Frascati Physics Series Vol. XXXVI (2004), pp.
DAΦNE 2004: Physics at meson factories – Frascati, June 7-11, 2004
Invited Review Talk in Plenary Session

RARE DECAYS AS WINDOW TO NEW PHYSICS

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Abstract

Rare decays of K mesons are reviewed from the perspective of testing the "ones" and "zeros" of the standard model. Decays $K^+ \to \pi^+\nu\overline{\nu}$ and $K_L \to \pi^0\nu\overline{\nu}$ probe the one-loop effective Hamiltonian for $s \to d\nu\overline{\nu}$, and can constrain the ρ, η coordinates of the unitarity triangle. Decays such as $K_L \to \pi^0 l^+ l^-$, $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+ l^+ l^-$ and $K_L \to \pi^+\pi^- e^+ e^-$ involve short-distance effects, as well as long-distance photon-induced contributions. Some comments are added on curious features of electroweak amplitudes in the "gaugeless" limit, and in the chiral electron limit $m_e \to 0$.

1 Ones and Zeros of the Standard Model

The study of rare decays may be regarded as a part of the endeavor to test the principles of symmetry and symmetry-breaking underlying the standard model of weak interactions. In any theory based on symmetries, the most important numbers are the "ones" and "zeros", the intensity rules and selection rules. In the case of the standard model, the ones and zeros are associated with the unitarity of the quark-mixing matrix, e.g.

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \text{ (ONE)} \tag{1}$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \text{ (ZERO)} \tag{2}$$

Physically, Eq. (1) expresses the universality of the lepton and hadron charged current couplings. The present status of this relation may be judged from the empirical results [1] $|V_{ud}| = 0.9738(5)$, $|V_{us}| = 0.2200(26)$, and $|V_{ub}| = (3.67(47)) \times 10^{-3}$, which satisfy Eq. (1) to within a deficit $\Delta = 0.0033(21)$.

The zero in Eq. (2) represents a *unitarity triangle*, and is one of six that encode the structure of CP violation in the weak nonleptonic Hamiltonian. These triangles have diverse shapes, corresponding to the diversity of the elements V_{ij}. There is, however, a unity in this diversity: all unitarity triangles have the same area A_Δ, as a consequence of the fact that 3×3 unitary matrices have an invariant property given by the Jarlskog parameter

$$J = \mathrm{Im}(\lambda_t \lambda_u^*) = \mathrm{Im}(\lambda_u \lambda_c^*) \tag{3}$$

where $\lambda_u = V_{us}V_{ud}^*$, $\lambda_t = V_{ts}V_{td}^*$, $\lambda_c = V_{cs}V_{cd}^*$ with $\lambda_u + \lambda_c + \lambda_t = 0$, and $|J| = 2A_\Delta$. This invariant is a universal measure of CP violation in weak phenomena. In addition, the existence of unitarity triangles implies a unification of CP-violating and CP-conserving observables. The sides of a triangle are determined by the moduli $|V_{ij}|$, measurable in CP-conserving processes. Knowledge of the sides fixes the angles, which are measures of CP violation. This property, as well as the universal area of unitarity triangles, is a feature specific to a world with three generations.

The zero in Eq. (2) has ramifications for flavour-changing neutral currents (FCNC). To order G_F, the weak neutral current has the structure

$$J_\mu^{\mathrm{NC}} = (\overline{d},\, \overline{s},\, \overline{b})\gamma_\mu \frac{1-\gamma_5}{2} V^\dagger V (d,\, s,\, b)^{\mathrm{tr}} \tag{4}$$

and the unitarity of the matrix V ensures the absence of non-diagonal terms. However, the symmetries which lead to the FCNC zero are broken in the standard model by Yukawa couplings of the scalar doublet $(\varphi^+,\, \varphi^0)$ to fermions.

For a typical doublet $(t,\, b)$, the Yukawa interaction is

$$\mathcal{L}_{\mathrm{Y}} = y_b(\overline{t}_L,\, \overline{b}_L) \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} b_R + y_t(\overline{t}_L,\, \overline{b}_L) \begin{pmatrix} \varphi^{0\dagger} \\ -\varphi^- \end{pmatrix} t_R \tag{5}$$

with $y_b = \sqrt{2}m_b/v$, $y_t = \sqrt{2}m_t/v$ (note that y_t is very nearly unity). These Yukawa couplings break chiral symmetry and give rise to a FCNC interaction like $(\overline{s}d)_{V-A}(\overline{\nu}\nu)_{V-A}$ at the level of one-loop (box and penguin) diagrams. Thus a typical FCNC amplitude has the form

$$A_{\mathrm{FCNC}} = G_F[0] + G_F\alpha \sum_{i=u,c,t} \lambda_i f(m_i). \tag{6}$$

2 Rare K Decays

2.1 Golden Modes: $K^+ \to \pi^+\nu\overline{\nu}$ and $K_L \to \pi^0\nu\overline{\nu}$

These two channels can be computed in an essentially model independent way from the effective Hamiltonian for $s \to d\nu\overline{\nu}$. The hadronic matrix element $\langle\pi^+|(\overline{d}s)_{V-A}|K^+\rangle$ can be related to the K_{l3} matrix element, and long-distance effects are negligible [2]. The effective Hamiltonian derived from the box and penguin diagrams is [3]

$$H_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi\sin^2\theta_W} \left[\lambda_c X_{NL} + \lambda_t X_t\right] (\overline{d}s)_{V-A}(\overline{\nu}\nu)_{V-A} \tag{7}$$

where X_{NL} is a small contribution due to c-quarks, and the dominant term is

$$X_t(x_t) = \frac{x_t}{8}\left[-\frac{2+x_t}{1-x_t} + \frac{3x_t-6}{(1-x_t)^2}\ln x_t\right] \tag{8}$$

with $x_t = m_t^2/m_W^2$. In a limited domain of m_t, X_t may be approximated as

$$X_t(x_t) = a + bx_t \tag{9}$$

The dominant term in the effective Hamiltonian Eq. (7) is then

$$H_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \frac{\lambda_t}{4\pi^2} \left[\frac{1}{2}ag^2 + by_t^2\right] (\overline{d}s)_{V-A}(\overline{\nu}\nu)_{V-A} \tag{10}$$

This expression reveals the two types of forces that are at work in FCNC decays: gauge forces associated with the gauge coupling $g\,(= e/\sin\theta_W)$ and Yukawa forces associated with the top-quark Yukawa coupling y_t. The latter force is independent of the gauge coupling, and exists even when g is switched off. It is the subtle interplay of these forces that one is testing in the study of FCNC processes.

A thorough analysis of the decays $K^+ \to \pi^+\nu\overline{\nu}$ and $K_L \to \pi^0\nu\overline{\nu}$ has been carried out by Buras et al. [3]. The first reaction can be used to obtain $|V_{ts}V_{td}^*|$ and hence $[(1-\rho)^2+\eta^2]^{1/2}$, the second determines the CP-violating parameter $\mathrm{Im}(V_{ts}V_{td}^*) \sim \eta$. The two together can localise the ρ, η coordinates of the unitarity triangle, and provide a consistency check of the $(\rho,\,\eta)$ domain delineated by B-decays. The predicted branching ratios are

$$Br(K^+ \to \pi^+\nu\overline{\nu}) = (7.8 \pm 1.2) \times 10^{-11}, \tag{11}$$

(to be compared with the experimental result $(14.7^{+13}_{-8.9}) \times 10^{-11}$ based on 3 events from the E949 and E787 experiments [4]), and

$$Br(K_L \to \pi^0\nu\overline{\nu}) = (3.0 \pm 0.6) \times 10^{-11}. \tag{12}$$

2.2 Decay Modes $K_L \to \pi^0 l^+ l^-$

These decays receive contributions from three sources: (a) a CP-violating short-distance interaction $s \to dl^+l^-$, (b) a CP-conserving two-photon contribution associated with the decay $K_L \to \pi^0\gamma\gamma$, (c) an indirect CP-violating contribution associated with a one-photon transition $K_1 \to \pi^0 l^+ l^-$. Accordingly, the decay amplitude has the structure

$$A \quad = \quad \underbrace{\eta\lambda^5 A_{sd}}_{\text{Direct } \not{C}\!P} \quad + \quad \underbrace{\alpha^2 A_{2\gamma}}_{CP\text{–conserving}} \quad + \quad \underbrace{\alpha\epsilon A_{1\gamma}}_{\text{Indirect } \not{C}\!P} \tag{13}$$

The coefficients $\eta\lambda^5$, α^2, $\alpha\epsilon$ have similar order of magnitude ($\eta \sim 0.3$, $\lambda \sim 0.2$, $\alpha \sim 10^{-2}$, $\epsilon \sim 10^{-3}$). Data on the branching ratio and $\gamma\gamma$ spectrum of $K_L \to \pi^0\gamma\gamma$ enable an estimate of $A_{2\gamma}$. The fact that the 2γ state appears to be mainly $J = 0$ implies that $A_{2\gamma}$ is of importance mainly for the $K_L \to \pi^0\mu^+\mu^-$ channel. The indirect CP-violating amplitude $A_{1\gamma}$ is fixed (up to a model-dependent sign) by the observed branching ratio for $K_S \to \pi^0 l^+ l^-$ [5]. A recent analysis obtains the prediction [6]

$$\begin{aligned} Br(K_L \to \pi^0 e^+ e^-) &= (3.7 \pm 1.0) \times 10^{-11} \\ Br(K_L \to \pi^0 \mu^+ \mu^-) &= (1.5 \pm 0.3) \times 10^{-11} \end{aligned} \tag{14}$$

2.3 Decay $K_L \to \mu^+\mu^-$

The decay $K_L \to \mu^+\mu^-$ is subject to a unitarity bound associated with the 2γ intermediate state [7], given by

$$R^K = \frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} \geq \frac{\alpha^2}{2\beta} \frac{m_\mu^2}{m_K^2} \left(\ln \frac{1+\beta}{1-\beta}\right)^2 = 1.2 \times 10^{-5} \tag{15}$$

where $\beta = (1 - 4m_\mu^2/m_K^2)^{1/2}$. The measured value of R^K is just 4% above the unitarity limit:

$$R^K_{\rm exp} = (1.238 \pm 0.024) \times 10^{-5} \tag{16}$$

This excess can be interpreted as an estimate of the quantity

$$|A_{\rm disp}(2\gamma) + A_{\rm s-d}|^2, \tag{17}$$

where $A_{\rm disp}(2\gamma)$ is the dispersive part of the 2γ contribution, and $A_{\rm s-d}$ is the contribution of the short-distance interaction $(\overline{s}d)(\bar{l}l)$. Such an analysis requires a model for the form factor of the two-photon vertex $K_L \to \gamma^*\gamma^*$ [8]. In principle, access to the real and imaginary parts of the $K_L \to \mu\overline{\mu}$ amplitude is also possible by studying the decay $K_L \to \mu^+\mu^-\gamma$ in the soft-photon region where the bremsstrahlung and Dalitz pair amplitudes for this process interfere [9].

2.4 Decay $K_L \to \pi^+\pi^-e^+e^-$

The decay $K_L \to \pi^+\pi^-e^+e^-$ is calculable in terms of empirical knowledge of the radiative transition $K_L \to \pi^+\pi^-\gamma$. It reveals a remarkable CP-violating, T-odd asymmetry, which is triggered by the small ϵ impurity in the K_L wavefunction [10].

The $K_L \to \pi^+\pi^-\gamma$ amplitude is the sum of a bremsstrahlung component, proportional to the CP-violating parameter η_{+-}, and a direct $M1$ term obtained by a fit to the photon energy spectrum. The e^+e^- pair in $K_L \to \pi^+\pi^-e^+e^-$ is interpreted as an internal conversion of the photon in $K_L \to \pi^+\pi^-\gamma$. The theoretical analysis leads to the prediction

$$\frac{d\Gamma}{d\phi} = \Gamma_1 \cos^2\phi + \Gamma_2 \sin^2\phi + \Gamma_3 \sin\phi\cos\phi \tag{18}$$

where ϕ is the angle between the $\pi^+\pi^-$ and e^+e^- planes. The last term is odd under CP as well as T, and gives rise to an asymmetry

$$A_\phi = \left(\int_0^{\pi/2} + \int_\pi^{3\pi/2} - \int_{\pi/2}^{\pi} - \int_{3\pi/2}^{2\pi}\right) \frac{d\Gamma}{d\phi} d\phi \bigg/ \int_0^{2\pi} \frac{d\Gamma}{d\phi} d\phi \tag{19}$$

The predicted value was 14% [10], and is in excellent agreement with the measured value [11]

$$A_\phi = \begin{cases} 13.7 \pm 1.4 \pm 1.5\% & \text{(KTeV)} \\ 14.2 \pm 3.6\% & \text{(NA48)} \end{cases} \tag{20}$$

In addition, the distribution of the $\pi^+\pi^-$ system in the final state confirms the presence of an s-wave amplitude, corresponding to a mean-square K^0 charge radius

$$\langle R^2 \rangle_{K^0} = \begin{cases} -0.077 \pm 0.014\, fm^2 & \text{(KTeV)} \\ -0.09 \pm 0.02\, fm^2 & \text{(NA48)} \end{cases} \tag{21}$$

in agreement with the theoretical expectation from vector meson dominance: $\langle R^2 \rangle_{K^0} = \frac{1}{2}\left[\frac{1}{m_\phi^2} - \frac{1}{m_\rho^2}\right] = -0.07\, fm^2$.

2.5 Decays $K^+ \to \pi^+ e^+ e^-$ and $K_S \to \pi^0 e^+ e^-$

These decays are determined mainly by the single photon intermediate state. The matrix elements have a similarity to that for the charged current decay $K^+ \to \pi^0 e^+ \nu$, and may be parametrized as

$$\begin{aligned} \mathcal{A}(K^+ \to \pi^0 e^+ \nu) &= \frac{G_F}{\sqrt{2}} \frac{f_+}{\sqrt{2}} \sin\theta_C (k+p)_\alpha \overline{\nu} \gamma^\alpha (1-\gamma_5) e \\ \mathcal{A}(K^+ \to \pi^+ e^+ e^-) &= a_+ \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} f_+ \sin\theta_C (k+p)_\alpha \overline{e} \gamma^\alpha e \\ \mathcal{A}(K_S \to \pi^0 e^+ e^-) &= a_S \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} f_+ \sin\theta_C (k+p)_\alpha \overline{e} \gamma^\alpha e \end{aligned} \tag{22}$$

An early analysis [12] yielded the prediction $a_+ = -0.7$, $a_S = 2.4$. A simple model of $K^+ \to \pi^+ e^+ e^-$ relates the matrix element to the weak two point vertex $K^+ - \pi^+$ and the charge radii of K^+ and π^+ [13]. A similar model was used a long time ago [14] to estimate the decay $K_S \to \pi^0 e^+ e^-$ in terms of the weak vertex $K_2 - \pi^0$ and the charge radius of the K^0 meson. The $K^+ - \pi^+$ and $K_2 - \pi^0$ vertices are given by current algebra and PCAC:

$$\langle \pi^0 | H_w | K_2 \rangle = -\langle \pi^+ | H_w | K^+ \rangle = 2 F_\pi g \tag{23}$$

where g is the coupling constant for $K_1 \to \pi\pi$, and $F_\pi = m_N g_A / g_{NN\pi} \approx 90\, MeV$. With these values, the measured branching ratios of $K^+ \to \pi^+ e^+ e^-$ and $K_S \to \pi^0 l^+ l^-$ are well reproduced.

3 Miscellaneous Remarks

As noted above, the standard model contains gauge couplings $\{g,\, g'\}$, which conserve chirality, and Yukawa couplings $\{y_f\}$ which are proportional to fermion masses and violate chirality. It is the interplay of those couplings that determines the strength of the FCNC interaction responsible for decays like $K^+ \to \pi^+ \nu \overline{\nu}$.

The reality of the Yukawa interaction as a force independent of gauge interactions is revealed if one considers the "gaugeless" limit of the standard model, viz. $g \to 0$ with $v = (\sqrt{2} G_F)^{-1/2}$ fixed. In this limit, studied by Bjorken [15], one has the remarkable consequence that the electron is unstable, with decay width

$$\Gamma(e^- \to \nu_e W^-) = \frac{\sqrt{2} G_F m_e^3}{16\pi} = \frac{y_e^2}{32\pi} m_e = (10.3\, ns)^{-1}. \tag{24}$$

Note that in the limit $g \to 0$, $m_W = gv/2 \to 0$. The electron decays purely by virtue of its Yukawa coupling $y_e = \sqrt{2} m_e / v$, and the massless (longitudinal) W it decays into is nothing but the massless Goldstone boson φ^- of the scalar sector.

In a similar spirit, one can investigate the behaviour of amplitudes in the limit $y_e \to 0$ with v fixed. A remarkable feature that emerges is that the electron chirality is not conserved. This is evident already at the level of QED: the cross section of helicity-flip Compton scattering is

$$\lim_{m_e \to 0} \sigma(\gamma + e_L^- \to \gamma + e_R^-) = 2\pi \frac{\alpha^2}{s} \tag{25}$$

Likewise, helicity-flip bremsstrahlung $e_L^- + \mathcal{N} \to e_R^- + \mathcal{N} + \gamma$ has the characteristic angular distribution [16]

$$d\sigma_{hf} \sim \alpha \left(\frac{m_e}{E} \right)^2 \frac{d\theta^2}{\left(\theta^2 + \frac{m^2}{E^2}\right)^2} \tag{26}$$

which, integrated over angles, gives a finite non-zero result in the limit $m_e \to 0$.

As a further interesting consequence [17] electrons in radiative muon decay $\mu^- \to e^- \overline{\nu}_e \nu_\mu \gamma$ are not purely left-handed in the limit $m_e \to 0$. Despite the $V - A$ structure of the weak interaction, there is a significant probability for electrons in μ-decay to be right-handed. Such right-handed electrons are typically accompanied by hard collinear photons. The contribution of these wrong-helicity electrons to the muon decay width is $\Gamma_R = \frac{\alpha}{4\pi}(G_F^2 m_\mu^5 / 192\pi^3)$.

The above curiosities in the gaugeless limit or in the limit of a massless fermion may be of some relevance when one contemplates the interplay of gauge couplings and Yukawa couplings in electroweak amplitudes.

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