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SPATIAL CORRELATION FOR LASER BEAM QUALITY EVALUATION

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Abstract

Low emittance beams are required for high brightness beam applications. Contributions to emittance degradations come from electromagnetic fields' non-linearity which can be reduced using a transversally and longitudinally uniform beam. Concerning the transverse analysis of a beam, statistical concept as mean and standard deviation are usually used. They describe non-uniformity of a beam without describing how non-uniformity is distributed: spatial correlation, developed in early 1960s for geostatistics studies, can be used at this purpose. The paper describes the meaning of spatial correlation applying it to few simple examples. Finally the concept is applied to the analysis of real images of a laser or of an electron beam.

1. Introduction

A high brightness beam is required in a lot of fields as for example free electron laser [Ref.1]. The state of the art points out that the best way to produce such a beam is using a photo-injector [Ref.2]. It is a device where a cathode illuminated by a laser generates electrons by photo-emission; the cathode is embedded in a radiofrequency (RF) cavity followed by a solenoid and by a booster which accelerates the beam.

High brightness means a high current and a low emittance beam [Ref.3]. Concerning the emittance its degradation is due to RF fields, space charge. Theoretical studies [Ref.4] demonstrated that a proper choice of a magnetic solenoid, placed outside the RF cavity, can completely compensate the emittance degradation due to longitudinal correlation of space charge and RF fields. On the contrary contributions due to non linearities of these fields cannot be corrected.

For these reasons a transverse and longitudinal uniform beam has been chosen world wide for high brightness beam applications to reduce non linearities. Such a choice means a laser longitudinally and transversally uniform where, because of the natural longitudinal and transverse Gaussian profile, sophisticated manipulations of the laser intensity [Ref.5] are applied before it hits the cathode surface.

Beam's non uniformity is anyway possible due to non perfect laser uniformity and different quantum efficiency of the cathode surface; so the evaluation of the laser and the extracted beam quality is indeed very important.

In this paper the concept of spatial correlation for transverse beam quality studies is analyzed.

2. Spatial correlation

Concepts such as mean, variance and standard deviation can be used to evaluate uniformity of a set of data distributed on a surface, as in the case of the transverse spot of an electron beam or of the laser itself.

The mean describes the central value of the data. In the case of a laser or of a beam cross section analysis, a matrix of pixel of certain intensity is given (each pixel representing the electrons or photons charge). For a matrix of elements the mean is obviously calculated as:

$$< a > = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{ij} \qquad (1)$$

where N and M are the matrix dimensions, $T=NM$ is the number of pixel involved, and a_{ij} the matrix element representing the generic sample, that is the pixel intensity.

The variance represents the distance from the central value, that is the spread, and for a 2D matrix it is calculated as follows:

$$\mathrm{var}(a) = \frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M}(a_{ij} - \langle a \rangle)^2 \qquad (2)$$

The obtained quantity is always positive so that the standard deviation can be defined as:

$$\sigma_a = \sqrt{\mathrm{var}(a)} \qquad (3)$$

which is a quantity whose dimensions are comparable to the mean. The argument $(a_{ij}-\langle a\rangle)$ defines a new matrix where every element represents the distance from the mean and $(a_{ij}-\langle a\rangle)^2$ is the variance matrix. It is obtained as a distance squared so that bigger differences are emphasized respect to the smaller ones.

For a perfectly uniformly charged beam cross section, normalized to the higher sample, $\langle a\rangle=1$ and $\sigma_a=0$. Of course more the cross section is non-uniform more the mean and the standard deviation will be far from the ideal values. The above parameters (but one could extend the analysis using other statistical parameters), describe non-uniformity without describing the way non-uniformity is distributed. It has been shown [Ref.6] and [Ref. 7] the importance of the distribution of the non-uniformity because it can give different results concerning the emittance degradation.

Spatial correlation describes such a property. The concept comes from spatial statistics, a subject developed to solve problems related to geo-statistical studies during early 1960s [Ref.8] and [Ref.9]; the problem was to predict the ore grades from a limited number of peripheral samples within an area to be mined. Anyway it can be applied to any kind of subject such as population density, rainfall, temperature that is where a sample value is expected to be affected by its position and its relationships with its neighbours.

It is necessary to introduce the covariance for a matrix point to define the spatial correlation. The quantity covariance answers the question whether a sample and its neighbour are at the same time different or not from the mean and it's defined as:

$$\mathrm{cov}(a,h)=\frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M}(a_{ij}-<a>)\cdot(a_{ijh}-<a>) \tag{4}$$

where a_{ijh} is the mean of the samples localized around the main sample a_{ij}. The argument $(a_{ij}-<a>)(a_{ijh}-<a>)$ is called the covariance matrix.

The samples can be taken in different ways depending also from the distance h from a_{ij} as represented in Figure 1:

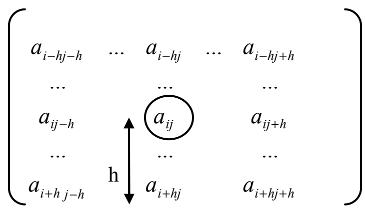

Figure 1.The a_{ij} is the generic sample whose variance is compared with the other samples' variance at a certain distance h.

As can be easly seen it results:

$$a_{ijh}=\frac{1}{(2h+1)^2-1}\left[\sum_{l=-h}^{h}\sum_{m=-h}^{h}a_{i+l\,j+m}-a_{ij}\right] \tag{5}$$

which is the mean of the samples around a_{ij}.

The distance h and the matrix dimensions N, M define the resolution of the spatial autocorrelation investigation: thus to make the spatial correlation for different matrix comparable, the ratio h/N or h/M has to be chosen about the same value.

The index Λ, which describes the spatial correlation, can be defined as the the covariance normalized to the standard deviation σ_a squared:

$$\Lambda(a,h)=\frac{\mathrm{cov}(a,h)}{\sigma_a^2} \tag{6}$$

which is a quantity whose value is between *-1* and *1*. The minus sign simply means most samples are lower than the mean.

The spatial correlation meaning is shown in the following examples. Let's consider the matrix represented in Figure 2:

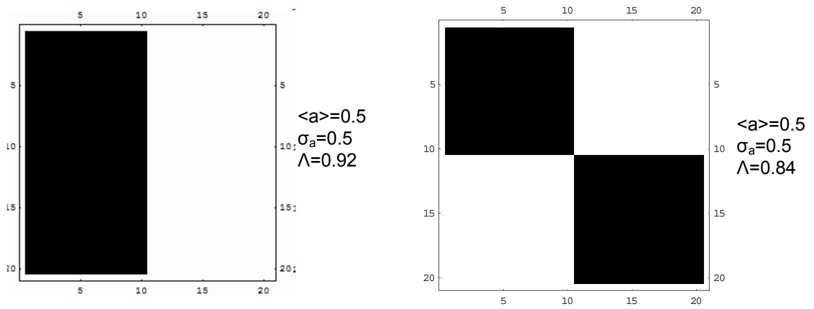

Figure 2.Two different samples distribution giving the same mean and standard deviation but different spatial correlation (*N/h=20*).

The results underline that, given the same mean and the same standard deviation, the spatial correlation maybe different: the matrix on the left hand side has a unique big spot whilst the matrix on the right hand side has two distributed spots of intensity, resulting in more distributed spots; in the first case it gives $\Lambda=0.92$ whilst in the second case, as expected, the spatial correlation is smaller ($\Lambda=0.84$).

It's worth noting that, as represented in Figure 3, the mean can be enhanced keeping the same spots distribution: in this case spatial correlation remains unchanged no matter of the intensity of the distribution.

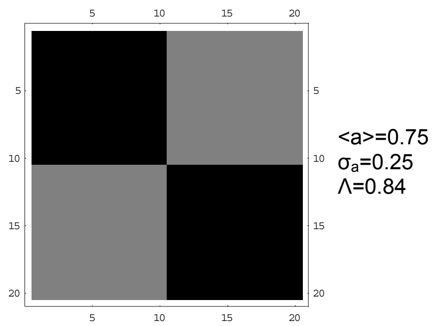

Figure 3.The media is increased, standard deviation decreased whilst the spatial correlation remains unchanged, compared to Figure 2 right hand side (*N/h=20*).

The calculation of the Λ for a uniform distribution of samples with a hole of intensity in the center and a hole of reversed intensity in the center are compared in Figure 4. As expected the mean changes whilst the standard deviation, which takes account of the contrast, doesn't change. The spatial correlation doesn't change either since it describes how spots are distributed.

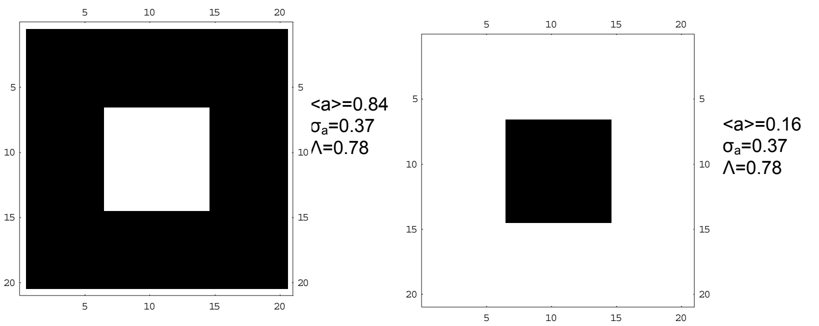

Figure 4.Examples of a uniform distribution with a hole of reverse intensity in the center (*N/h=20*).

Spatial correlation anyway decreases as the spots of intensity become more random. This is shown in Figure 5 where on the right hand side is depicted a completely random distribution of samples.

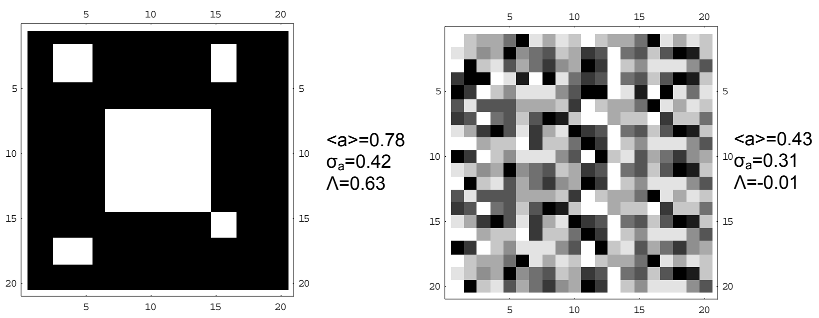

Figure 5.Matrix with spots of intensity and a random distribution of spots. The Λ decreases as the distribution becomes more random (*N/h=20*).

3. Application of the spatial correlation concept to beam quality studies

Concerning the evaluation of the beam quality, it is clear from the previous examples that a well-behaving beam has a high mean and a low standard deviation while the spatial correlation has to be as low as possible. This property has been verified studying the effects of beam charge in-homogeneities on the emittance[Ref.7].

The charge distribution extracted from the cathode has been modeled as a sine and cosine function having a frequency *n* and a charge intensity *δ*. For there are no particular differences between the sine and cosine case, the latter will be presented in details here. Figure 6 shows the matrix representing a non-uniform beam as the frequency *n* increases. The generic matrix element is represented by the following function:

$$\rho(i,j) = \rho_0(1+\delta\cos k_n i)(1+\delta\cos k_n j) \quad (7)$$

where $k_n = 2\pi n/r_p$ and r_p is the beam radius.

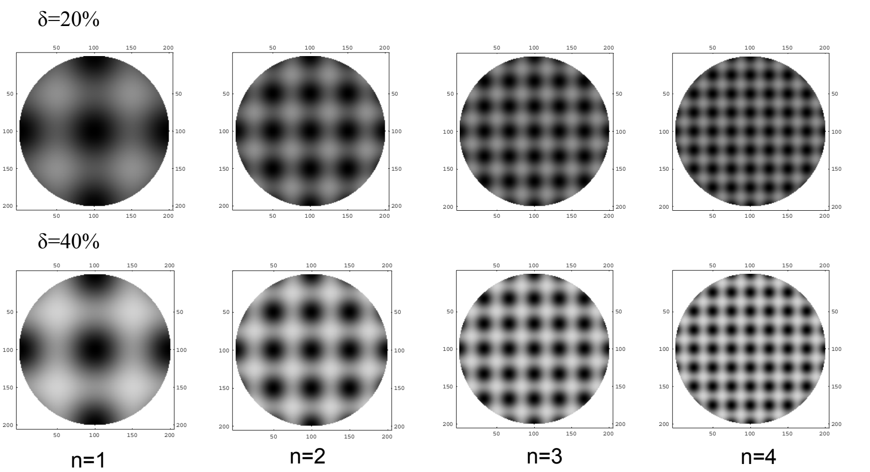

Figure 6.Matrix representation of eq.8 showing non-uniform distribution versus *n* and for different *δ* (*δ=40%* and *δ=20%*). In this case $\rho_0=1$, $r_p=100$.

Such distributions have been analyzed, concerning the emittance degradation with the Parmela code where the accelerator machine set up is the one used for the SPARC project [Ref.10]. The emittance degradation as a function of *n* and for different *δ* is represented on the left hand side of Figure 7.

The right hand side represents the emittance degradation versus the calculated index Λ for different σ_a being the standard deviation a measure of δ. As explained earlier Λ doesn't change increasing or decreasing the intensity δ, anyway it changes as the frequency n increases. It's interesting to note also that all the distributions with the same δ have the same mean and standard deviation (σ_a=0.21 for a δ=40%, σ_a=0.14 for a δ=20%) underling the spatial correlation is the only parameter able to distinguish different distributions.

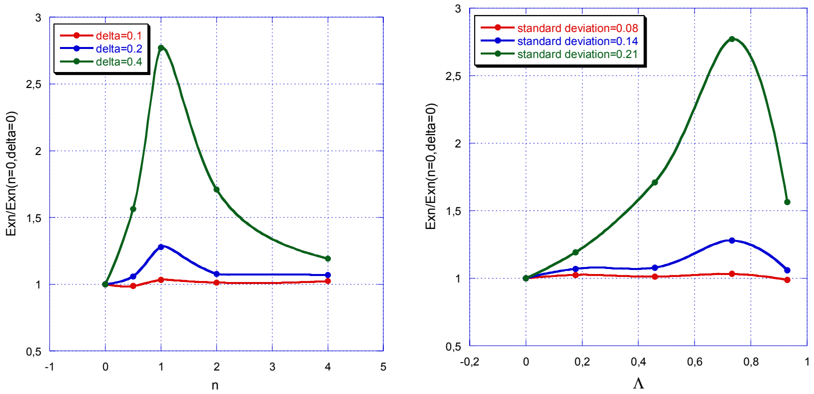

Figure 7.On the left hand side emittance behaviour vs n at $z=1.25$ (that is on the first emittance minimum in its evolution along z). On the right hand side emittance behaviour vs spatial correlation Λ ($N/h=M/h=20$).

Table 1 and Table 2 show the obtained spatial correlation for different frequency n and the standard deviation for different intensity of non uniformity δ. The best situation is obtained when both spatial autocorrelation and standard deviation are low that is below 0.4 for the Λ and below 0.14 for the σ_a.

Table 1. Spatial autocorrelation Λ for different frequency n of the non-uniformity distribution. The ratio is $N/h=M/h=20$.

n	1	2	3	4
Λ	0.92	0.71	0.45	0.18

Table 2.Standard deviation σ for different intensity δ of the non-uniformity distribution.

δ	10%	20%	30%	40%
σ	0.08	0.14	0.18	0.21

It is interesting to show in Figure 8 a plot of the spatial autocorrelation as a function of the frequency n for different distances h: when h is small (or the ratio $N/h=M/h$ is high) the Λ variation is so small that the frequency $n=1$ and $n=4$ cannot be distinguished. It means the a_{ij} and a_{ijh} are so far from the mean $<a>$ (see eq.4) that the Λ is always high. Anyway a_{ijh} can be chosen as obtained from a bigger number of samples (that is choosing a bigger h) thus the distance a_{ijh} from the $<a>$ decreases, decreasing the overall covariance. This gives the possibility to distinguish between the frequency $n=1$ and $n=4$. The distance h has to be high enough with an upper limit given by the frequency n to be investigated that is:

$$k_n h < \pi$$

or substituting k_n

$$\frac{N}{h} > 4n \tag{8}$$

In this case eq.8 leads to h=10.

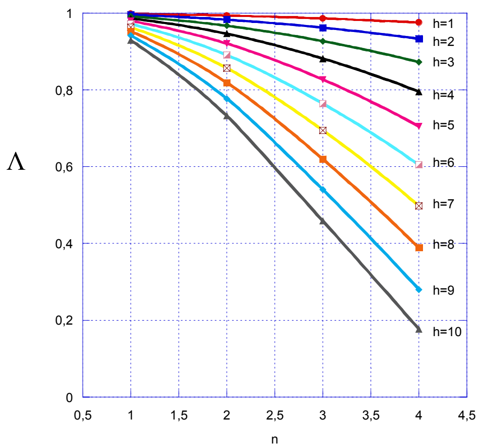

Figure 8.Spatial autocorrelation vs n for different distances h.

A program devoted to the calculation of spatial correlation has been built using the Mathematica software. Briefly the algorithm reads the image in tiff format coming directly from a camera acquisition and it changes it in a matrix whose elements represent the intensity of the pixels. It performs a filtering action eliminating the bias and those pixels which haven't a physical meaning: this is the filtered matrix. The algorithm normalizes the matrix and calculates the radius of the beam choosing those values which are above a certain threshold.

The threshold is chosen making the mean of the pixels around the barycentre of the filtered distribution and lowering it of a percentage: in this way the beam boundary are established.

The spatial correlation calculation is executed simply applying the formulas mentioned at the beginning of this paper on the filtered matrix where the pixels inside the spot of the beam are untouched. In this way the spatial correlation depends on the threshold chosen only because the threshold is responsible of the chosen radius. Because of boundary problems, elements outside the established radius of the beam are treated as being equal to the mean of the distribution. The distance h is automatically chosen by the algorithm keeping the ratio *N/h* fixed and equal to *20* as shown earlier. Finally the mean and the standard deviation are calculated, as well as the barycentre of the obtained beam, the standard deviation of the radius and the eccentricity.

The situation described in Figure 6 has been further on studied by evolving the $n=1$ and $\delta=40\%$ case with the Parmela code (Figure 9 beam spot at $z=0$ and Figure 10 beam spot along z) and calculating the obtained index Λ.

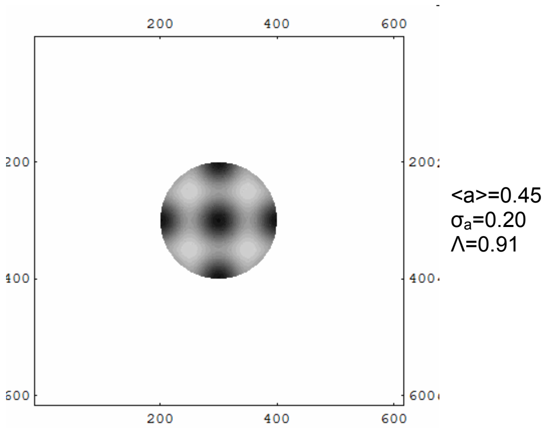

Figure 9.Image of the beam evolved along *z* with Parmela (SPARC configuration: radius=*1 mm*, Q=*1 nC*, flat top pulse with FWHM=*10 psec* and rise time=*1 psec*, ε_{th}=*0.*, number of particles used= *300000*).

Figure 11 shows the calculated spatial correlation versus z and the obtained rms beam spot size compared with the Parmela one. The index Λ first oscillates because the space charge spreads the spots intensity; on the contrary the solenoid increases the spots intensity. Finally far from the waist the propagation is dominated by space charge and the index Λ decreases.

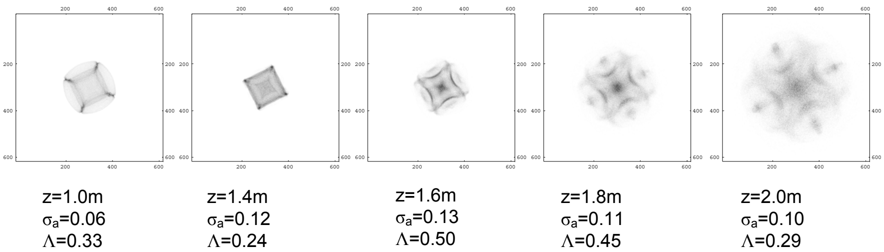

Figure 10.Samples of the beam images evolving along *z*.

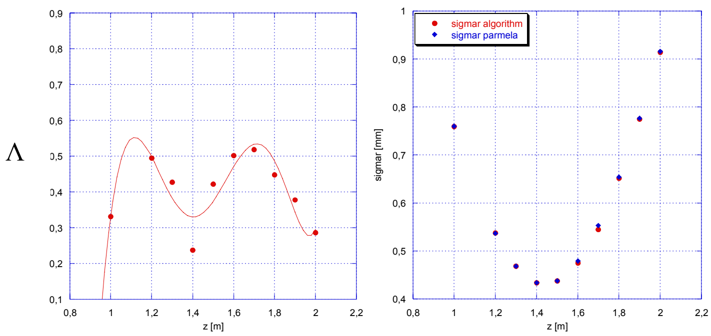

Figure 11.On the left hand side spatial correlation vs *z*. On the right hand side the beam radius vs *z* as calculated by the program and as obtained from Parmela.

One more example is analyzed: the evolution along the horizontal dimension *z* of a uniform beam. Figure 12 shows the beam spot at $z=0$. The standard deviation σ_a is below 0.14 and the spatial correlation is about zero as well because the distribution shows a random distribution of intensity as can been seen looking at Figure 12.

Figure 13 shows samples of the beam spot evolution and Figure 14 shows the beam spot size calculated by the program and Parmela. Again the spatial correlation oscillates because the beam itself oscillates. Finally Λ decreases due to the spread induced by the space charge.

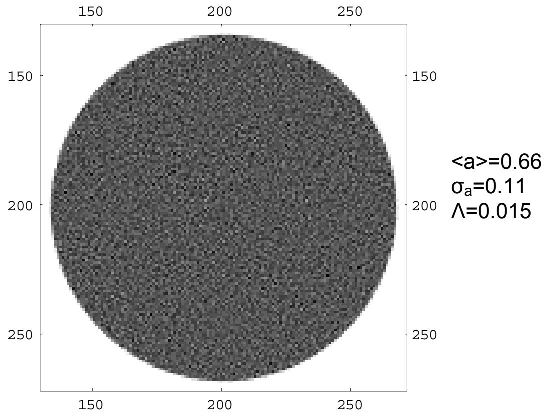

Figure 12.Image of a uniform beam as obtained from Parmela (SPARC configuration: radius=*1 mm*, Q=*1 nC*, flat top pulse with FWHM=*10 psec* and rise time=*1 psec*, ε_{th}=*0.6 mm mrad*, number of particles used= *150000*).

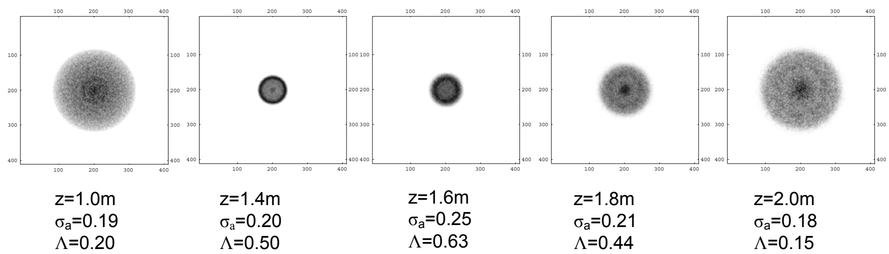

Figure 13.Samples of the beam images evolving along *z*.

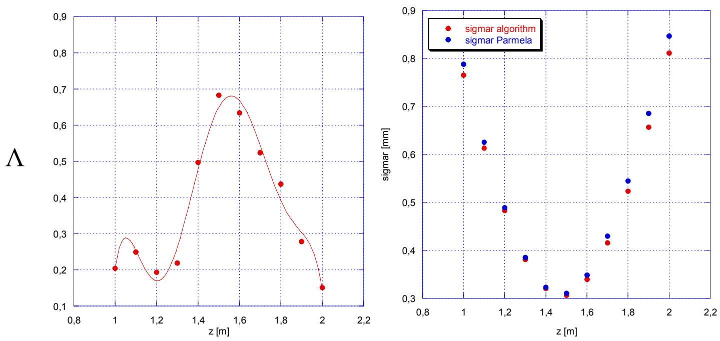

Figure 14.On the left hand side spatial correlation vs *z*. On the right hand side the beam radius vs *z* as calculated by the program and as obtained from Parmela.

Figure 15 shows an example of a real laser beam analysis. The image taken on the cathode and the processed image with the proper radius are shown together with the matrix representing the distance from mean and the variance matrix that is the distance squared.

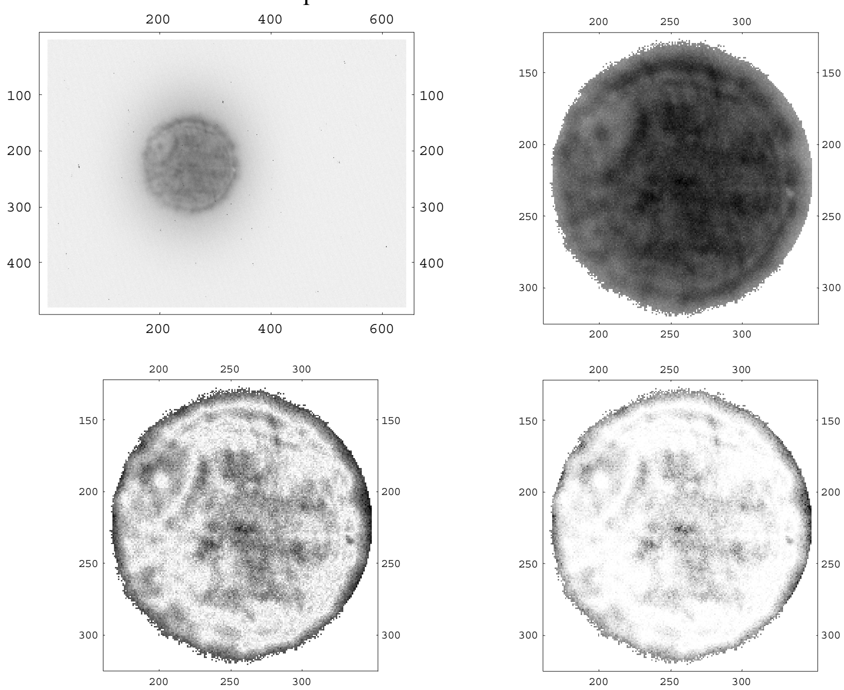

Figure 15.From up left to right: laser beam image on the cathode, filtered image, distance from the mean (left hand side) and the variance.

Figure 16 is a coloured version of the beam image with a dot representing the calculated barycentre.

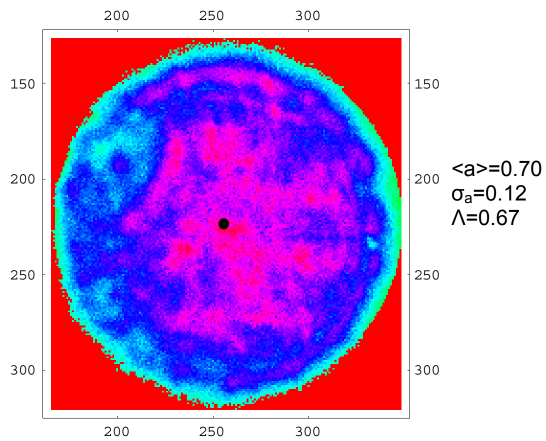

Figure 16.Laser beam image with a central dot representing the barycentre, the mean, standard deviation and spatial correlation as calculated by the algorithm.

The high spatial correlation (above 0.4) demonstrates the laser image exhibits spots which are not random but quite concentrate. Luckily the standard deviation is low (below 0.14) that is there is no much contrast between the mean and the spots.

Next example shows the behavior, in terms of spatial correlation, of the same beam evolving along the longitudinal direction *z* in a photo-injector for high brightness beam applications. In particular the analyzed images are the results of the emittance meter measurements in the SPARC photo-injector [Ref.10]. This is shown in Figure 17 whilst Figure 18 shows also the obtained spatial correlation as a function of *z* and the calculated radius of the beam. As previously described spatial correlation oscillates because of focusing (solenoid) and defocusing force (space charge); then, because of the only space charge, defocusing, spots tend to partially compensate assuming a random aspect and the index decreases.

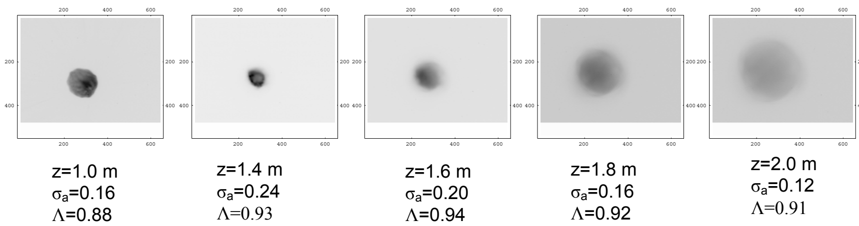

Figure 17.Samples of the beam images evolving along z.

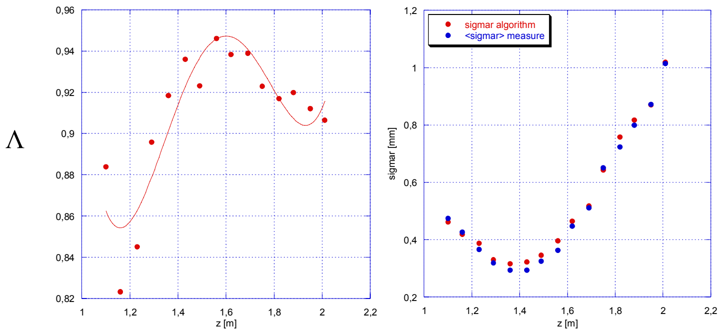

Figure 18.On the left hand side spatial correlation vs z. On the right hand side the beam radius vs z as calculated by the algorithm and as obtained from the measures.

4. Conclusions

Quantities like means, variance and standard deviation give details on non-uniformity of a beam without showing how non-uniformity is distributed. Spatial correlation depicts this important property of a laser or of a beam. The paper reports and develops the spatial correlation concept with the help of few simple examples. Finally spatial correlation is applied to a real beam giving an addition tool to evaluate a beam quality.

As a conclusion a well behaving beam displays a standard deviation and a spatial correlation which have to be as low as possible. If the standard deviation is high but the spatial autocorrelation is low it means beam non-uniformity is distributed randomly and previous studies demonstrated this causes lower emittance degradation. Viceversa if the spatial autocorrelation is high but the standard deviation is low it means the laser or the beam shows spots of intensity nonetheless the contrast between spots is low and the beam can be considered good.

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