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KAONIC ATOMS IN QCD

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Abstract

In this talk, I comment on the theoretical and experimental status of kaonic atoms, in particular $\bar{K}\pi$ and $\bar{K}p$ bound states.

1 Introduction

Kaonic atoms are particular examples of *hadronic atoms*. They are of the type $\bar{K}X$, with $X = \pi, K$; p; d; ^{3}He; ^{4}He.... Kaonic atoms are by definition bound by electromagnetic interactions, so a more precise title of my talk would be *Kaonic atoms in QCD + QED*. On the other hand, deeply bound kaonic nuclear states are of a different variety - as far as I understand, they are predicted to exist already in the framework of QCD [1, 2], electromagnetic forces are not required for their formation. I do not consider these systems here (nor $\bar{K}K$ bound states [3]). The reason to investigate *hadronic atoms* in general is the

following: as just said, they are formed by electromagnetic forces, which are well known. Strong interactions - mediated by QCD - have two effects: they i) distort the spectrum, and ii) let the atoms decay. As we will see below, strong interactions may be considered a small perturbation in some cases, and it is then possible to calculate their effect. Indeed, as is known since fifty years [4], the energy shift and the lifetime of hadronic atoms are in general related to the pertinent T- matrix element in QCD at threshold. Therefore, measuring the spectra amounts to measure these amplitudes. Classic applications of this procedure to determine strong amplitudes are: *pionic hydrogen at PSI* [5] $\leftrightarrow$ $T_{\pi N}$*; pionium at DIRAC* [6] $\leftrightarrow$ $T_{\pi\pi}$*; kaonic Hydrogen at DEAR* [7] $\leftrightarrow$ $T_{\bar{K}N}$.

Data on hadronic atoms have therefore the potential to replace low energy experiments on $\pi N \to \pi N \leftrightarrow T_{\pi N}$; $\pi\pi \to \pi\pi \leftrightarrow T_{\pi\pi}$; $\bar{K}N \to \bar{K}N \leftrightarrow T_{\bar{K}N}$ that are difficult (or impossible) to perform. All in all, hadronic atoms allow one to confront high precision, low energy QCD predictions with data. As a now classic example I mention $\pi\pi$ scattering lengths, where the theoretical predictions are [8, 9]

$$a_0 = 0.220 \pm 0.005\,,\; a_0 - a_2 = 0.265 \pm 0.004\,, \tag{1}$$

to be confronted with e.g. data from K_{e4} decay [10],

$$a_0 = 0.216 \pm 0.013\,(\text{stat.}) \pm 0.002\,(\text{syst.}) \pm 0.002\,(\text{theor.})\;. \tag{2}$$

Data on $\pi\pi$ scattering from the DIRAC experiment are discussed in Tauscher's contribution to this conference [6]. Furthermore, a high statistics K_{e4} experiment is underway at NA48 [11]. As Cabibbo has pointed out at this conference, $K^\pm \to \pi^\pm\pi^0\pi^0$ decays may provide the possibility to determine the combination $a_0 - a_2$ with high precision [12, 13].

The procedure to confront QCD predictions with data on atomic spectra consists of two steps: First, one relates the spectra to QCD scattering amplitudes at threshold [4]. The precision of this calculation must match the accuracy of the data, which requires in many cases to go beyond the relation provided in [4]. Second, one calculates QCD amplitudes using effective field theories, lattice calculations ... , and compares with what one obtains from step one.

The experimental and theoretical situation for kaonic atoms is summarized in table 1.

	experiment	*theory*
$\bar{K}\pi$	Letter of Intent [14]	[15, 16]
$\bar{K}p$	DEAR [7]	[17, 18]
$\bar{K}d$	SIDDHARTA [19, 20]	[21]

Table 1: *Kaonic atoms: status of theory and experiment.*

The DEAR experiment is presently the only place where there is overlap between theory and data in *kaonic atoms*. Let us hope that the situation changes in the future.

2 $\bar{K}\pi$ atoms

$\bar{K}\pi$ atoms are interesting, because the hadronic effects in the spectrum are related to $SU(3) \times SU(3)$ chiral perturbation theory (ChPT) in the meson sector, which works, as far as is known today, very well. The modern way to interrelate the spectrum and QCD works as follows. First, one observes that the momenta of the constituents as well as of the decay products are small, of the order of 1 MeV or less. Therefore, it is advisable to use a non relativistic field theory framework for the calculation [22, 23] - for a relativistic approach see [24]. In order to verify that a perturbative calculation is reasonable, we note that the Coulomb binding energy of the ground state is $E_B \simeq 2.9$ keV, whereas the strong shift of the energy level is about -9 eV [15] - a tiny effect. Further, the lifetime of the ground state turns out to be about $4 \cdot 10^{-15}$ sec. An estimate of the number of orbits performed before decaying, $\tau \cdot E_B \simeq 1.8 \cdot 10^4$, reveals that the atom may be considered as nearly stable. I conclude that the calculation is self consistent - $\bar{K}\pi$ atoms belong to a class of systems where the perturbation of the QED spectrum by the strong interaction among the constituents is small. Next, we consider the decay channels allowed. The mass differences are

$$M_{K^-} + M_{\pi^+} = M_{K^0} + M_{\pi^0} + 0.60\text{MeV}\,, \tag{3}$$

as a result of which possible decay channels are

$$A_{K^-\pi^+} \to \bar{K}^0\pi^0, \bar{K}^0 + n\gamma, \ldots \tag{4}$$

One expands the decay width in powers of the isospin breaking parameters[1] α and $m_d - m_u$, that are counted as quantities of order δ. For the ground state, the leading and next-to-leading terms are due to the decay into $\bar{K}^0\pi^0$:

$$\Gamma_G = \underbrace{a\,\delta^{7/2} + b\,\delta^{9/2}}_{\bar{K}^0\pi^0} + \underbrace{\mathcal{O}(\delta^5)}_{\bar{K}^0\pi^0+\text{other channels}} \quad . \tag{5}$$

The formula for the decay width of the ground state at next-to-leading order has recently been worked out by Julia Schweizer [15],

$$\Gamma_G = 8\alpha^3 \mu_c^2 p^* [a_0^-]^2 (1+\epsilon) + \mathcal{O}(\delta^5)\,, \tag{6}$$

where a_0^- is the isospin odd S-wave scattering length in elastic πK scattering, p^* denotes the relative 3-momentum of the $\bar{K}^0\pi^0$ pair in the final state, and μ_c stands for the reduced mass of the charged mesons. Finally, the quantity ϵ is a correction due to isospin breaking, known at order δ [15]. Therefore, a measurement of the decay width of the ground state provides a_0^-,

$$\Gamma_G \quad \to \quad a_0^- \quad \leftrightarrow \quad \text{low energy QCD} \; . \tag{7}$$

We note that a_0^- is the scattering length in pure QCD, purified from electromagnetic corrections, evaluated at $m_u = m_d$, with $M_K = 493.7$ MeV. Using the value of a_0^- determined recently in a dispersive analysis [25] gives

$$\tau_G = (3.7 \pm 0.4) \cdot 10^{-15}\text{sec} \; . \tag{8}$$

The main open problem here concerns the experimental verification of this result, and an investigation of whether one may obtain in this manner more information on the LECs that occur in the chiral expansion of the scattering lengths [26].

For an exhaustive discussion of the various decay channels and energy shifts, I refer the interested reader to the work of Julia Schweizer [15]. I conclude with the observation that the theory of πK atoms very well understood. On the other hand, experiments are sadly missing.

3 Kaonic hydrogen

Here, I discuss properties of kaonic hydrogen, a system investigated in the last years at DEAR [7]. Let us first again discuss orders of magnitudes. The

[1] We denote the fine structure constant by $\alpha \simeq 1/137$.

Coulomb binding energy of the ground state is about 8.6 keV, the strong shift about .2 keV [7] - the perturbation is still small. The width is $\Gamma \simeq 250$ eV [7], such that the system performs about $\tau \cdot E_B \simeq 35$ orbits before decaying, considerably less than in the case of the πK atom, but still reasonably many. Note, however, that this number becomes $\simeq 10$ for the width found in [18] from unitarized ChPT - which is surprisingly small.

3.1 Theory

Some of the decay channels of kaonic hydrogen are

$$A_{\bar{K}p} \to \pi\Sigma, \Sigma\pi\gamma, \Sigma\pi e^+e^-, \Sigma\gamma, \ldots \tag{9}$$

Note that it cannot decay into an $\bar{K}^0 n$ pair for kinematic reasons: in our world, the value of the up and down quark masses are such that $M_{K^-} + M_p < M_{\bar{K}^0} + M_n$. This is in contrast to what happens in the $\bar{K}\pi$ atom, where the main decay channel is into the neutral pair $\bar{K}^0\pi^0$.

The necessary steps to get the pertinent formula for the energy shift and decay width have been performed recently by Meißner, Rusetsky and Raha [18] in a very nice piece of work in the framework of effective field theory, that accounts for a systematic expansion in isospin breaking effects. A different approach has been used in [17]. In order to illustrate the difficulties one is faced with in this system, I display in figure 1 the analytic properties of the forward $\bar{K}p \to \bar{K}p$ amplitude at $\alpha \neq 0, m_u \neq m_d$. The various branch points and cuts have to be taken into account properly in the derivation of the result, and this amplitude must then be related to the one in pure QCD, where e.g. the branch points at $\bar{K}p$ and $\bar{K}^0 n$ coincide, and where the $\Sigma\gamma$ cut is absent.

The main observation is the following [18]: there are large isospin breaking effects in the final formula, as large as the uncertainty in present DEAR data. Whereas this observation is not new [27, 28], the authors of [18] have shown how to sum up the most singular pieces, such that the remainder is of next order in isospin breaking and therefore expected to be small. The result for the energy shift and level widths of the S-states is similar in structure to the $\bar{K}\pi$ atom considered above, however considerably more complicated - I refer the interested reader to the original article [18] for the explicit formula. The main point is that the shift and width can be calculated, once the $I = 0, 1$ scattering lengths $a_{0,1}$ in $\bar{K}p \to \bar{K}p$ scattering are known in pure QCD, at $m_u = m_d$.

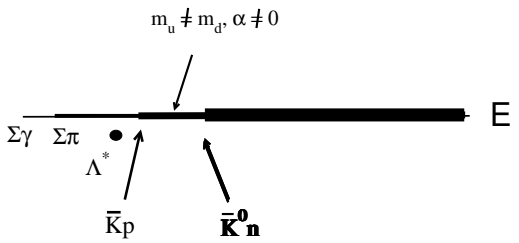

Figure 1: *The analytic properties of the forward amplitude for $\bar{K}p \to \bar{K}p$ scattering in the presence of isospin breaking interactions. Indicated are some of the branch points in the amplitude. The filled circle denotes the $\Lambda^*(1405)$ pole on the second Riemann sheet.*

Vice versa, if the shift and width is known, one may determine these scattering lengths.

3.2 Comparison with data

The scattering lengths $a_{0,1}$ have been calculated in [29] - see also [31, 32] - by use of unitarized ChPT. The comparison with the data from the DEAR collaboration is provided in Ref. [18], to which I refer the reader for details, see in particular their figure 3, that illustrates the large isospin breaking present in this system. The theoretical prediction [29] does not agree with the measurement performed at DEAR - although it must be said that the calculation of the scattering lengths in [29] does not include an error analysis of the final result. The reason for this disagreement has not yet been investigated [18, 30]. It is interesting to compare the scattering lengths in [29] with ChPT in the standard loop expansion. The relevant calculation had been performed by Kaiser [33]. It turns out that the one loop result for the isospin zero amplitude is completely off the correct answer, as a result of which the predicted energy shift in the ground state of kaonic hydrogen has the wrong sign. This shows that, due to the nearby resonance $\Lambda^*(1405)$, one has to go beyond a pure loop expansion. This is what has been done [31, 32, 29]. However, the procedure is not without pitfalls: the authors of e.g. Ref. [32] have provided scattering lengths that are in sharp conflict with the DEAR data. The reason for this failure is explained

in [18].

Once data on kaonic hydrogen energy shift and width will be available at the eV level, it will be even more dramatic to compare theoretical prediction with these data - I am rather curious to see whether unitarization procedures will pass this test. Needless to say that it would be comforting to have a precise prediction from theory, including uncertainties attached, before our experimental colleagues have done their job.

Finally, I shortly remind the reader that it would be, in my opinion, a theoretically tremendous effort to derive a precise relation between the scattering lengths determined through the measurement of kaonic hydrogen, and the kaon nucleon sigma terms [34].

4 More complicated systems

The are more complicated systems than the ones we have considered so far, e.g., kaonic deuterium. There are plans to investigate this system with SIDDHARTA, see the contributions by Iliescu and Jensen to this conference [19, 20]. The investigation of the relevant spectra can provide information on the $\bar{K}p, \bar{K}n$ scattering amplitude at threshold. Of course, one needs the corresponding formula, relating the scattering lengths to the spectrum. One may compare this with pionic deuterium, where first theoretical investigations using effective field theories are already available [35] or underway [36]. The $\bar{K}d$ system is even more complicated [21]. Whether a theoretically sound analysis in the framework of effective field theories is possible remains to be seen.

5 Conclusions

1. *Hadronic atoms* are a wonderful tool to measure QCD amplitudes at threshold.

2. $\bar{K}\pi$ *atoms* are theoretically well understood [15] [16]. The relevant $\bar{K}\pi$ scattering amplitude is now available to two loops in ChPT [26], and an analysis invoking Roy-Steiner equations has been performed as well [25]. On the other hand, the precise connection between the vacuum properties of QCD and $\bar{K}\pi$ scattering is still an open question, and experiments on the atom are absent.

3. The ground state of *kaonic hydrogen* has been investigated in a beautiful experiment at DEAR [7]. Data are available, the S-states state of the atom are theoretically understood [18, 17].

4. On the other hand, the theory of $\bar{K}p$ scattering leaves many questions open. More precise data will reveal whether present techniques are able to describe the complicated situation properly.

5. Concerning *kaonic deuterium*, experiments are planned [19, 20]. Whether this systems allows for a theoretically sound analysis in the framework of effective field theory remains to be seen.

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