Frascati Physics Series Vol. XXXVI (2004), pp.
DAΦNE 2004: Physics at meson factories – Frascati, June 7-11 , 2004
Short Talk in Plenary Session

$K \to 3\pi$ DECAYS IN CHIRAL PERTURBATION THEORY

Fredrik Borg
Department of Theoretical Physics, Lund University,
Sölvegatan 14A, SE - 223 62 Lund, Sweden

1 Introduction

A kaon decaying into three pions is an example of a weak process. However, since the quarks are confined into mesons, the strong interaction also plays an important part. The reaction is a low-energy one, meaning that it takes place in the non-perturbative region of QCD. In that region perturbative QCD doesn't give you any answers and other methods have to be used. The one we use is called Chiral Perturbation Theory.

2 Chiral Perturbation Theory

ChPT is an effective field theory describing the low-energy interactions of the kaons, the pions and the eta. It can be used for strong as well as non-leptonic weak interactions. The Chiral Lagrangian is based on the spontaneous breaking of chiral symmetry.

Chiral symmetry is the separate symmetry between the left- and right-handed quarks. In the theory only the u, d and s quarks are included which means that the symmetry group is $SU(3)_L \times SU(3)_R$. This symmetry is spontaneously broken by the vacuum condensate into $SU(3)$. Since the symmetry is only *approximate* (exact if $m_u = m_d = m_s = 0$), this generates 8 *light* (not massless) Goldstone particles, identified as the kaons, pions and eta. From the knowledge of interactions between Goldstone particles one then constructs the Chiral Lagrangian.

The Chiral Lagrangian is organized in terms of importance. However, since it deals with low-energy processes, α_S can not be used for this purpose. Instead it is written as an expansion in p and m, the momenta and the masses of the pseudoscalars (K, π, η). Properly normalized these quantities are small and can be used as perturbation expansion parameters. Lowest order then means p^2 and m^2 and next-to-leading order p^4, m^4, p^2m^2 and so on. If one includes also isospin breaking, the unit charge, e, is also considered an expansion parameter.

3 Isospin Symmetry

Calculations are often performed in the isospin limit, where the u and d quarks are treated as being identical. In practice this means setting $m_u = m_d$ and neglecting electromagnetism.

In our first paper [1] the calculation was made in the isospin limit. In the second paper [2] we took into account strong isospin breaking, ie. the quark mass difference $m_u - m_d$ as well as the local electromagnetic effects. Work is in progress to evaluate the other electromagnetic corrections as well.

4 Results

There are five different CP-conserving decays of the type $K \to 3\pi$. The K^- decays are not treated since they are counterparts to the K^+ decays.

A full isospin limit fit was made in [1] taking into account all data published before May 2002. One of the reasons for the further investigation of isospin breaking effects is to see whether isospin violation can solve the discrepancies in the quadratic slope parameters found there. A new full fit will be done after all the electromagnetic contributions have been included in the

$$
\begin{array}{rcl}
K_L & \to & \pi^0\,\pi^0\,\pi^0 \\
K_L & \to & \pi^+\,\pi^-\,\pi^0 \\
K_S & \to & \pi^+\,\pi^-\,\pi^0 \\
K^+ & \to & \pi^0\,\pi^0\,\pi^+ \\
K^+ & \to & \pi^+\,\pi^+\,\pi^-
\end{array}
$$

amplitudes (work in progress).

4.1 Results with and without strong isospin breaking

Our main result up to now is the comparison between the amplitudes in the isospin limit and including first order strong isospin breaking. In Fig. 1 we show the phase space boundaries for the five different decays and the three curves along which we compared the squared amplitudes with and without first order strong isospin breaking

In general the differences are of the size to be expected from this type of isospin breaking. For $K_L \to \pi^0\pi^0\pi^0$ the central value of the amplitude squared increases by about 3% when strong isospin breaking is included. The change in the quadratic slope is similar but the total variation over the Dalitz plot is small so the total decay rate increases by about 3% as well. The squared amplitude $K_L \to \pi^+\pi^-\pi^0$ increases by about 2.5%. The decay rate and the changes in the Dalitz plot slopes are of similar size. For the decay $K_S \to \pi^+\pi^-\pi^0$ the amplitude in the center of the Dalitz plot vanishes because of CP-asymmetry. The amplitude and the slopes increase by about 3%, see Fig. 2. The decay $K^+ \to \pi^0\pi^0\pi^+$ has the largest increase. The squared amplitude in the center changes by about 11%. The linear slopes decrease somewhat leading to an increase of about 8% to the total decay rate when compared with the isospin conserved case. The decay $K^+ \to \pi^+\pi^+\pi^-$ has a change of about 7.5% upwards in the center of the Dalitz plot and a similar change in the decay rate. The slopes decrease somewhat. For more figures and detailed results, see [2].

5 Conclusions

We have calculated the $K \to 3\pi$ amplitudes to next-to-leading order in ChPT. A first calculation was done in [1] in the isospin limit, but we have now also

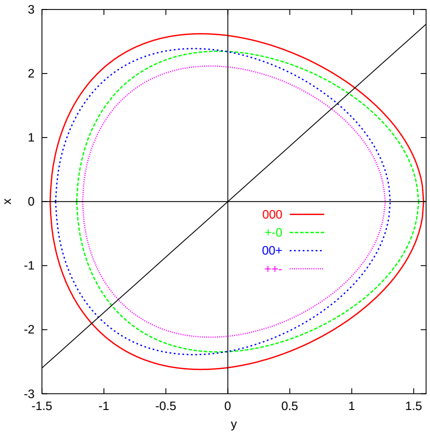

Figure 1: em The phase space boundaries for the five different decays and the curves along which we will compare the amplitudes.

included effects from $m_u \neq m_d$ and local electromagnetic isospin breaking in [2]. This was done partly because it is interesting in general to see the possible importance of isospin breaking in this process, but also to investigate whether isospin violation will improve the fit to experimental data made in [1]. We have tried to estimate the effects of the breaking by comparing the squared amplitudes with and without isospin violation. The effect seems to be at a few percent level, and probably not quite enough to solve the dicrepancies. However, to really investigate this a new full fit has to be done, including the explicit photon diagrams and the new data published after [1] as well. This is work in progress and will be presented in future papers.

6 Acknowledgements

The program FORM 3.0 has been used extensively in these calculations. This work is supported in part by the Swedish Research Council and European Union TMR network, Contract No. HPRN-CT-2002-00311 (EURIDICE).

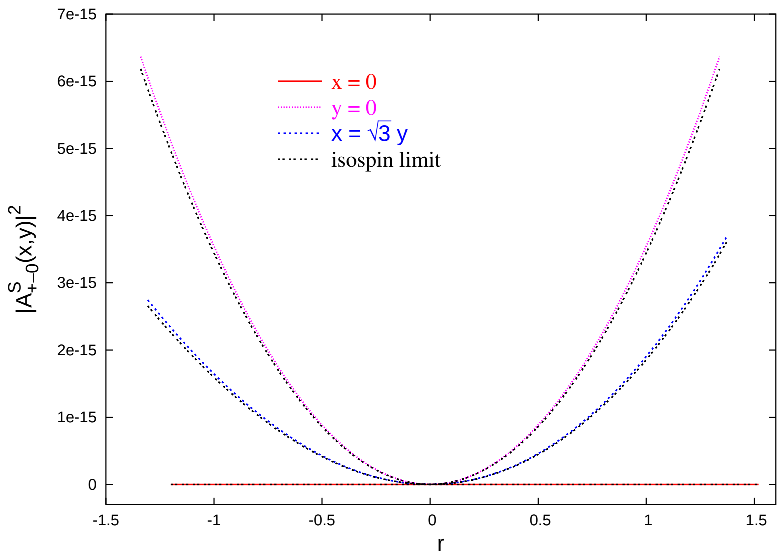

Figure 2: $K_S \to \pi^+\pi^-\pi^0$ *with and without strong isospin breaking.*

References

1. J. Bijnens, P. Dhonte and F. Persson, Nucl. Phys. B 648 (2003) 317, [hep-ph/0205341].

2. J. Bijnens and F. Borg, to be published in Nucl. Phys. B, [hep-ph/0405025].