employed in the KLOE experiment [24].

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