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ON EXPECTED VALUE OF CP EFFECTS IN DECAY OF CHARGED KAONS INTO 3 PIONS

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ABSTRACT

The amplitudes of the $K^{\pm} \to 3\pi$ and $K \to 2\pi$ decays are expressed in terms of different combinations of one and the same set of CP-conserving and CP-odd parameters. Extracting the magnitudes of these parameters from the data on $K \to 2\pi$ decays, we estimate an expected CP-odd difference between the values of the slope parameters g^{+} and g^{-} of the energy distributions of "odd" pions in $K^{+} \to \pi^{+}\pi^{+}\pi^{-}$ and $K^{-} \to \pi^{-}\pi^{-}\pi^{+}$ decays.

1 Introduction

The observation of CP effects in $K^{\pm} \to 3\pi$ decays would allow to understand better how the mechanisms of CP violation work.

Now the Collaboration NA48/2 began a search for such effect with accuracy $\delta(\frac{g^{+}-g^{-}}{g^{+}+g^{-}}) \leq 2 \times 10^{-4}$.

Contrary to the case of $K_L \to 2\pi$ decay, where CP violates both in $\Delta S = 2$ and $\Delta S = 1$ transitions, in the $K^\pm \to 3\pi$ decays, only the last (the so-called "direct") CP violation takes place. Experimentally, an existence of the direct CP violation in $K_L \to 2\pi$ decays, predicted by Standard Model (SM) and characterized by the parameter ε', is established: $\varepsilon'/\varepsilon = (1.66 \pm 0.16) \times 10^{-3}$.

What is expected for CP effects in $K^+ \to \pi^\pm\pi^\pm\pi^\mp$ decay? To give an answer, it is necessary to understand the role of the electroweak penguin (EWP) operators in both decays and get rid of the large uncertainties usual for the theoretical calculations. The real scale of these uncertainties is characterized by the following predictions obtained before the above experimental result:

$$\tfrac{\varepsilon'}{\varepsilon} = (17^{+14}_{-10}) \times 10^{-4} \;\; ^{1)}, \qquad \tfrac{\varepsilon'}{\varepsilon} = (1.5 \div 31.6) \times 10^{-4} \;\; ^{2)}.$$

To avoid the uncertainties arising in the theoretical calculation of the ingredients of the theory, we use the following procedure. We express the amplitudes of $K \to 2\pi$ and $K^\pm \to 3\pi$ decays in terms of one and the same set of parameters, and calculating $g^+ - g^-$, we use the magnitudes of these parameters extracted from data on $K \to 2\pi$ decays.

2 The scheme of calculation

A theory of $\Delta S = 1$ non-leptonic decays is based on the effective Lagrangian [3]

$$L(\Delta S = 1) = \sqrt{2} G_{\rm F} \sin\theta_{\rm C} \cos\theta_{\rm C} \sum_i c_i O_i \,, \tag{1}$$

where O_{1-6} are the 4-quark operators represented in [3] and O_{7-8} are the 4-quark operators generated by electroweak penguin (EWP) diagrams

$$O_7 = \frac{3}{2}\bar{s}\gamma_\mu(1+\gamma_5)d(\sum_{q=u,d,s} e_q \bar{q}\gamma_\mu(1-\gamma_5)q) \qquad (\Delta I = 1/2, 3/2); \tag{2}$$

$$O_8 = -12 \sum_{q=u,d,s} e_q(\bar{s}_L q_R)(\bar{q}_R d_L), \quad e_q = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) \quad (\Delta I = 1/2, 3/2). \tag{3}$$

Representing $M(K \to 2\pi)$ in the form

$$M(K_1^0 \to \pi^+\pi^-) = A_0 e^{i\delta_0} - A_2 e^{i\delta_2}, \tag{4}$$

$$M(K_1^0 \to \pi^0\pi^0) = A_0 e^{i\delta_0} + 2A_2 e^{i\delta_2}, \tag{5}$$

$$M(K^+ \to \pi^+\pi^0) = -\frac{3}{2} A_2 e^{i\delta_2}, \tag{6}$$

and using the relations expressing the diquark operators in terms of pseudoscalar fields [4] we find

$$A_0 = G_{\rm F} F_\pi \sin\theta_{\rm C}\cos\theta_{\rm C}\frac{m_K^2 - m_\pi^2}{\sqrt{2}}[c_1 - c_2 - c_3 + \frac{32}{9}\beta({\rm Re}\tilde{c}_5 + i{\rm Im}\tilde{c}_5)]; \qquad (7)$$

$$A_2 = G_{\rm F} F_\pi \sin\theta_{\rm C}\cos\theta_{\rm C}\frac{m_K^2 - m_\pi^2}{\sqrt{2}} \cdot [c_4 + i\frac{2}{3}\beta\Lambda^2{\rm Im}\tilde{c}_7(m_K^2 - m_\pi^2)^{-1}] \qquad (8)$$

where

$$\tilde{c}_5 = c_5 + \frac{3}{16}c_6, \quad \tilde{c}_7 = c_7 + 3c_8, \quad \beta = \frac{2m_\pi^4}{\Lambda^2(m_u+m_d)^2}, \quad \Lambda \approx 1 \text{ GeV}$$

The contributions from $\tilde{c}_7 O_7$ into ${\rm Re}A_0$ and ${\rm Im}A_0$ are small because $\tilde{c}_7/\tilde{c}_5$ is proportional to the electromagnetic constant α and we neglected these corrections. From data on widths of $K \to 2\pi$ decays we obtain

$$c_4 = 0.328; \quad c_1 - c_2 - c_3 + \frac{32}{9}\beta{\rm Re}\tilde{c}_5 = -10.13. \qquad (9)$$

At $c_1 - c_2 - c_3 = -2.89$ [3, 5] we obtain

$$\frac{32}{9}\beta{\rm Re}\tilde{c}_5 = -7.24. \qquad (10)$$

Using the general relation

$$\varepsilon' = ie^{i(\delta_2-\delta_0)}\left[-\frac{{\rm Im}A_0}{{\rm Re}A_0} + \frac{{\rm Im}A_2}{{\rm Re}A_2}\right]\cdot|\frac{A_2}{A_0}| \qquad (11)$$

and the experimental value $|\varepsilon'| = (3.78 \pm 0.38) \times 10^{-6}$, we come to the relation

$$-\frac{{\rm Im}\tilde{c}_5}{{\rm Re}\tilde{c}_5}\left(1 - \Omega_{\eta,\eta'} + 24.36\frac{{\rm Im}\tilde{c}_7}{{\rm Im}\tilde{c}_5}\right) = (1.63 \pm 0.16) \times 10^{-4}, \qquad (12)$$

where $\Omega_{\eta,\eta'}$ takes into account the effects of $K^0 \to \pi^0\eta(\eta') \to \pi^0\pi^0$ transitions.

Introducing the notations

$$-\frac{{\rm Im}\tilde{c}_5}{{\rm Re}\tilde{c}_5} = x\frac{{\rm Im}\lambda_t}{s_1}, \qquad \frac{24.36}{1-\Omega_{\eta,\eta'}}\cdot\frac{{\rm Im}\tilde{c}_7}{{\rm Im}\tilde{c}_5} = -y \qquad (13)$$

and using [6]

$$({\rm Im}\lambda_t)/s_1 \approx s_2 s_3 \sin\delta = \frac{(1.2 \pm 0.2)\times 10^{-4}}{0.223}, \qquad (14)$$

we can write Eq.(11) for $\Omega_{\eta,\eta'} = 0.25 \pm 0.08$ in the form

$$x(1-y) = 0.40 \times (1 \pm 0.22). \tag{15}$$

In the last two equations s_i and δ are the parameters of CKM matrix. The Eq.(15) depends on the variables x and y representing the contribution of QCD penguin and relative contribution of EWP, respectively. To move farther, we are enforced to apply to existing theoretical estimates of one of these variables.

In terms of notations in [7, 8, 9]

$$y = \frac{\Pi_2}{\omega}/\Pi_0(1-\Omega_{\eta,\eta'}). \tag{16}$$

According to [7], $y \approx 0.3$ and hence $x = 0.57 \pm 0.12$. But $\varepsilon'/\varepsilon = 2.2 \cdot 10^{-3}$ or by 30% is larger than the experimental value . In [9], the central value of y is $y \approx 0.5$ and, consequently, $x = 0.80 \pm 0.18$. From [10] $x = 0.71 \pm 0.27$; from [11] $x = 1.4 \pm 0.28$; from [12] $x = 2$; from [13] $x = 2.8$; from [14] $x = 5.5$. Such difference of the theoretical estimates of x makes very desirable an investigation of CP-effects in $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ decays, where, contrary to $K_L \to 2\pi$ decays, the EWP contributions increase CP effects .

3 Decay $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$

The slope parameters g^+ and g^- are defined by the relation

$$|M(K^\pm(k) \to \pi^\pm(p_1)\pi^\pm(p_2)\pi^\mp(p_3))|^2 \sim [1 + g^\pm Y + ...], \qquad Y = (s_3 - s_0)/m_\pi^2, \tag{17}$$

where $s_3 = (k-p_3)^2$ and $s_0 = m_K^2/3 + m_\pi^2$. Our calculations give [15]:

$$R_g \equiv \frac{g^+ - g^-}{g^+ + g^-} = \frac{a(b_{KM} - a_{KM})}{1 + ab} \tag{18}$$

where a_{KM} and b_{KM} are induced by the Kobayashi-Maskawa phase:

$$a_{KM} = \left[\frac{32}{9}\beta \mathrm{Im}\tilde{c}_5 + 4\beta \mathrm{Im}\tilde{c}_7 \left(\frac{3\Lambda^2}{2m_K^2} + 2\right)\right]/c_0, \tag{19}$$

$$b_{KM} = \left[\frac{32}{9}\beta \mathrm{Im}\tilde{c}_5 + 8\beta \mathrm{Im}\tilde{c}_7\right]/(c_0 + 9c_4). \tag{20}$$

$$c_0 = c_1 - c_2 - c_3 - c_4 + \frac{32}{9}\beta \mathrm{Re}\tilde{c}_5 = -10.46. \tag{21}$$

The quantities a and b are the CP-even imaginary parts produced by rescattering of the pions (see [15]). To leading order $a = 0.121, \quad b = 0.714$. At the fixed above numerical values of the parameters and $\Omega_{\eta,\eta'} = 0.25$ we obtain to leading p^2 approximation

$$R_g = 0.030 \frac{\mathrm{Im}\tilde{c}_5}{\mathrm{Re}\tilde{c}_5}(1 - 14.9 \frac{\mathrm{Im}\tilde{c}_7}{\mathrm{Im}\tilde{c}_5}) = = -(2.44 \pm 0.44) \times 10^{-5} x \left(1 - \frac{0.13 \pm 0.03}{x}\right). \tag{22}$$

The higher-order corrections lead to 20% larger result

$$\begin{aligned} (R_g)_{corr} &= 0.039 \tfrac{\mathrm{Im}\tilde{c}_5}{\mathrm{Re}\tilde{c}_5}\left(1 - 11.95 \tfrac{\mathrm{Im}\tilde{c}_7}{\mathrm{Im}\tilde{c}_5}\right) = \\ &= -(3.0 \pm 0.5) \times 10^{-5} x \left(1 - \tfrac{0.11 \pm 0.025}{x}\right). \end{aligned} \tag{23}$$

Therefore, the corrections to the result obtained in the framework of conventional chiral theory to the leading approximation are not so large, as it was declared in [16].

4 Conclusion

From Eqs.(11), (15) and (23), it follows that EWP contributions diminish ε'/ε and increase R_g. The EWP corrections cancel one half of the QCD penguin contribution into ε'/ε at $x = 0.8$ and cancel 80% of QCD penguin contribution at $x = 2$. In both cases ε'/ε is the same.

In the case of $K^{\pm} \to 3\pi$ decays, the *direct* influence of EWP corrections themselves on CP effects is not so crucial as in $K_L \to 2\pi$ decays. But if a cancellation between the contribution of QCD and electroweak penguins in ε'/ε is large, the factor x in Eq.(23) is also larger than 1. So, for $x = 2$, the predicted R_g must be 2.5 times larger than at $x = 0.8$.

Therefore, measuring R_g, one obtains a possibility to determine the true relation between QCD and EWP contributions into CP violation in kaon decays.

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