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THE $a_0 - a_2$ PION SCATTERING LENGTH FROM $K^+ \to \pi^+\pi^0\pi^0$ DECAY

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ABSTRACT

I present a new method for determining the pion-pion scattering length combination $a_0 - a_2$ through the analysis of the Dalitz plot of the $K^+ \to \pi^+\pi^0\pi^0$ decay in the region where the invariant mass of the $\pi^0\pi^0$ pair is in the vicinity of the $\pi^+\pi^-$ threshold.

1 Pion-pion scattering lengths

An impressive development of the last few years is the emergence of technologies which allow for very high statistics experiments. The prime example of this trend is offered by BaBar and Belle which together have collected $\sim 10^8$ well

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measured $\mathrm{B}-\bar{\mathrm{B}}$ pairs. High statistics means the possibility of studying subtle effects, such as the one I will discuss, which can lead to a precision measurement of the $\pi\pi$ scattering length combination $a_0 - a_2$, a measurement made possible by the availability in NA48 of $\sim 10^8$ well-analyzed $K^+ \to \pi^+\pi^0\pi^0$ events.

Weinberg's predictions of the S-wave $\pi\pi$ scattering lengths [1], or the modern improved computations, are justly considered as one of the principal results of Chiral Perturbation Theory in its purest form: that of a theory of low energy (soft) pions and their interactions. Pion-pion S-wave scattering lengths are now predicted in Chiral Perturbation Theory with a very small uncertainty [2, 3]:

$$\begin{aligned} a_0 m_{\pi^+} &= 0.220 \pm 0.005 \\ a_2 m_{\pi^+} &= -0.0444 \pm 0.0010 \\ (a_0 - a_2) m_{\pi^+} &= 0.265 \pm 0.004 \end{aligned} \tag{1}$$

Can the accuracy of these predictions be matched by experimental results? It was long recognized [4] that the angular distributions in $K^+ \to \pi^+\pi^- e^+\nu$ are sensitive to the $\pi\pi$ phase shifts, and can be used to obtain informations on the S-wave scattering lengths [5] [6]. The first results by the Geneva-Saclay experiment [7], leading to $a_0 m_{\pi^+} = 0.26 \pm 0.05$, were recently improved by the E865 collaboration at Brookhaven [8] that quotes a result: $a_0 m_{\pi^+} = 0.216 \pm 0.013$ (stat.)± 0.002 (syst.)± 0.002 (theor). Data on K_{e4}, with a large statistics, are currently being analyzed by the NA48 collaboration at CERN.

The K_{e4} decay yields values of the phase shift difference $\delta_0^0 - \delta_1^1$ as a function of the $\pi\pi$ invariant mass $M_{\pi\pi}$ in the range $2m_{\pi^+} < M_{\pi\pi} < M_K - m_{\pi^+}$, but the best data lies in the range > 310 MeV. The extraction of a value for a_0 requires an extrapolation to the threshold region and a substantial theoretical input, whence the interest in alternative methods which permit the determination of the scattering lengths through measurements that are directly sensitive to $\pi\pi$ scattering in the threshold region, $M_{\pi\pi} \sim 2m_{\pi^+}$. An example of this is the decay rate of the pionic atom $\pi^+\pi^-$. This decay is due to the $\pi^+\pi^- \to \pi^0\pi^0$ transition at an energy that essentially coincides with the $\pi^+\pi^-$ threshold, so that its rate is directly proportional to $(a_0 - a_2)^2$. The DIRAC experiment at CERN [9] [10] is now attempting a measurement of the pionic atom decay rate.

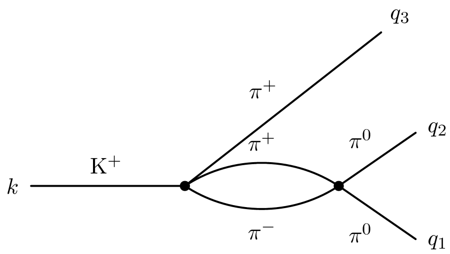

Figure 1: *Contribution of* $\pi^+\pi^- \to \pi^0\pi^0$ *to* $K^+ \to \pi^+\pi^0\pi^0$.

2 A new measurement of $a_0 - a_2$

An entirely new method [11] for measuring the value of $a_0 - a_2$ is based on a subtle modification that $\pi\pi$ re-scattering introduces in the $K^+ \to \pi^+\pi^0\pi^0$ Dalitz plot when the $\pi^0\pi^0$ invariant mass is close to the $\pi^+\pi^-$ threshold. The new method can lead to a measurement of $a_0 - a_2$, and is also sensitive to the sign of this quantity.

The contribution of the $\pi^+\pi^- \to \pi^0\pi^0$ rescattering can be graphically represented by the diagram in fig. 1. The effect of this diagram can be directly computed in chiral perturbation theory, but the result is really independent from this theory, since in the vicinity of the $\pi^+\pi^-$ threshold the result can be directly derived from the unitarity of the **S**-matrix, $\mathbf{S}^\dagger\mathbf{S} = 1$, which determines the absorptive part of the amplitude above the threshold, and the analyticity of the **S**-matrix elements as a function of the external momenta, which determines their analytic continuation from the region above the threshold, where the amplitude of the diagram is absorptive, to the region below the threshold, where the amplitude is dispersive.

If we write the amplitude for the $K^+ \to \pi^+\pi^0\pi^0$ decay as

$$\mathcal{M} = \mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = \mathcal{M}_0 + \mathcal{M}_1 \tag{2}$$

with $\mathcal{M}_0$ the "unperturbed" amplitude, and $\mathcal{M}_1$ the contribution of the re-scattering graph, above the $\pi^+\pi^-$ threshold we obtain an imaginary value,

$$\mathcal{M}_1 = i2\frac{(a_0 - a_2)m_{\pi^+}}{3}\mathcal{M}_{+,\mathrm{thr}}\sqrt{(s_{\pi\pi} - 4m_{\pi^+}^2)/s_{\pi\pi}} \tag{3}$$

which turns into a real value below the threshold,

$$\mathcal{M}_1 = -2\frac{(a_0 - a_2)m_{\pi^+}}{3}\mathcal{M}_{+,\mathrm{thr}}\sqrt{(4m_{\pi^+}^2 - s_{\pi\pi})/s_{\pi\pi}} \tag{4}$$

where $\mathcal{M}_{+,\mathrm{thr}}$ is the amplitude of the $K^+ \to \pi^+\pi^+\pi^-$ decay evaluated at the $\pi^+\pi^-$ threshold. If now we look at the square of the amplitude in eq. (2), below threshold there is an interference term:

$$|\mathcal{M}|^2 = (\mathcal{M}_0)^2 + (\mathcal{M}_1)^2 + 2\mathcal{M}_0\mathcal{M}_1$$

which is absent above the threshold, where

$$|\mathcal{M}|^2 = (\mathcal{M}_0)^2 + |\mathcal{M}_1|^2$$

The interference term is proportional to $a_0 - a_2$. Taking the theoretical pre-

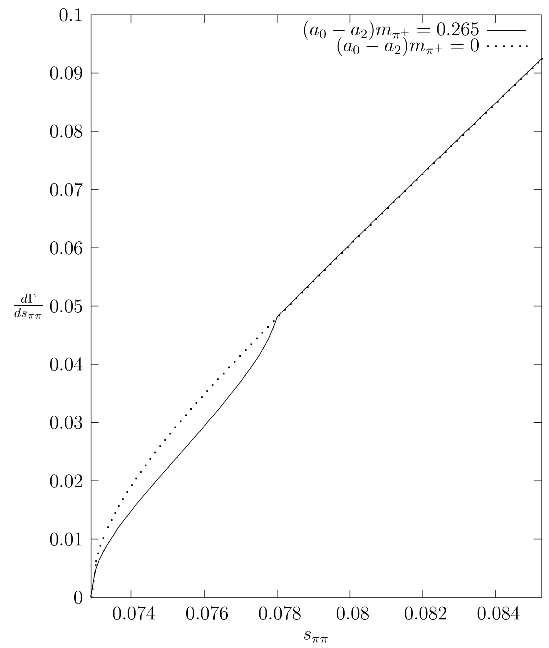

Figure 2: *The $s_{\pi\pi}$ invariant mass distribution with/without the re-scattering correction, in the threshold region, arbitrary units.*

dictions for $a_0 - a_2$, and the PDG values for the $K \to \pi\pi\pi$ decays one obtains the $\pi^0\pi^0$ spectrum in fig. 2, which displays a $\sim$10% deviation below the $\pi^+\pi^-$ threshold. The region below the threshold contains only $\sim$ 3% of the events, but with $\sim 10^8$ $K^+ \to \pi^+\pi^0\pi^0$ events this 10% deviation can be determined with a $\sim$ 1.5% statistical error!

With such a small statistical error, the accuracy of the result will critically depend on a firm control of the systematics. From the theoretical side many aspects require more work. First of all we should include the effects of coulomb corrections, which may become significant close to the $\pi^+\pi^-$ threshold, and of radiative corrections. One would also like to revisit the derivation of the predictions in eq. (1) to take into account the fact that we are working in a region — between the $\pi^0\pi^0$ and the $\pi^+\pi^-$ thresholds — where in a sense isospin invariance is maximally broken. We should finally take into account the fact that both $\mathcal{M}_0$ and $\mathcal{M}_{+,\mathrm{thr}}$ receive small imaginary contributions from other rescattering diagrams [12]; these contributions, although not singular at the $\pi^+\pi^-$ threshold, modify the interference terms with $\mathcal{M}_1$ both above and below the $\pi^+\pi^-$ threshold.

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