Frascati Physics Series Vol. XXXVI (2004), pp.
DAΦNE 2004: Physics at meson factories – Frascati, June 7-11, 2004
Invited Review Talk in Plenary Session

THE THEORY OF CP-VIOLATION – IN AS MUCH OF A NUTSHELL AS WILL FIT ON 8 PAGES

Patricia Ball
IPPP, University of Durham, Durham DH1 3LE, UK

Abstract

Did you know that CP violation is intrinsically linked to the scalar sector of the Standard Model and its extensions? If yes, you need read no further — if no, you may turn over the title page and start reading now.

It is difficult to do justice to a topic as vast and complex as CP-violation in a 30-minutes conference talk — and even more so in a 8-pages contribution to the proceedings. Well, practitioners in teaching & learning do know that nothing is impossible, and so I shall try to stand up to the challenge and concentrate on a less common viewpoint on the subject than is to be found in most textbooks[1], in the hope the reader may find it as entertaining as enlightening.

[1]Everything you ever wanted to know about CP-violation (and more) can be found in Ref. [1)].

It is actually very surprising that CP should be violated at all. Many gauge-theories preserve C(harge conjugation symmetry) and P(arity) naturally & separately, the probably most prominent ones being (massless) QED and QCD. Even more contrived theories, especially designed to violate parity, like the chiral gauge-theory

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_L i\sigma D \psi_L, \tag{1}$$

where only the left-handed (Weyl) fermions ψ_L interact with gauge-bosons[2], are still invariant under CP transformations, which implies that *CP is a natural symmetry of massless gauge theories.* So where does CP-violation come in? The catch is that, as the mass term

$$m\bar{\psi}\psi \equiv m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \tag{2}$$

violates gauge-symmetry, it is forbidden in $\mathcal{L}$ and hence left-handed fermions must be massless — at obvious variance with experiment. If the theory (1) is to serve as model for parity-violating interactions, it has to be amended in some ingenious way as to give mass to the fermions (and gauge-bosons), but at the same time preserve gauge-invariance.

In the Standard Model (SM), this objective is being achieved by adding a scalar (Higgs) sector which generates a nontrivial ground-state (vacuum) of the theory. In general, this vacuum-state is less symmetric than the full theory — a phenomenon usually referred to as spontaneous symmetry breaking (SSB), which in the case of gauge-theories is dubbed Higgs mechanism and allows gauge-bosons (and chiral fermions) to become massive. The Lagrangian of the SM can be written as

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{gauge}}(\psi_L, \psi_R, W, \phi) + \mathcal{L}_{\mathrm{Higgs}}(\phi) + \mathcal{L}_{\mathrm{Yukawa}}(\psi_L, \psi_R, \phi), \tag{3}$$

where the first term on the right-hand side, the equivalent of (1), contains the kinetic terms of the fields involved, i.e. left- and right-handed fermions ψ_L and ψ_R, gauge-bosons W and scalar (Higgs) fields ϕ, as well as their gauge-interactions. The second term is the potential felt by the scalar fields and is responsible for some of them to acquire a nonzero vacuum expectation value

[2] Whereas their right-handed counterparts are "sterile" and hence omitted from the theory.

(VEV) which gives rise to SSB. The third term describes interactions between fermionic and scalar fields, which after SSB induce fermion mass terms. In the SM, $\mathcal{L}_{\rm Higgs}$ is automatically CP-invariant[3], which leaves us with $\mathcal{L}_{\rm Yukawa}$ as the only possible source of CP-violation in the SM[4]. It is given by

$$\mathcal{L}_{\rm Yukawa} = -\lambda_{ij}^d \bar{Q}_L^i \cdot \Phi d_R^j - (\lambda_{ij}^d)^* \bar{d}_R^j \Phi^\dagger \cdot Q_L^i + \dots, \tag{4}$$

where the indices i, j run over the three generations and the dots denote terms with up-type quarks. Q_L^i denotes the $\mathrm{SU_L}(2)$ quark doublet (u_L^i, d_L^i) and Φ the $\mathrm{SU_L}(2)$ Higgs doublet (ϕ^+, ϕ^0). The second term on the right-hand side of (4) is the complex conjugate of the first one — as required by the condition that the Lagrangian be a Hermitian operator.

So how does $\mathcal{L}_{\rm Yukawa}$ transform under CP? The P-transformation exchanges L (left) and R (right) indices, the C-transformation exchanges particles (d etc.) and antiparticles ($\bar{d}$ etc.), so that

$$CP: \quad \bar{Q}_L^i \cdot \Phi d_R^j \to \bar{d}_R^j \Phi^\dagger \cdot Q_L^i. \tag{5}$$

Comparing with (4), we see that $\mathcal{L}_{\rm Yukawa}$ is CP-invariant if $\lambda \equiv \lambda^*$. Hence, a necessary (but not sufficient) condition for CP-violation is that the Yukawa couplings $\lambda^{u,d}$ are complex.

What does all that actually mean? Well, one conclusion is that *CP-violation happens in the scalar sector* — at least in the SM. What about extensions? The statement stays evidently true for "simple" extensions of the SM with just an enlarged gauge- and scalar-field content (e.g. two Higgs-doublet model), and it also applies to theories where CP is not violated explicitly by complex couplings, but by spontaneous symmetry breaking — which by definition is related to the scalar sector. What about supersymmetry? Again, CP is conserved in theories with unbroken SUSY, for the same reasons as above, but complex couplings occur after SUSY-breaking. Another conclusion is that studying CP-violation means probing the scalar sector — which is also one of the main objectives of the Tevatron and the LHC. In this sense the measurement of CP-asymmetries in K and B decays is complementary to the direct searches for Higgs *et al.* at high-energy colliders.

---

[3]The reason being that there is only one Higgs-doublet; CP-violation in $\mathcal{L}_{\rm Higgs}$ can occur, however, in models with more than one Higgs-doublet.

[4]Note that the QCD θ-term $\theta_{\rm QCD} g_s^2/(64\pi^2)\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$ can be set to 0 if all quarks are massless.

What about CP-violation in the SM? Well, after SSB the Yukawa couplings $\lambda^{u,d}$ induce 3×3 mass matrices for u and d-type quarks which are eigenstates under weak interactions. If the theory is to be expressed in terms of states with definite mass, these matrices have to be diagonalized. The resulting transformation from the basis of weak eigenstates to that of mass eigenstates,

$$u_i^{(\mathrm{weak})} = U_{ij}^{(u)} u_j^{(\mathrm{mass})}, \qquad d_i^{(\mathrm{weak})} = U_{ij}^{(d)} d_j^{(\mathrm{mass})}, \tag{6}$$

has no effect on neutral interactions[5], $\bar{u}_i^{(\mathrm{weak})} u_i^{(\mathrm{weak})} \equiv \bar{u}_i^{(\mathrm{mass})} u_i^{(\mathrm{mass})}$, but profoundly changes charged interactions:

$$\bar{u}_i^{(\mathrm{weak})} d_i^{(\mathrm{weak})} \to \bar{u}_i^{(\mathrm{mass})} (U^{(u)})^\dagger U^{(d)} d_i^{(\mathrm{mass})}. \tag{7}$$

The matrix $V \equiv (U^{(u)})^\dagger U^{(d)}$ describes the strength of d-type quarks decaying into u-type quarks and is nothing else but the well-known CKM matrix. As $U^{(u,d)}$ just rotate the quark basis, they are unitary, and so is V. Any 3×3 unitary matrix can be parametrised in terms of three angles (the familiar Euler angles of three-dimensional rotations) and six complex phases. In the present case, however, not all six phases are physical: five of them can be "rotated away" by redefining the phases of the quark fields — which leaves three angles and one phase to describe the CKM matrix V. It is this complex phase that is the *one and only source of CP-violation in the SM.*

The fact that V is unitary allows one to express the conditions for CP-violation in the SM in an intuitively appealing form: unitarity means

$$\sum_j V_{ij} V_{kj}^* = \delta_{ik}, \tag{8}$$

which, for $i \neq k$, implies three complex numbers to add up to zero. This relation can be represented by a triangle in the complex plane, as shown in the left half of Fig. 1. For three generations, there are six of these triangles in total. This statement is true for arbitrary unitary matrices; the CKM matrix with only one complex phase (instead of six in the general case) is distinguished by the fact that all these six triangles have the same area, which consequently is a measure of the strength of CP-violation in the SM. From an experimental point of view, four of the triangles are rather difficult to explore: one side is much

[5]That is: there are no tree-level flavour-changing neutral interactions in the SM. Such interactions (e.g. $b \to s$) show up only at loop-level.

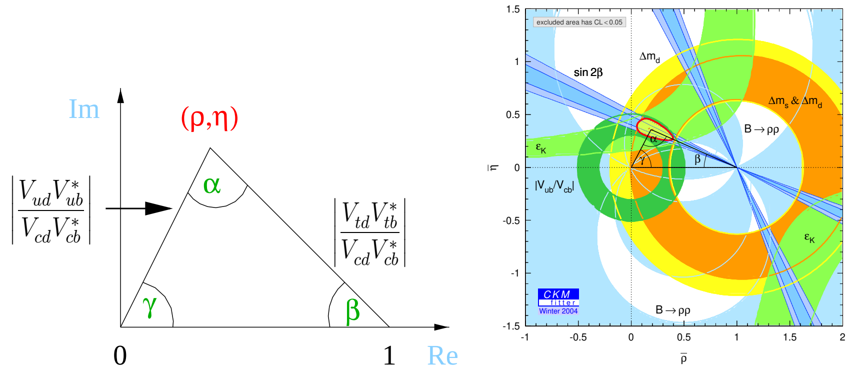

Figure 1: *Left: the B_d unitarity triangle (UT). The apex is labelled (ρ, η), which refers to the Wolfenstein parametrisation of the CKM-matrix. Right: the present (early 2004) experimental status of the UT* [2]*.*

smaller than the others, which makes it difficult to measure the area (or angles) of these triangles with sufficient precision. The two remaining triangles, with $i \in \{d, s\}$ and $k = b$, have sides of comparable length, so that all their sides and angles are, in principle, accessible in experiment: the *bd* triangle is presently being studied at the B factories Babar and Belle and its current experimental status is shown in Fig. 1, right. The various constraints depicted in this figure are discussed in other contributions to these proceedings. The *bs* triangle will be the subject of experimental scrutiny at the LHC. The objective of all these studies is to overconstrain the triangles by measuring their sides and angles from various channels and possibly refute the SM picture of CP-violation. Figure 1 shows that significant discrepancies have yet to be found.

The experimental determination of the sides and angles of the UT is nothing less than trivial and will be the subject of other contributions to these proceedings. Rather than embarking on a discussion of the respective merits and shortcomings of various methods aiming to master the all-important (and, in general, yet unmastered) nonperturbative QCD effects in K and B decays, I would like to spend the remaining three pages of this note on a discussion of the bigger picture in which to embed any non-standard results on CP-violation.

So what are the alternatives to the SM picture of CP-violation? I mentioned a few of them already; a more complete list includes

- complex couplings in the Higgs potential (e.g. multi Higgs-doublet models);
- complex couplings in the effective low-energy Lagrangian obtained from a fundamental theory by SSB (e.g. soft SUSY-breaking terms);
- CP-violation from spontaneous symmetry breaking.

The latter scenario is rather attractive from the theorists' point of view as it relieves us from the task of coming up with clever explanations for where the complex couplings come from — other than the standard excuse that they are there because there is nothing to forbid them. If CP-violation is the result of SSB, the underlying fundamental theory must be manifestly CP-invariant, which requires the addition of (at least) an $\mathrm{SU_R}(2)$ gauge-group to the SM. This type of theories goes by the name of left-right symmetric models [3] and has been studied rather extensively. CP-violation occurs as consequence of the SSB $SU_\mathrm{L}(2)\times SU_\mathrm{R}(2)\times U(1)\to SU_\mathrm{L}(2)\times U(1)$. Like in the SM, fermion masses are generated from Yukawa interactions, but $\mathcal{L}_\mathrm{Yukawa}$ is now a bit more involved and includes a Higgs-bidoublet Φ, that is a doublet under both $\mathrm{SU_L}(2)$ and $\mathrm{SU_R}(2)$. CP-violation occurs as the VEV of Φ can carry a complex phase:

$$\langle\Phi\rangle = \left(\begin{array}{cc} v & 0 \\ 0 & w\,e^{i\alpha}\end{array}\right). \tag{9}$$

The phenomenology of this model has been recently studied in Ref. [4], for the quark sector; the main prediction, a small value of $\sin 2\beta$, one of the angles of the *bd* UT, has not been confirmed by experiment. The other main prediction is large CP-violation in B_s decays, which will be tested at the LHC. One major problem of left-right symmetric models is the generically large value of the electric dipole moment of the neutron, which is a two-loop electroweak effect in the SM and hence exceedingly small, but occurs at one-loop level and is dangerously large in left-right symmetric models (and other models with additional sources of flavour-violation, including SUSY). At present public opinion is rather in disfavour of left-right models, but it is to be hoped that their more attractive features, in particular the possibility of spontaneous CP-violation, will eventually lead to their revival in an up-to-date form.

Figure 2: *One rather weighty consequence of CP-violation.*

The last point I would like to stress in this note is the truly cosmic implication of CP-violation: as Sakharov has shown in 1967 [5], the fact that the Universe is dominated by matter, and antimatter suspiciously absent, can only be explained if

1. fundamental interactions violate baryon number conservation;
2. the Universe has undergone non-equilibrium processes (phase-transitions) in its youth;
3. there is CP-violation, which allows Nature to distinguish baryons from antibaryons.

Do we understand the origin of the cosmic matter-antimatter asymmetry? Well, not *really*. Sakharov's conditions give us the minimum ingredients, but don't tell us the recipe to use for cooking up the asymmetry. Ever since 1967 creative *maîtres d'* have come up with ingenious compositions (e.g. GUT baryogenesis, leptogenesis, electroweak baryogenesis), but none of them seems to get it quite right. One result, however, does have emerged: CP-violation as observed in weak interactions is not strong enough to explain the scale of the observed asymmetry — which leaves us with the exciting certainty that new physics must be out there, longing to be discovered.

Let me conclude this *tour de force* by summarizing the messages I want to convey to you:

- CP-violation occurs in the scalar sector of the SM and its extensions;
- in the SM, all CP-violation is related to one single complex phase in the CKM-matrix V;
- the fact that V is unitary and complex allows a simple visualisation of CP-violation in the SM: the unitarity triangle;
- CP-violation is a phenomenon that does not only occur in the subatomic regime, but has profound consequences on the world we live in and is at the heart of the matter-antimatter asymmetry of the Universe.

Acknowledgments

I would like to thank the organisers of the workshop for providing a pleasant and stimulating atmosphere. I also thank J.M. Frère who first introduced me to the marvels of CP-violation.

References

1. G.C. Branco, L. Lavoura and J.P. Silva, International Series of Monographs on Physics, No. 103, Oxford University Press (1999); I. Bigi and A.I. Sanda, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. **9** (2000) 1.
2. Cf. the CKMfitter homepage `http://ckmfitter.in2p3.fr/`.
3. R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975) 566; Phys. Rev. D **11** (1975) 2558; G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12** (1975) 1502.
4. It is difficult to withstand the temptation of self-citations, so here they are: P. Ball, J.M. Frere and J. Matias, Nucl. Phys. B **572** (2000) 3 [arXiv:hep-ph/9910211]; P. Ball and R. Fleischer, Phys. Lett. B **475** (2000) 111 [arXiv:hep-ph/9912319]; P. Ball, arXiv:hep-ph/0004245.
5. A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967) 24].