

Laser Comb μ -Workshop

INFN-LNF May 14th, 2007

14.30 Introduction (M. Ferrario)

15.00 PARMELA Simulations (M. Boscolo)

15.30 Laser System(S. Cialdi)

16.00 Thz FEL (I. Boscolo)

16.30 FEL Simulations (V. Petrillo)

17.00 Discussion

A TRAIN OF MICRO-BUNCHES FOR PWFA EXPERIMENTS
PRODUCED BY RF PHOTONINJECTORS

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MOPCH025

Proceedings of EPAC 2006, Edinburgh, Scotland

**LASER COMB: SIMULATIONS OF PRE-MODULATED E⁻ BEAMS AT THE
PHOTOCATHODE OF A HIGH BRIGHTNESS RF PHOTONINJECTOR**

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**NUCLEAR
INSTRUMENTS
& METHODS
IN
PHYSICS
RESEARCH**

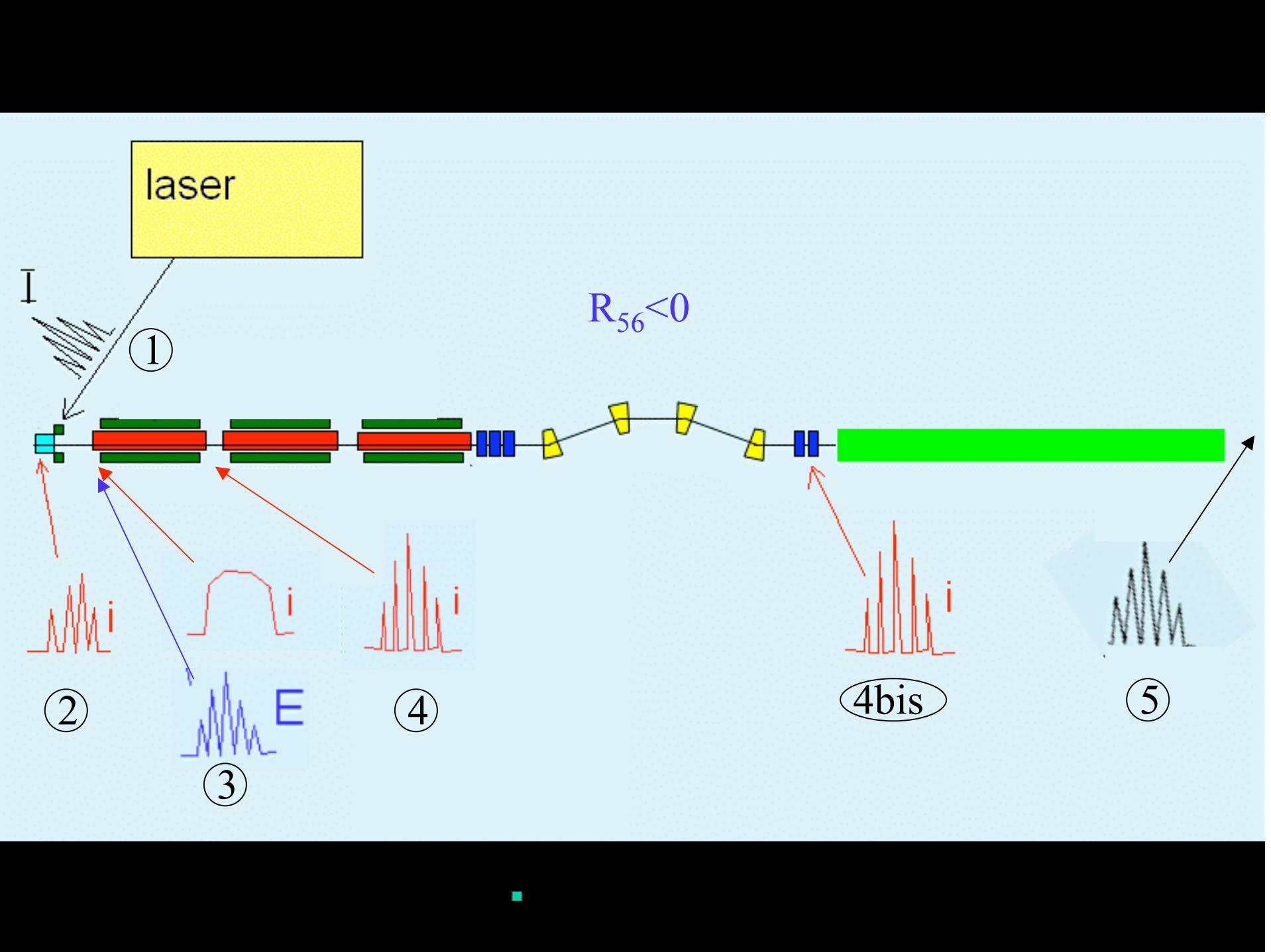
Section A: accelerators, spectrometers, detectors
and associated equipment

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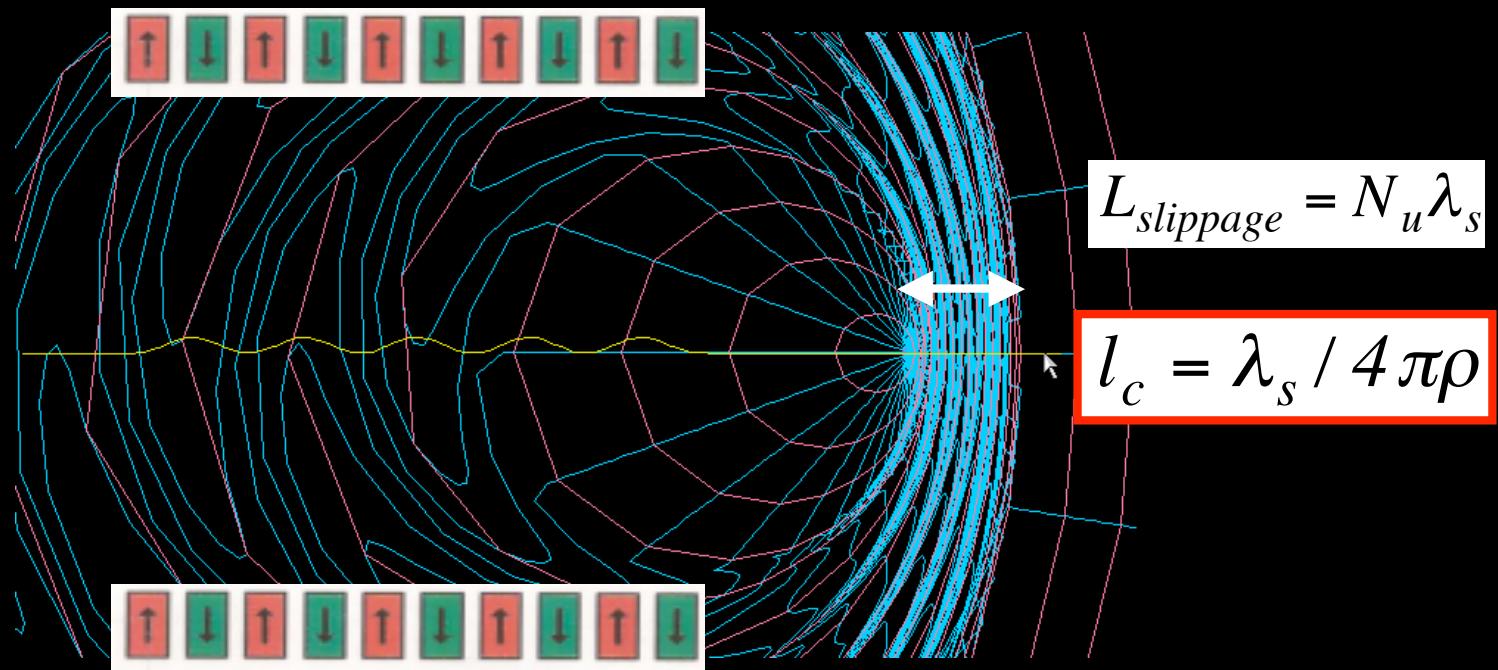
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Possible applications

- Longitudinal Beam Dynamics studies @ SPARC
- Pump and Probe with FEL spike train,
 $l_{\text{bunch}} \sim l_{\text{cooperation}}$, @ SPARC
- THz Coherent Synchrotron Radiation
- Plasma Wakefield Acceleration Experiments
@SLAC
- Pump and Probe wakefields measurements in high frequency rf structures (X-band) @ SPARC

FEL clean spike train



$$L_{slippage} = N_u \lambda_s$$

$$l_c = \lambda_s / 4\pi\rho$$

l_{bunch} $\begin{cases} > l_c \Rightarrow \textit{Many Spikes} \\ \leq l_c \Rightarrow \textit{Single Spike} \\ < \lambda_s \ll l_c \Rightarrow \textit{Coherent Synchrotron Radiation} \end{cases}$

Spectrum, Temporal Structure, and Fluctuations in a High-Gain Free-Electron Laser Starting from Noise

R. Bonifacio,^{1,2} L. De Salvo,¹ P. Pierini,² N. Piovella,¹ and C. Pellegrini³

$$\ell_c = \lambda_s / 4\pi\rho$$

$$l_b \approx l_c$$

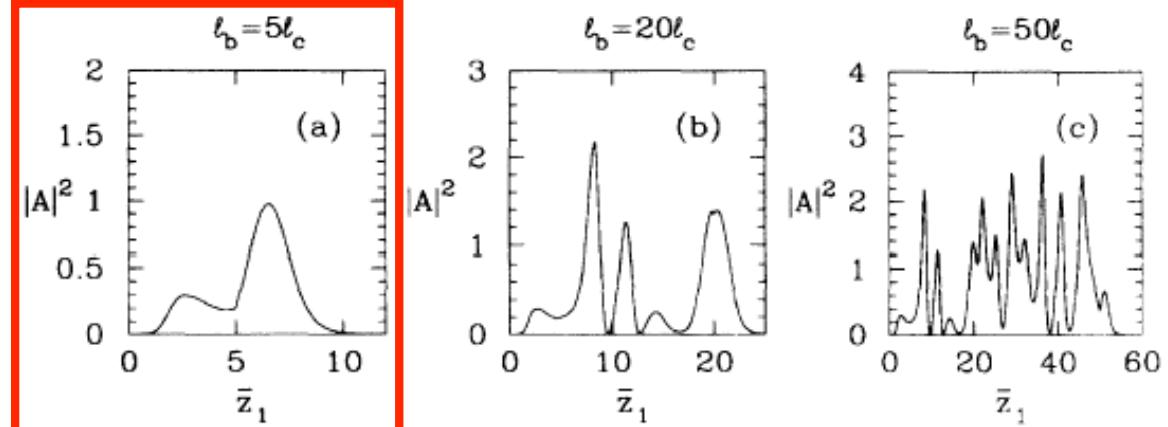


FIG. 1. Results of the numerical model: temporal structure of the radiated pulse, $|A|^2$ vs \bar{z}_1 , at the first saturation, for the initial noise pattern $b_0(\bar{z}_1)$.

Each spike exhibits the superradiant scaling of intensity as the square density of the electron beam, as has been numerically tested. If the pulse is shorter than or of the order of $2\pi\ell_c$, only one “clean” spike occurs, as can be seen in Figs. 1 and 3.

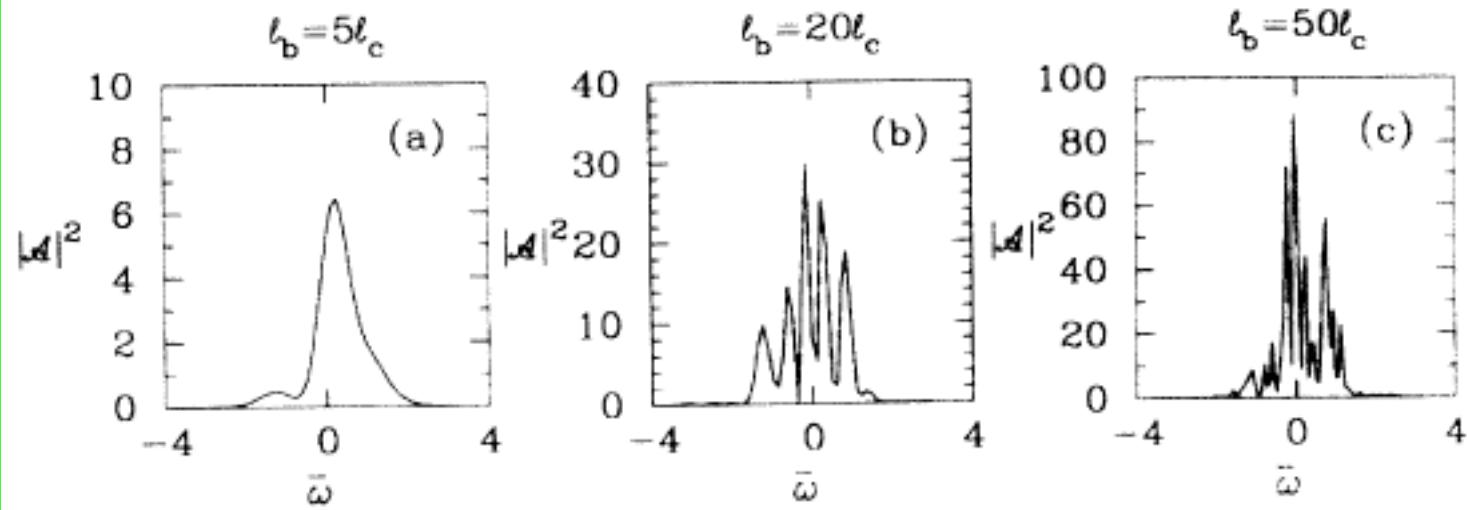


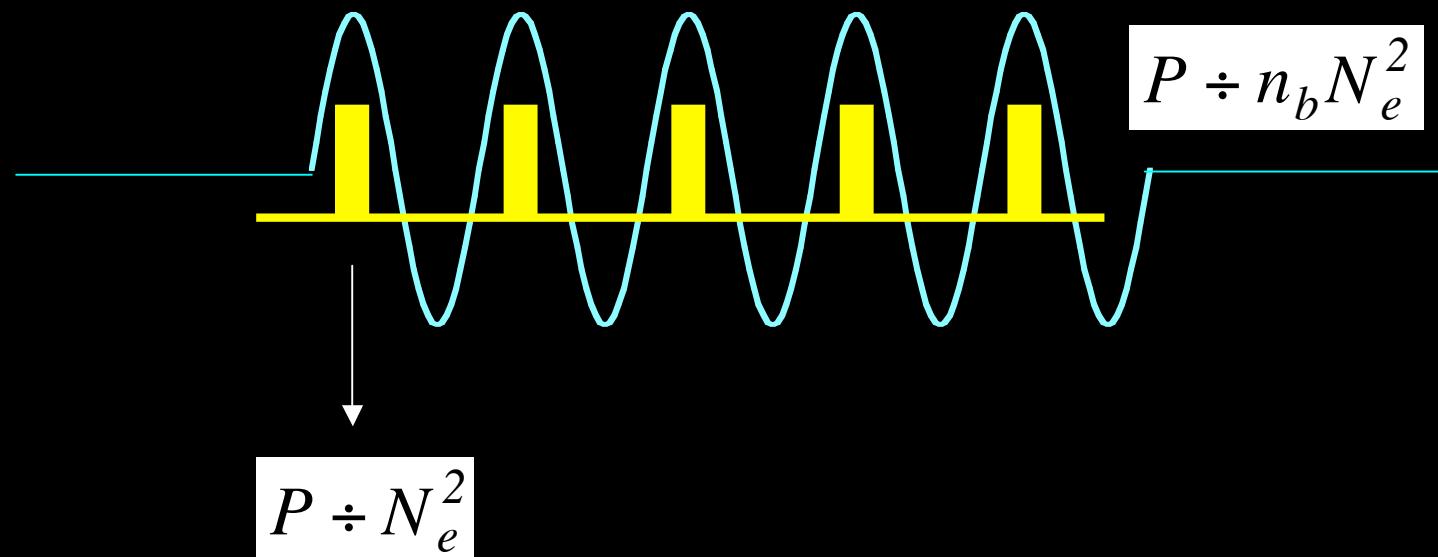
FIG. 3. Spectrum of the radiated pulses, for the same cases of Fig. 1; here $\bar{\omega} = \Delta\omega/2\rho\omega$.

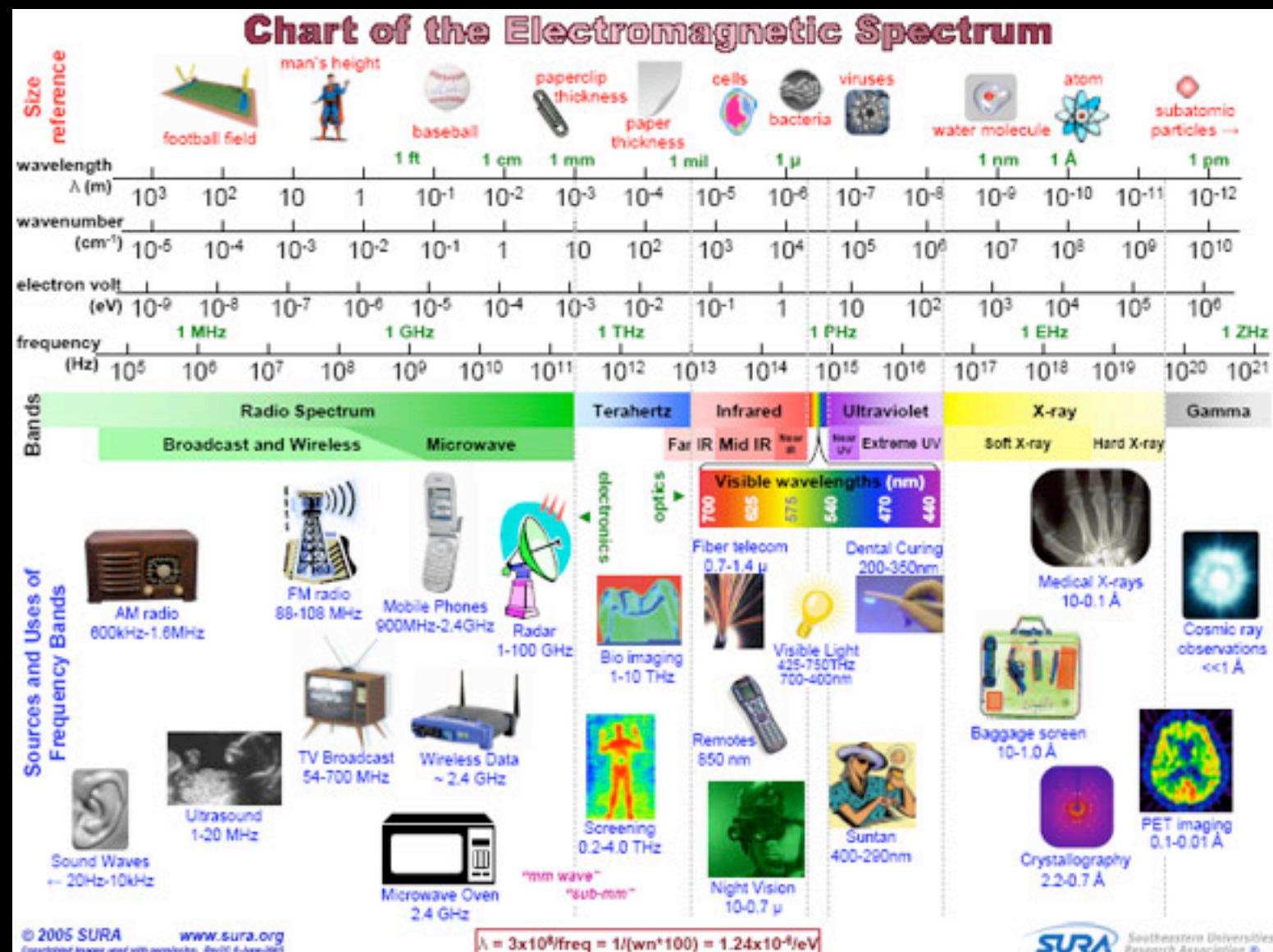
THz Coherent Synchrotron Radiation

$$l_{bunch} < \lambda_s \ll l_c$$

$$W_{tot}(\omega) = W_1(\omega)(N_e + N_e(N_e - 1)f(\omega))$$

$$f(\omega) = \left| \int d\vec{r} S(\vec{r}) e^{i \frac{\omega \wedge \vec{n}}{c} \cdot \vec{r}} \right|^2$$





Plasma Accelerators

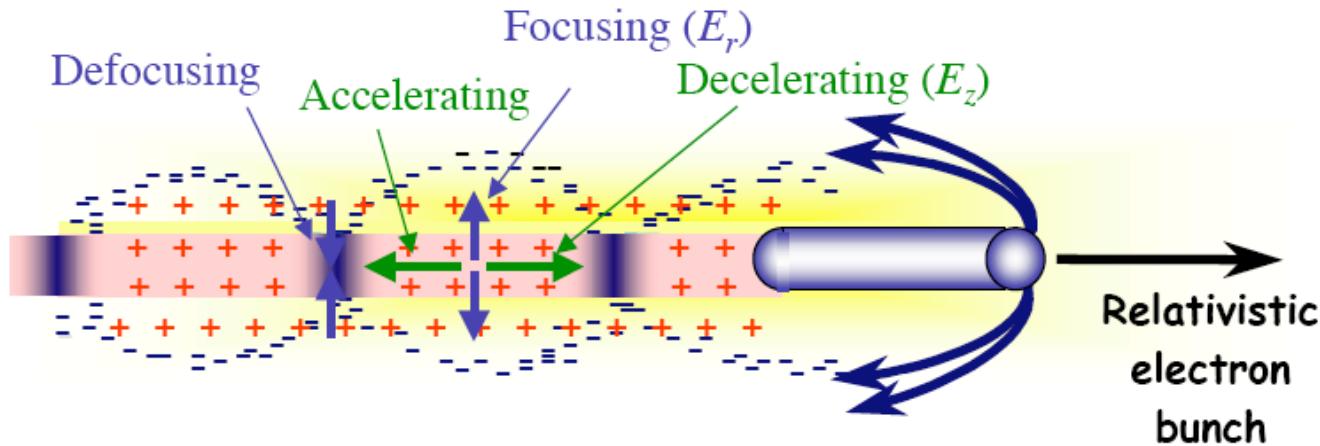
$$l_{bunch} < \lambda_p$$



PLASMA WAKEFIELD ACCELERATOR (e^-)



PWFA = beam-driven plasma accelerator



Plasma oscillation wavelength and longitudinal field value can be estimated to be

$$\lambda_p \approx \sqrt{\frac{10^{15} \text{ cm}^{-3}}{n_0}} \text{ (mm)} \quad \text{and} \quad E_z \approx 100\sqrt{n_0} \text{ (V m}^{-1}\text{)}, \quad (3.1.9)$$

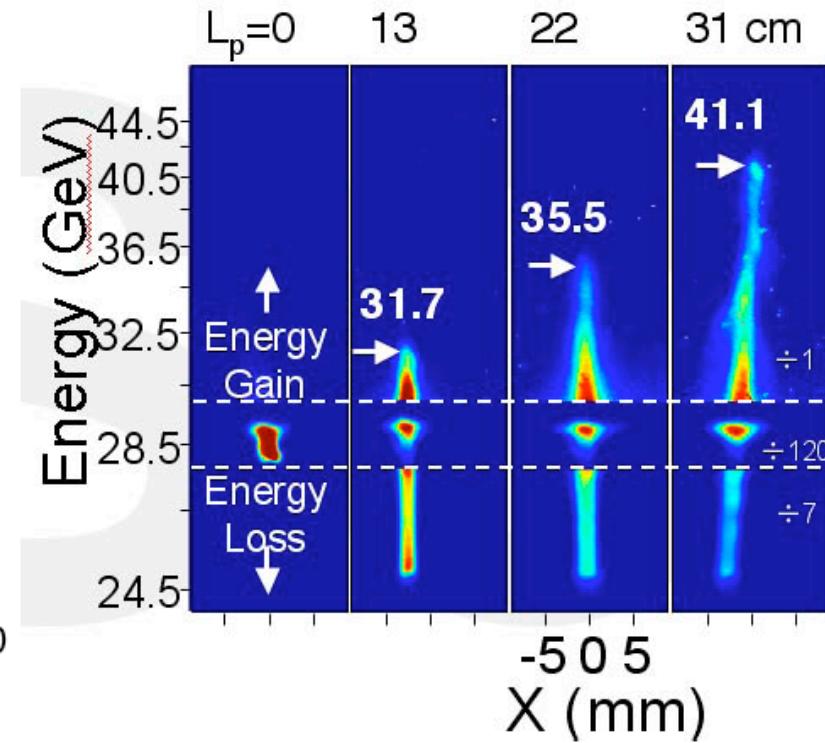
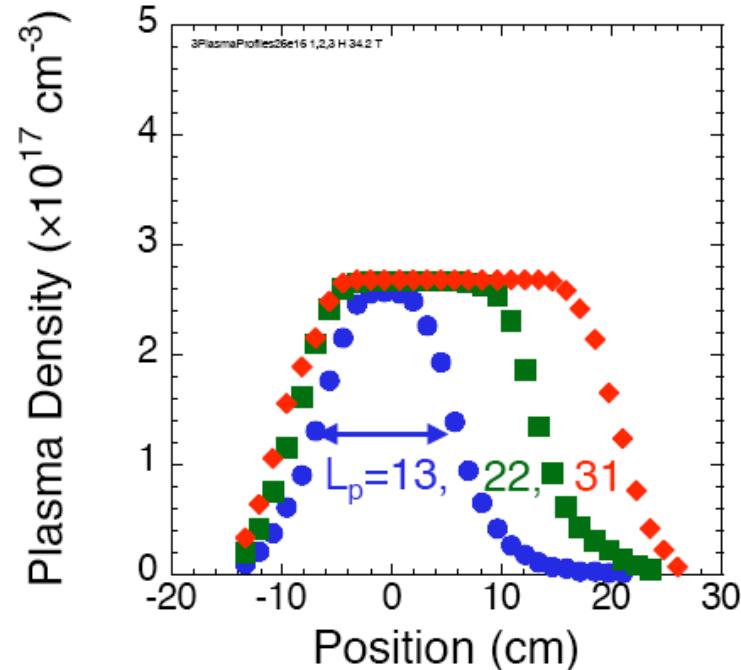
respectively, n_0 being the plasma density. The equations show that bunches to be accelerated must be rather short: as an example, with $n_0 = 10^{14} \text{ cm}^{-3}$ one has $\lambda_p = 3.3 \text{ mm}$ and $E_z = 1 \text{ GV m}^{-1}$. Femto-second long bunches should thus be injected with sub-fs timing accuracy.



ENERGY GAIN VS. PLASMA LENGTH



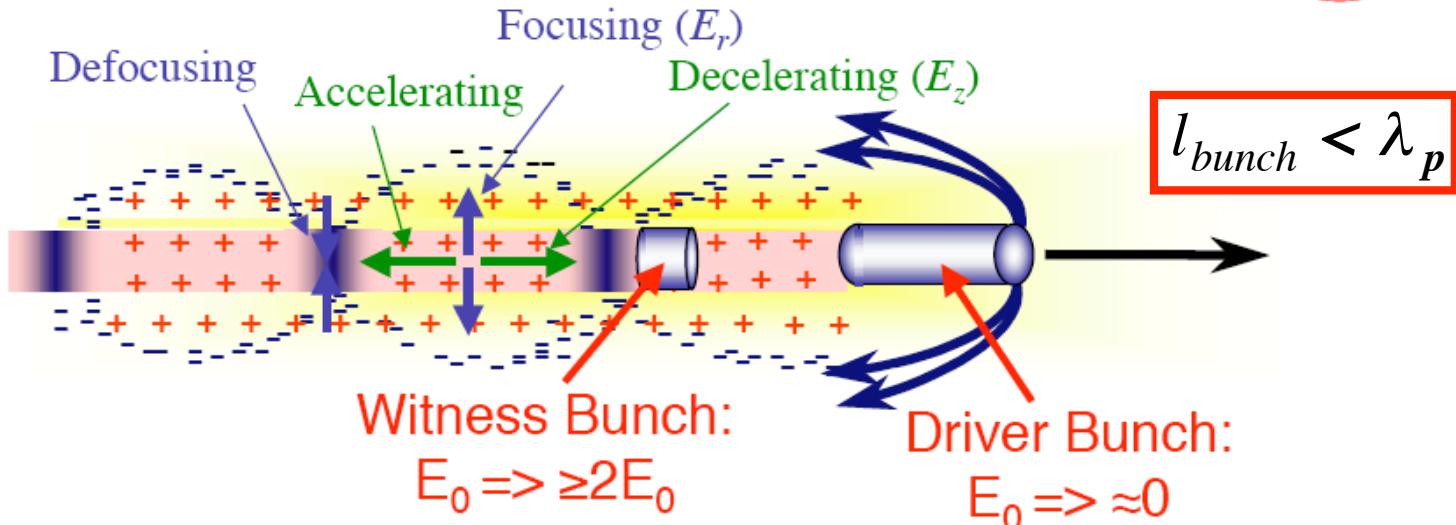
$E_0=28.5 \text{ GeV}$, $n_e=2.7 \times 10^{17} \text{ cm}^{-3}$



- Energy gain increases with plasma length (L_p)
- Energy gain reaches 13.6 GeV with $L_p=31 \text{ cm}!$

FUTURE EXPERIMENT

- 2-Bunch PWFA



- Driver bunch: high-charge ($3N$), modest emittance, shaped?
- Witness bunch: lower charge (N), good emittance
beam loading for $\Delta E/E \ll 1$
- Typical 2-bunch PWFA parameters:

$$n_e \approx 10^{16} \text{ cm}^{-3}, f_{pe} \approx 900 \text{ GHz}, \lambda_{pe} \approx 300 \mu\text{m}$$

$$G \approx 10-20 \text{ GeV/m}$$

$$N \approx 0.5 \times 10^{10} \text{ e}^-$$

$$\sigma_D \approx 60 \mu\text{m}, \sigma_W \approx 30 \mu\text{m}, \Delta t \approx 150 \mu\text{m}$$

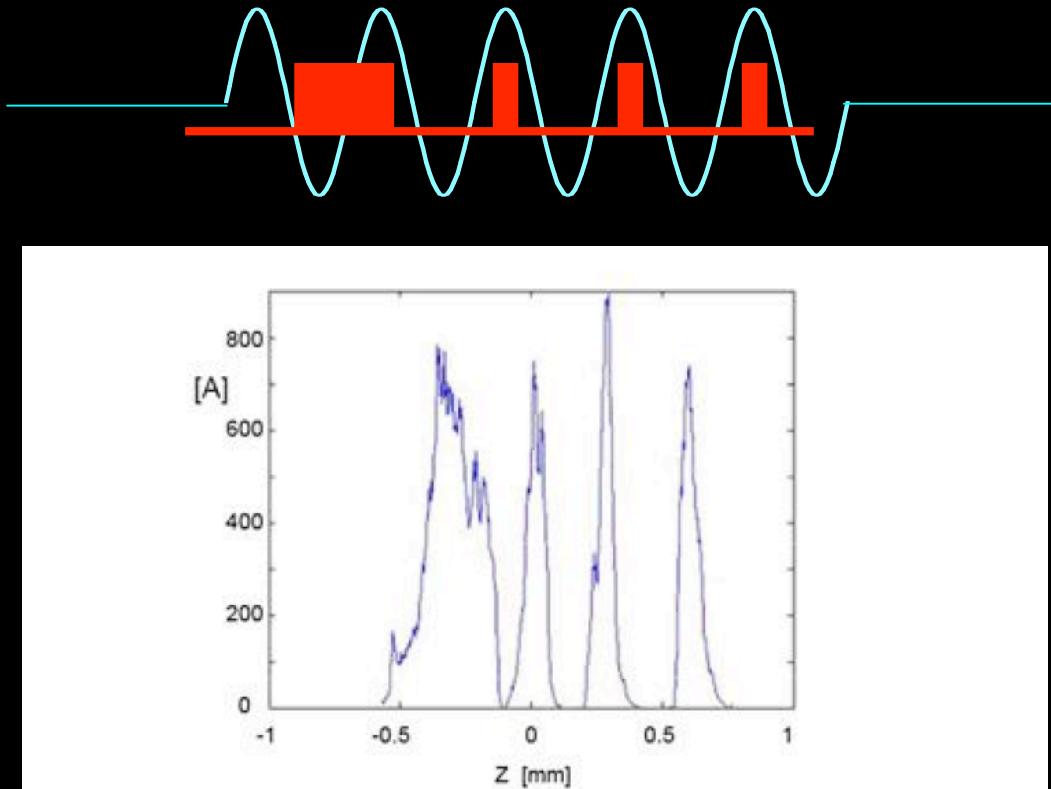


Fig. 7. Beam current at the end of three TW structures, 100 MeV energy.

three micro-bunches with σ_z 35 μm corresponding to 750 A peak current, with a population of 10^9 electrons, separated by 300 μm . The first three micro-bunches could excite coherently a $\lambda_p=300 \mu\text{m}$ plasma wave corresponding to a plasma density of 10^{16} cm^{-3} producing hundreds of MV/m accelerating field probed by the fourth bunch. Transverse emittance has not been optimized in the simulations

SIMULATIONS AND EXPERIMENTS OF ELECTRON BEAMS PRE-MODULATED AT THE PHOTOCATHODE*

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G. L. Carr, W.S. Graves[#], H. Loos[†], T. Shaftan, B. Sheehy, Y. Shen, Z. Wu,
BNL, Upton, NY 11973, USA

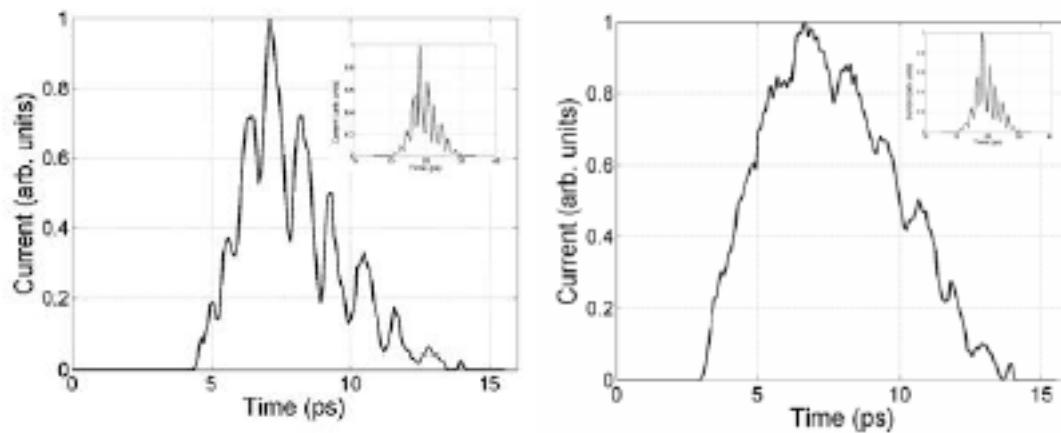


Figure 4: Electron beam longitudinal density after acceleration to 34 MeV for (left) 20 pC and (right) 200 pC.

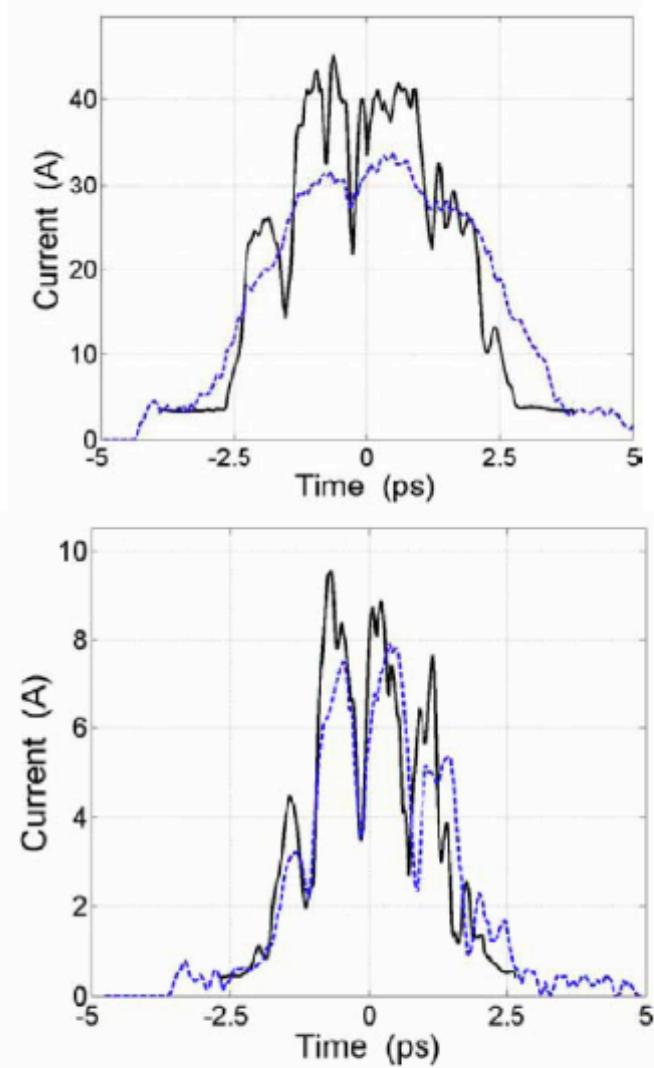


Figure 7: Longitudinal profile measurements (solid black) compared with simulations (dashed blue) for a 160 pC beam (top) and a 20 pC beam (bottom).

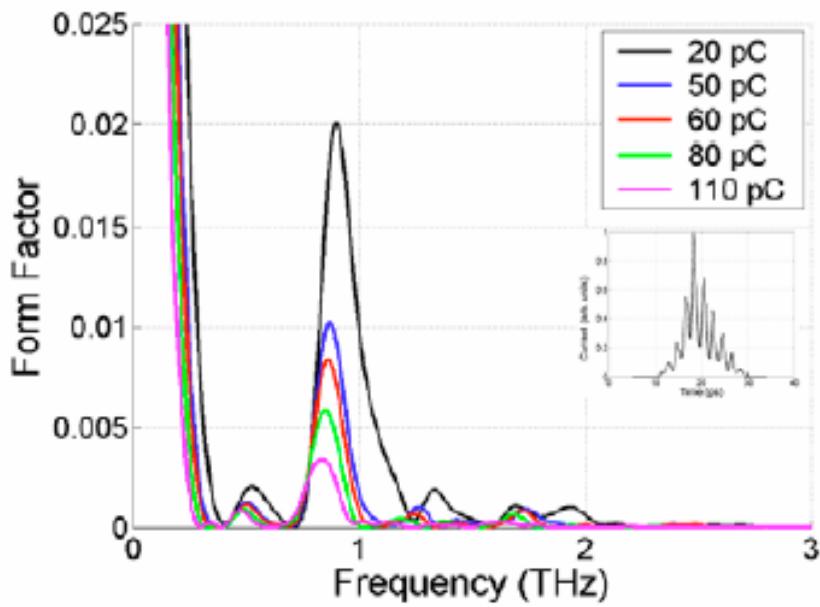


Figure 5: Variation of form factor with charge for pre-modulated electron beams accelerated to 34 MeV in the PARMELA simulation.

$$W_{tot}(\omega) = W_1(\omega)(N_e + N_e(N_e - 1)f(\omega))$$

$$f(\omega) = \left| \int d\vec{r} S(\vec{r}) e^{i \frac{\omega}{c} \vec{n} \cdot \vec{r}} \right|^2$$

Experimental investigation of the longitudinal beam dynamics in a photoinjector using a two-macroparticle bunch

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The photoemission electron source offers a convenient way of producing a two-macroparticle bunch: the photocathode drive-laser pulse is split into two and then recombined in such a way that a time delay is introduced between the two pulses. The delay can be remotely varied from ~ 7 to ~ 35 ps. A calibrated potentiometer provides a readout for the delay between the two pulses. An example of a double uv pulse is presented in Fig. 2. When such a double-pulse impinges the photocathode, it produces two electron bunches with a time separation much smaller than the rf period ($T_{rf} = 769$ ps). Hence both macroparticles fall into the same rf bucket and can be treated as a single bunch.

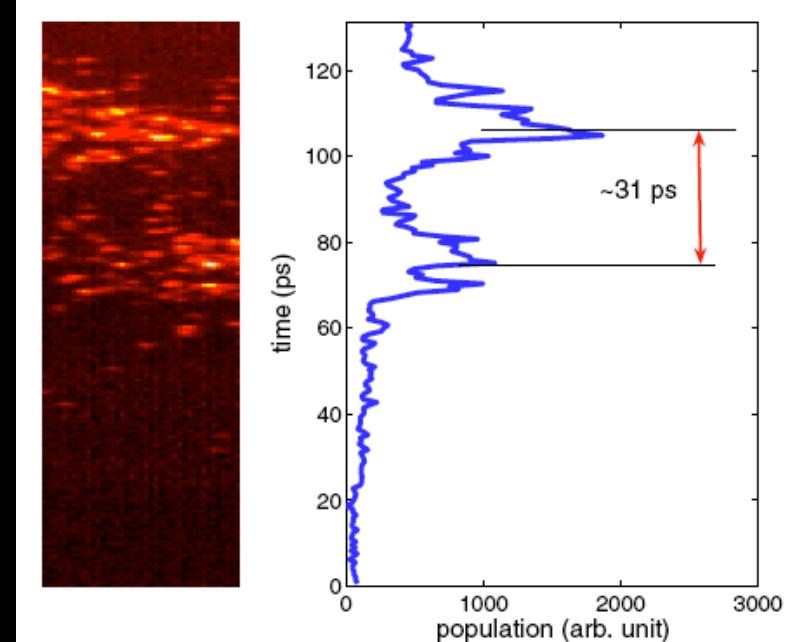


FIG. 2. (Color) Streak camera image (left) and corresponding time-profile (right) of a two-pulse laser. In this example the separation between the two pulses is approximately 31 ps. The apparent intensity difference between the two pulses (noticeable on the profiles) is an artifact of the measurement (slight misalignment of the second pulse on the streak camera entrance slit).

Generation of Short Bunch Trains in a RF Photoinjector- Beam Dynamics

M. Boscolo

Outline

- Description of the guiding idea
- Physics
- Simulations
- Perspectives , Open questions
- Tests at SPARC

Presentation based on:

- Nucl. Instrum. and Methods A
M. Boscolo et al. "Generation of Short THz bunch trains in a RF Photoinjector" <http://dx.doi.org/10.1016/j.nima.2007.04.129>
- Intern. Journal of Modern Physics B vol. 21 nos.3-4 p.415:
M. Ferrario et al. "A Train of micro-bunches for PWFA Experiments Produced by RF Photoinjectors"
- EPAC06:
M. Boscolo et al., "Laser Comb:Simulations pf pre-modulated e-beams at the photocathode of a high brightness RF photoinjector"

Introduction

trains of electron pulses
 $L \sim$ sub-ps; rep.freq. \sim THz; $Q \sim 100$ pC;
interesting for
FEL experiments,
plasma accelerators and
efficient generation of THz radiation

Idea is to produce these pulse trains
in a RF photoinjector

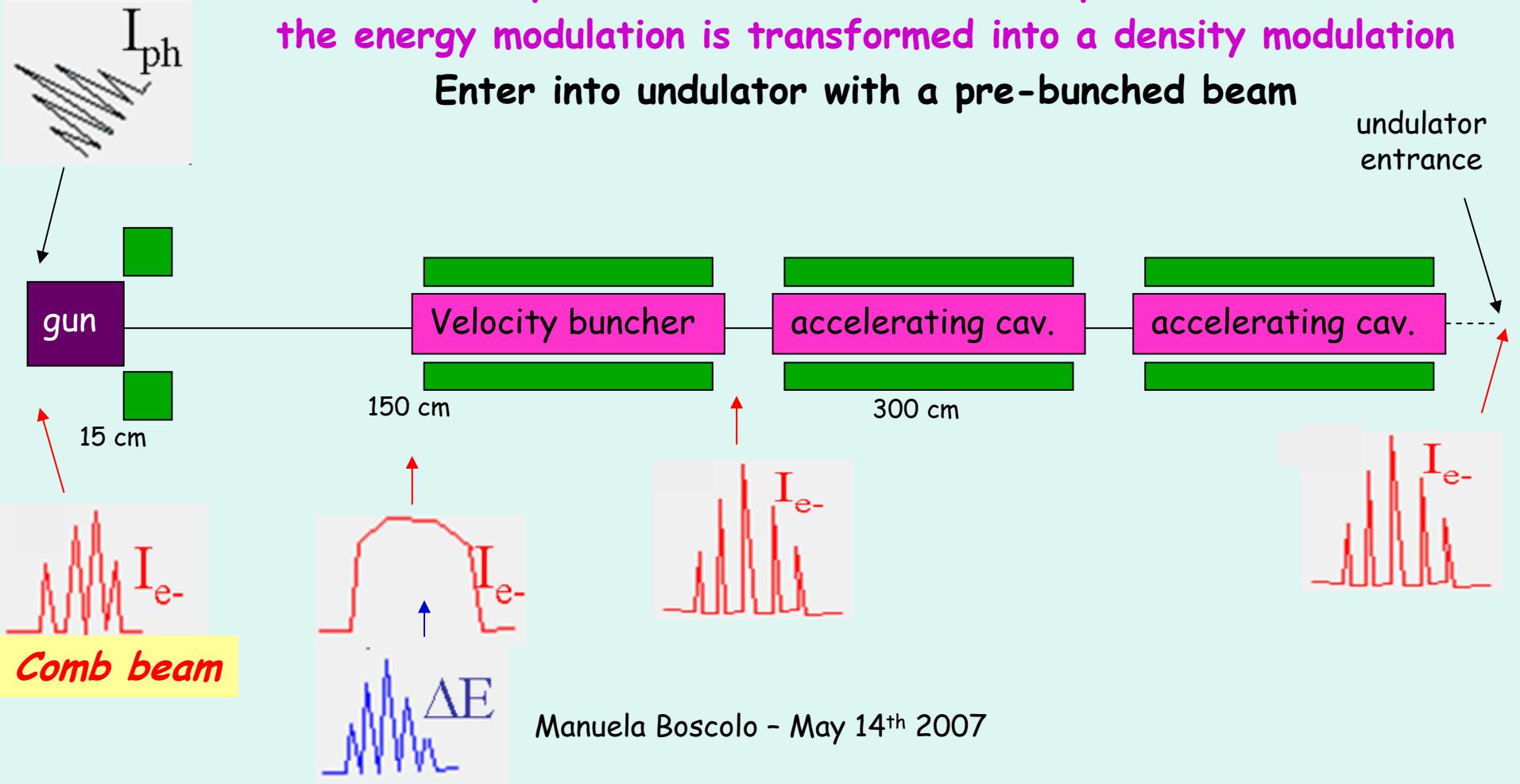
Experimental Scheme

Trains of subps e- pulses by illuminating the photocathode with trains of laser pulses

The disk train evolves toward a slug with a sawtooth energy modulation

In the rf compressor the initial multi-disk profile is recovered: the energy modulation is transformed into a density modulation

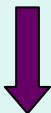
Enter into undulator with a pre-bunched beam



Challenging task:
see next talk by Simone



A comb-like laser pulse illuminates the cathode
generating a comb-like electron pulse



Electrons in each disk experience a large longitudinal space charge

They expand longitudinally travelling along gun and drift

The work done by the space charge force produces an energy modulated e- beam **with a sawtooth profile** of energy modulation with $\Delta E \sim 0.4$ MeV

This energy modulation is transformed into a density modulation in a magnetic or rf compressor

SPARC as a reference injector

Nominal parameters RF GUN

Gradient [MV/m]	120
Energy [MeV]	5.6
Phase ϕ [deg]	32
Solenoid field [G]	2730
length [cm]	15

Total macro-pulse length 10 ps

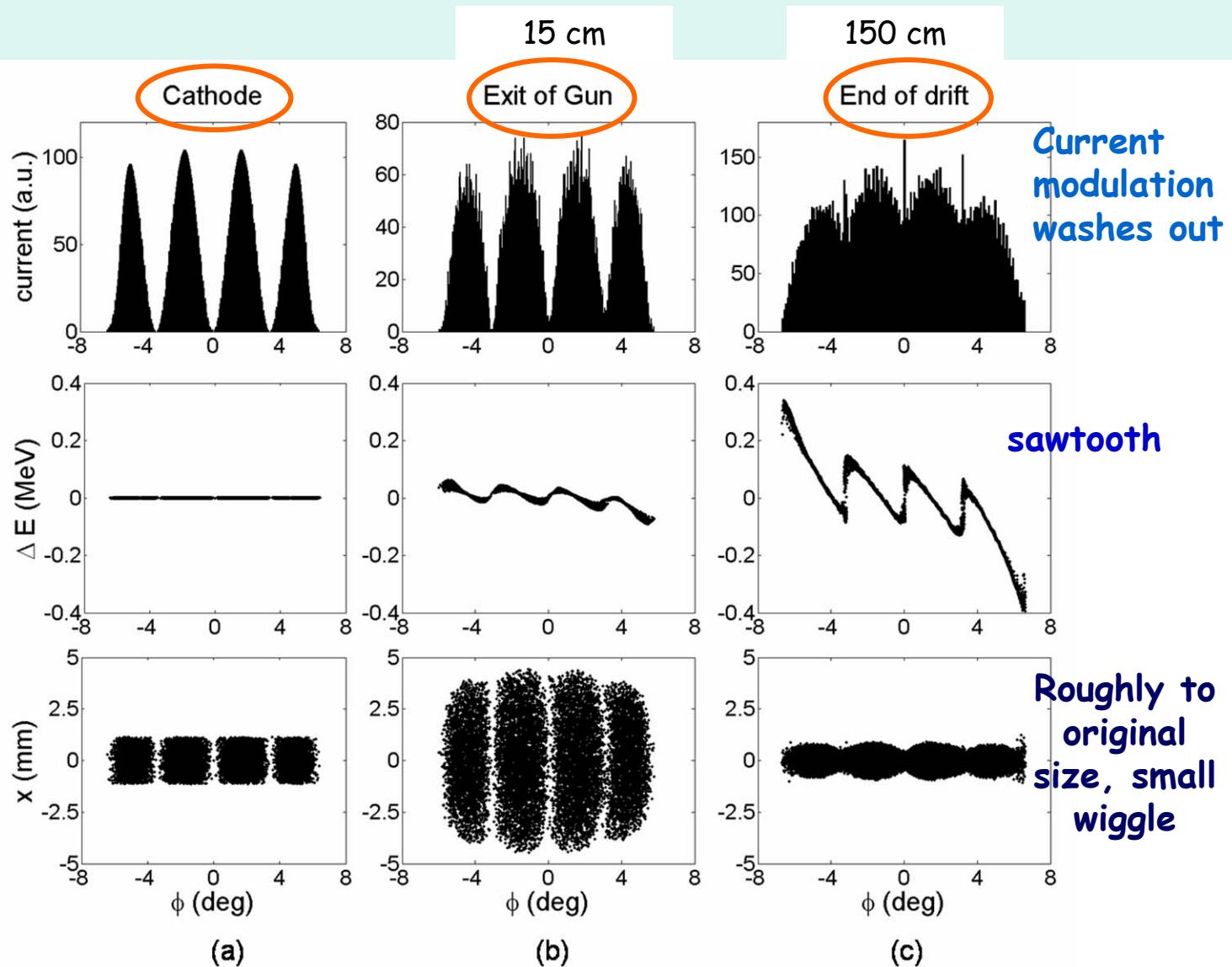
Total macro-pulse charge $Q_{tot} = 1.1 nC$

$$Q_{\text{micro-pulse}} = Q_{\text{tot}} / N$$

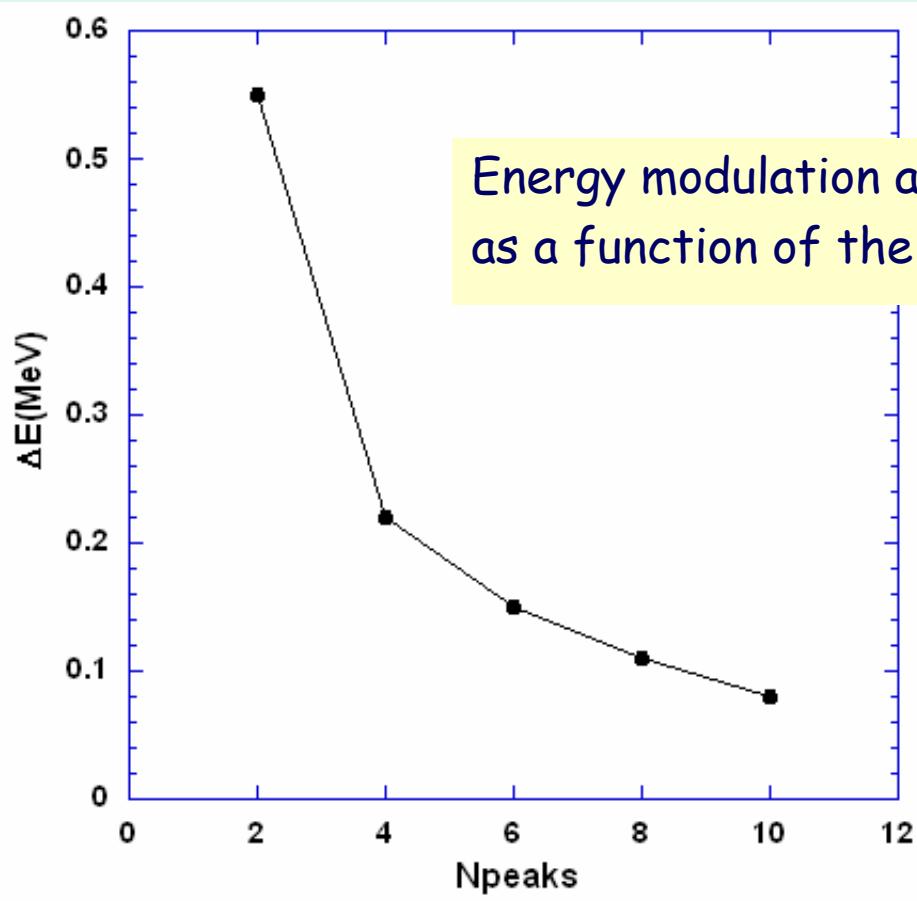
N=number of micro-pulses

Example of typical behavior: $N = 4$

longitudinal profile



The higher the number of micro-pulses
the lower is the energy modulation



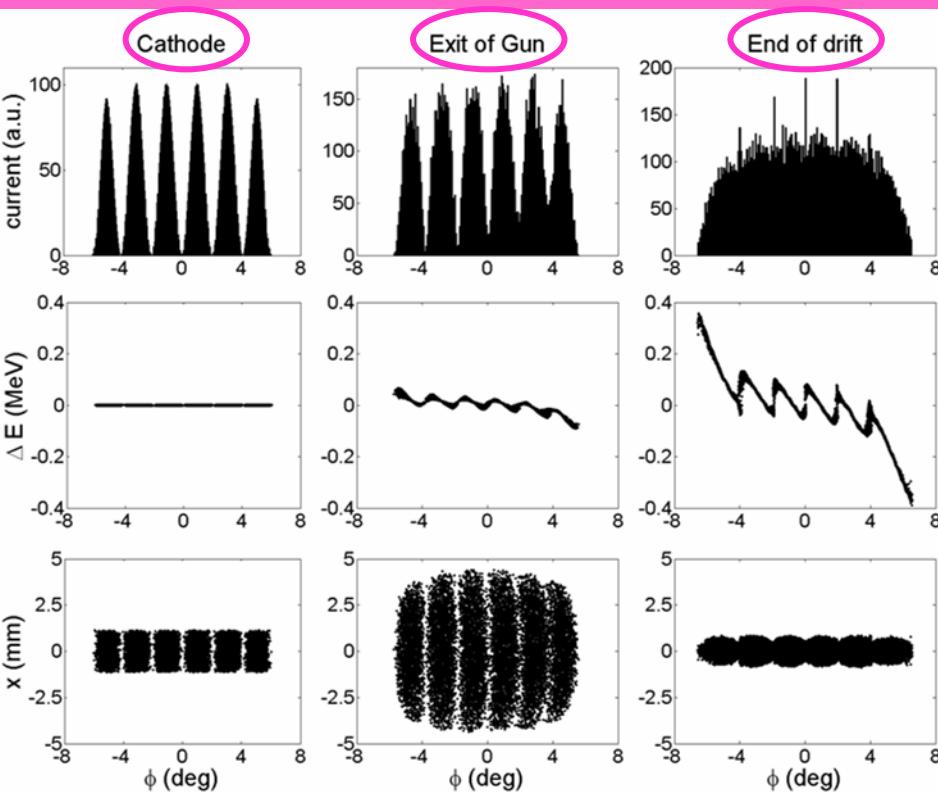
Energy modulation at the exit of the rf-gun
as a function of the number of sinusoidal peaks

as the $N_{\text{micro-pulses}}$ increases
each micro-bunch has less charge
and micro-bunches are closer

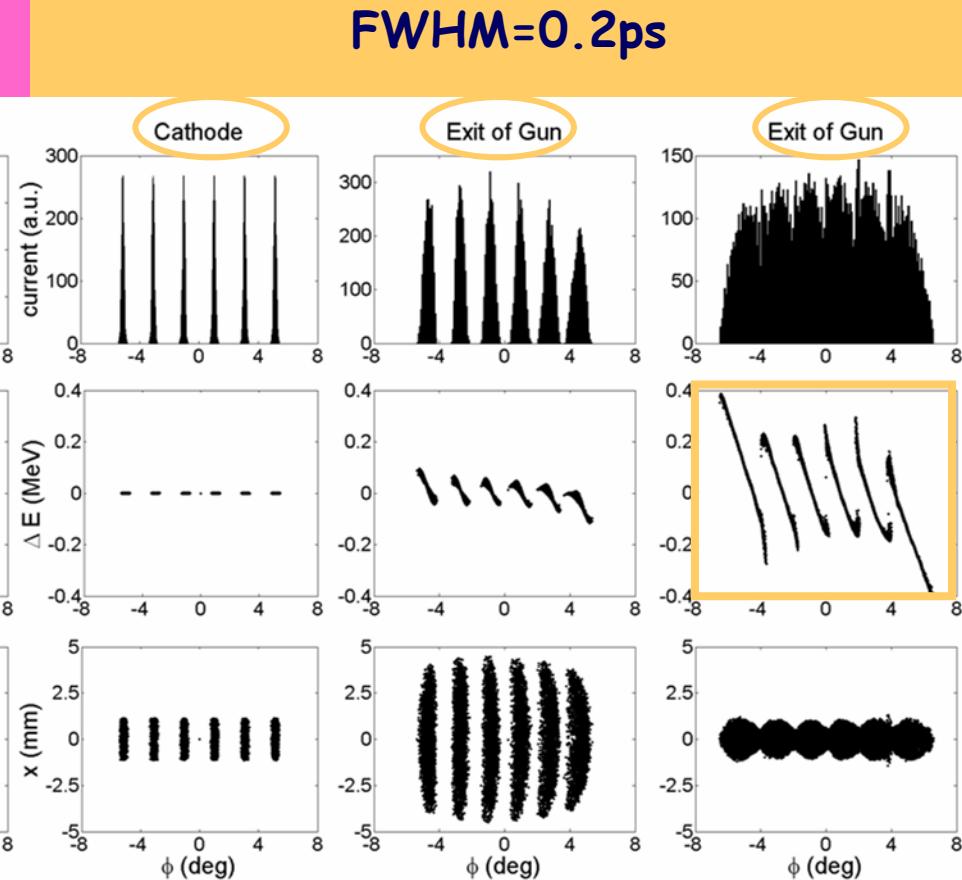
$N = 6$

The narrower the micro-pulses
the higher is the energy modulation
and the nearer the energy separation between pulses

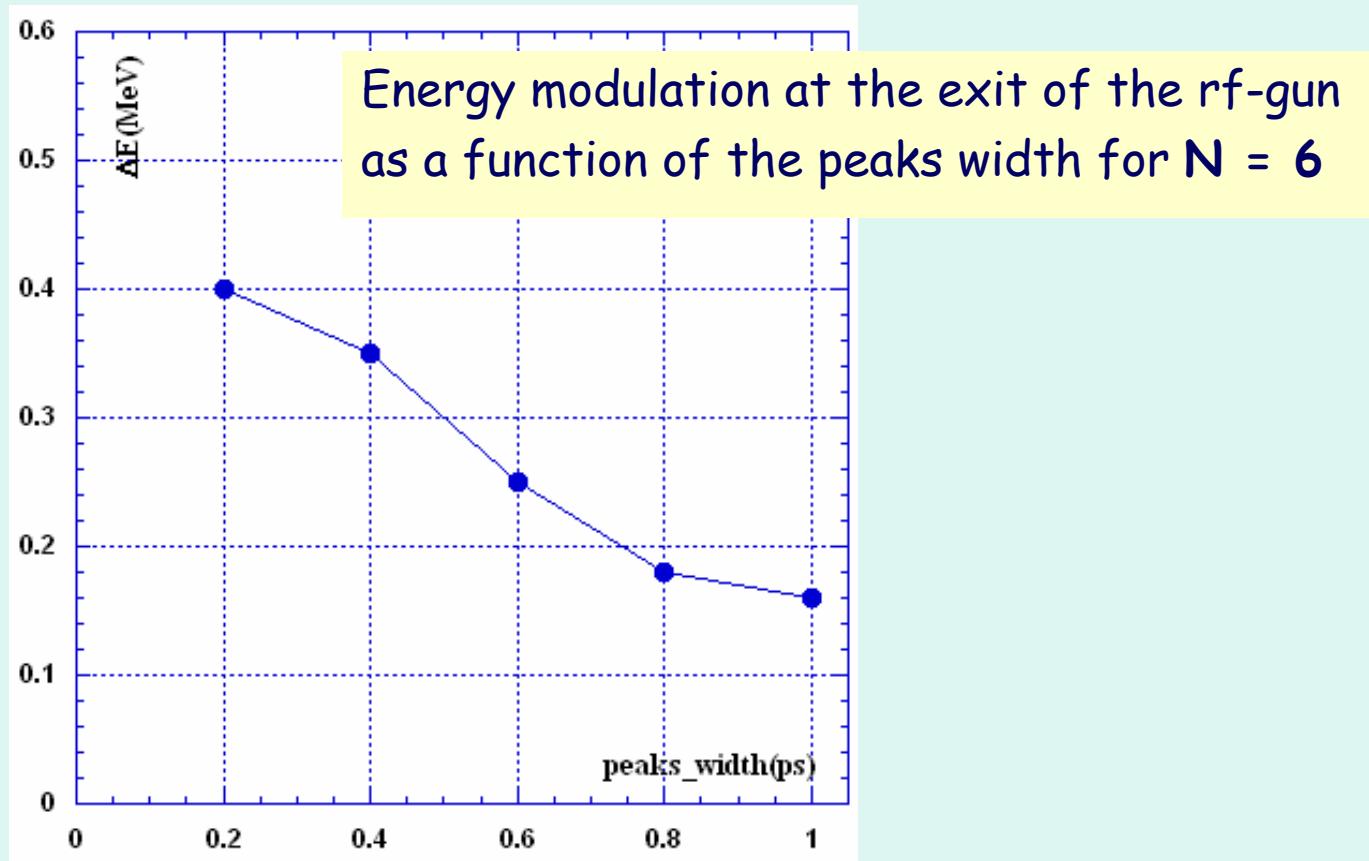
FWHM=1ps



FWHM=0.2ps



The narrower the micro-pulses
the higher is the energy modulation
and the nearer the energy separation between pulses

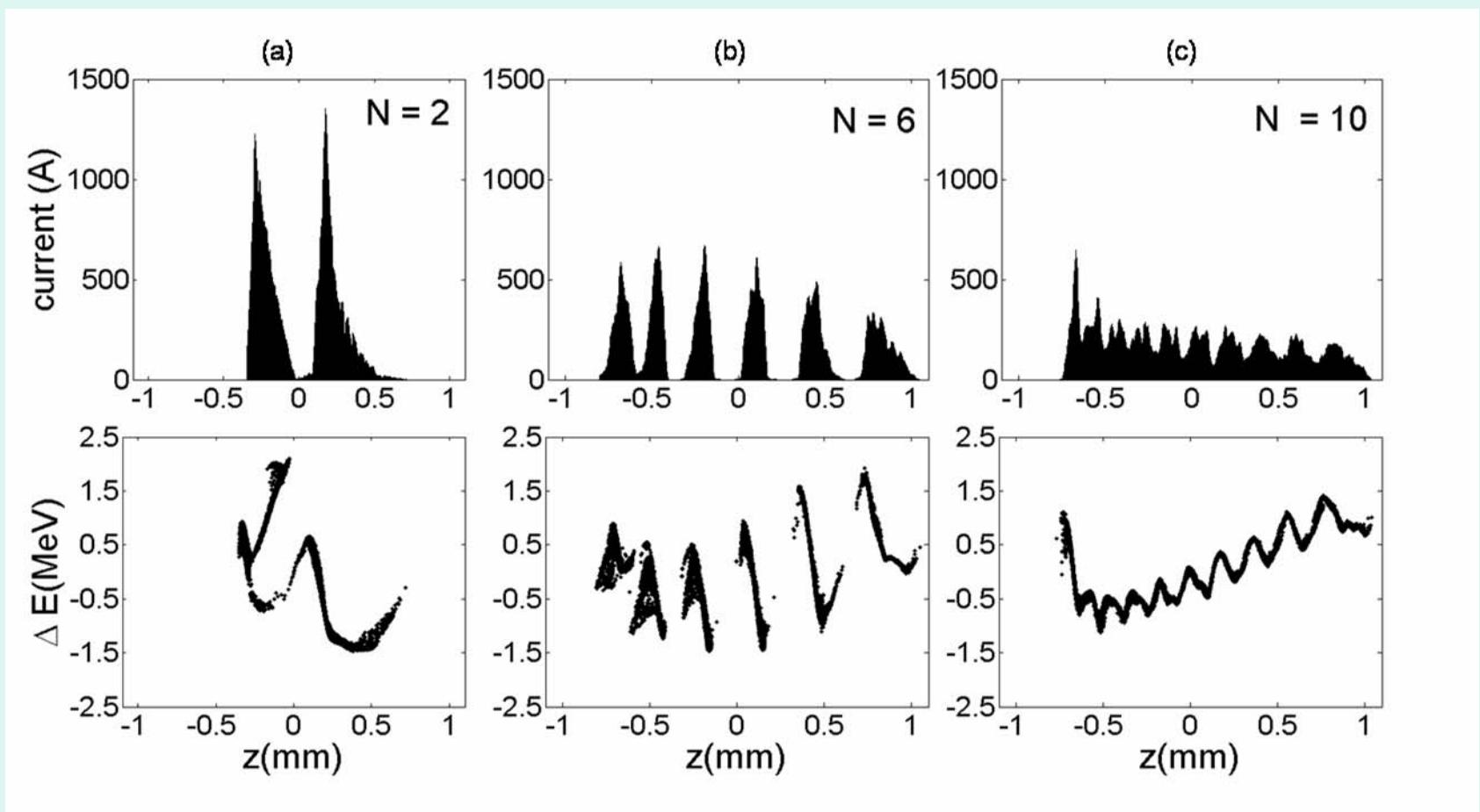


Evolution within the velocity buncher and accelerator

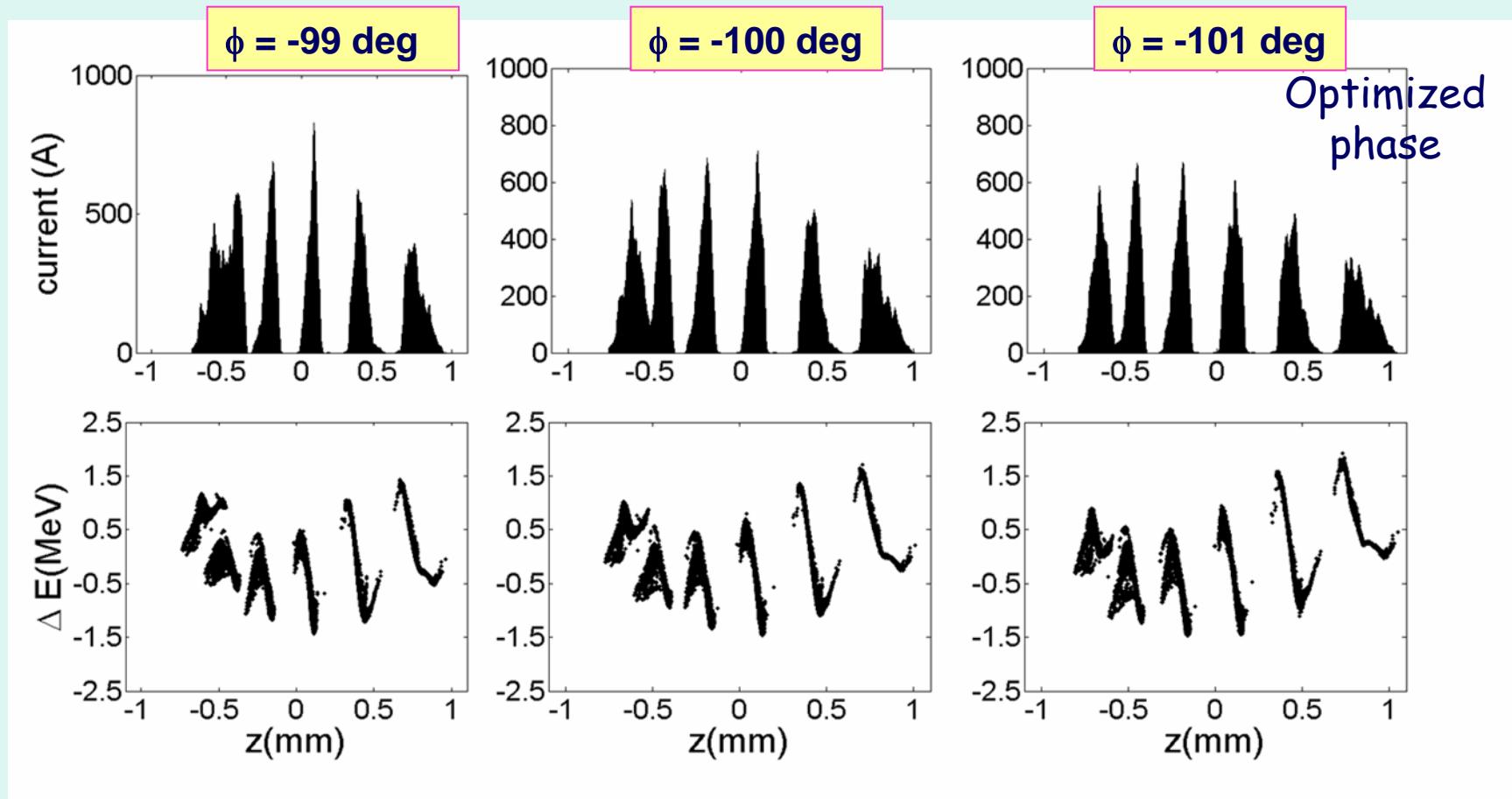
	Velocity buncher	II TW section	III TW section
Gradient [MV/m]	25	12.5	12.5
Energy [MeV]	17	55	88
Phase ϕ [deg]	-99/-101	on crest	on crest
Solenoid field [G]	615	0	0
length [cm]	300	300	300

First investigations with magnetic compressor,
beam quality better preserved with rf compression

The intensity modulated electron comb beam at the end of the accelerator
after velocity buncher and 2 TW cavities



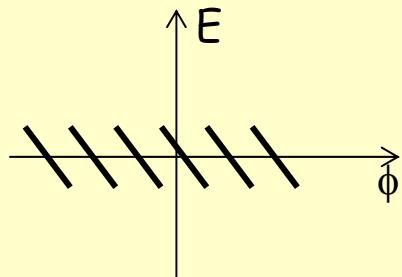
Rf compression for N = 6 and FWHM=0.2ps



the two leading micropulses move from a partial overlap to a complete separation decreasing the phase of 2°

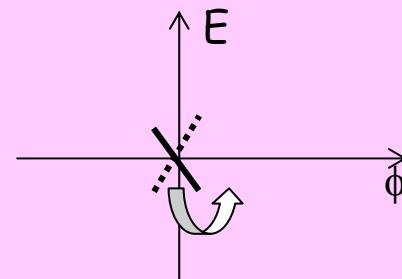
Proper rotation that recovers
initial comb beam is in the over-
compression region
for $\phi = -101$ deg

at the entrance of the
rf compressor

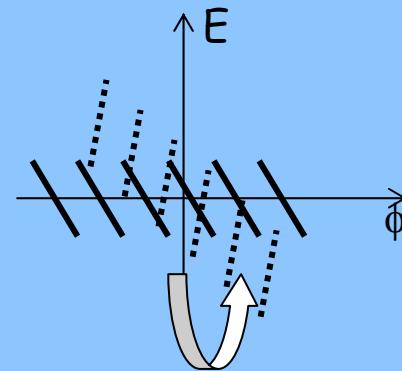


Comb beam : sawtooth

At exit of rf compressor:



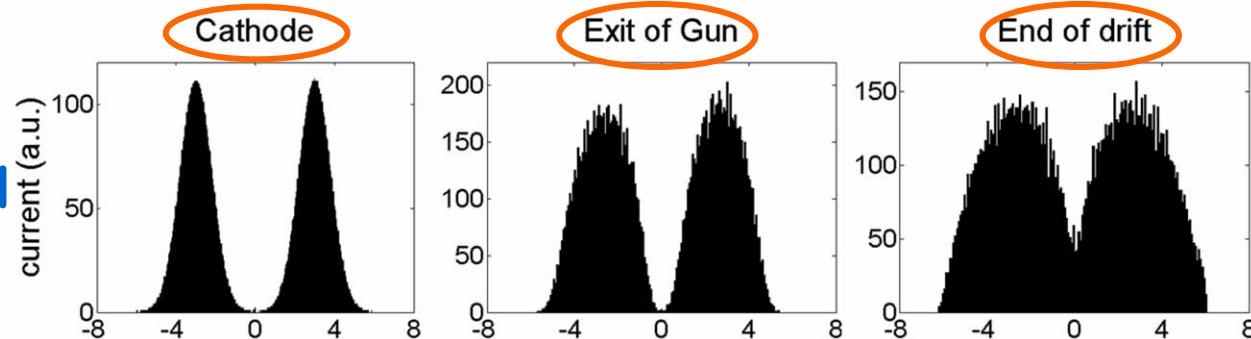
each segment:
rotation angle= $(3/4)\pi$



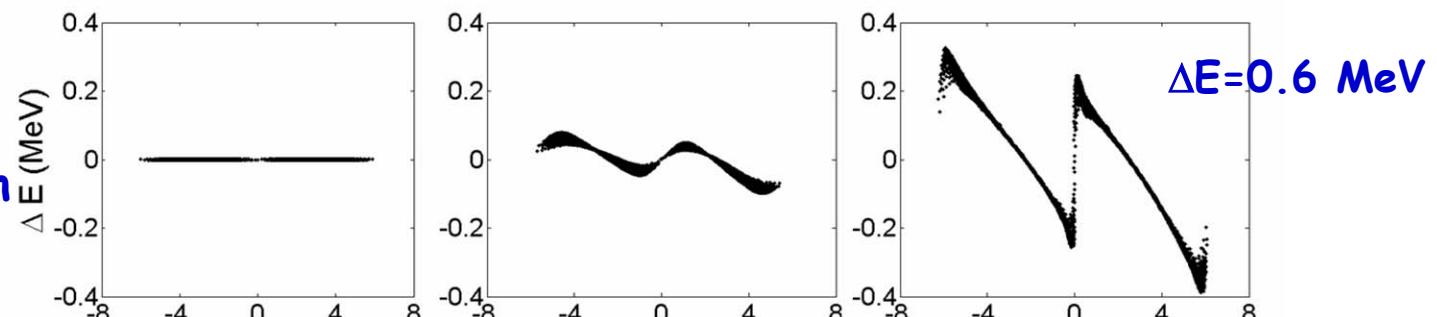
Entire train:
Rotation angle= $(5/4)\pi$

$N = 2$

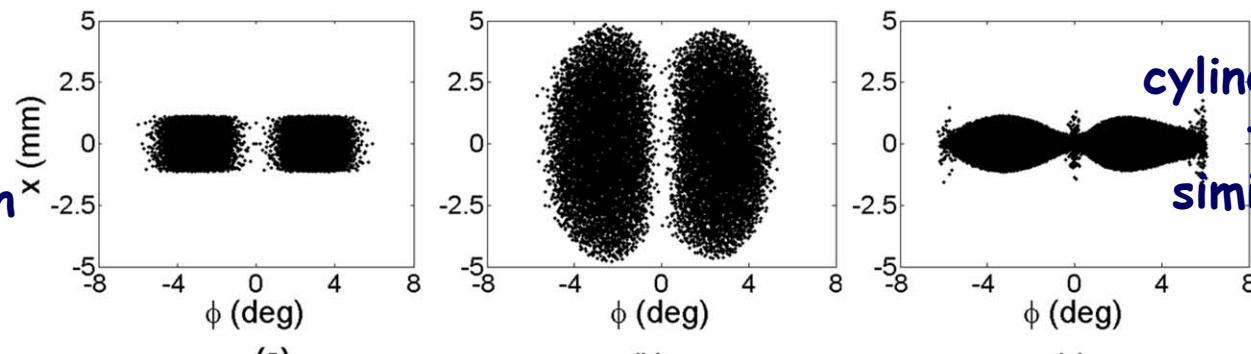
longitudinal profile



energy modulation



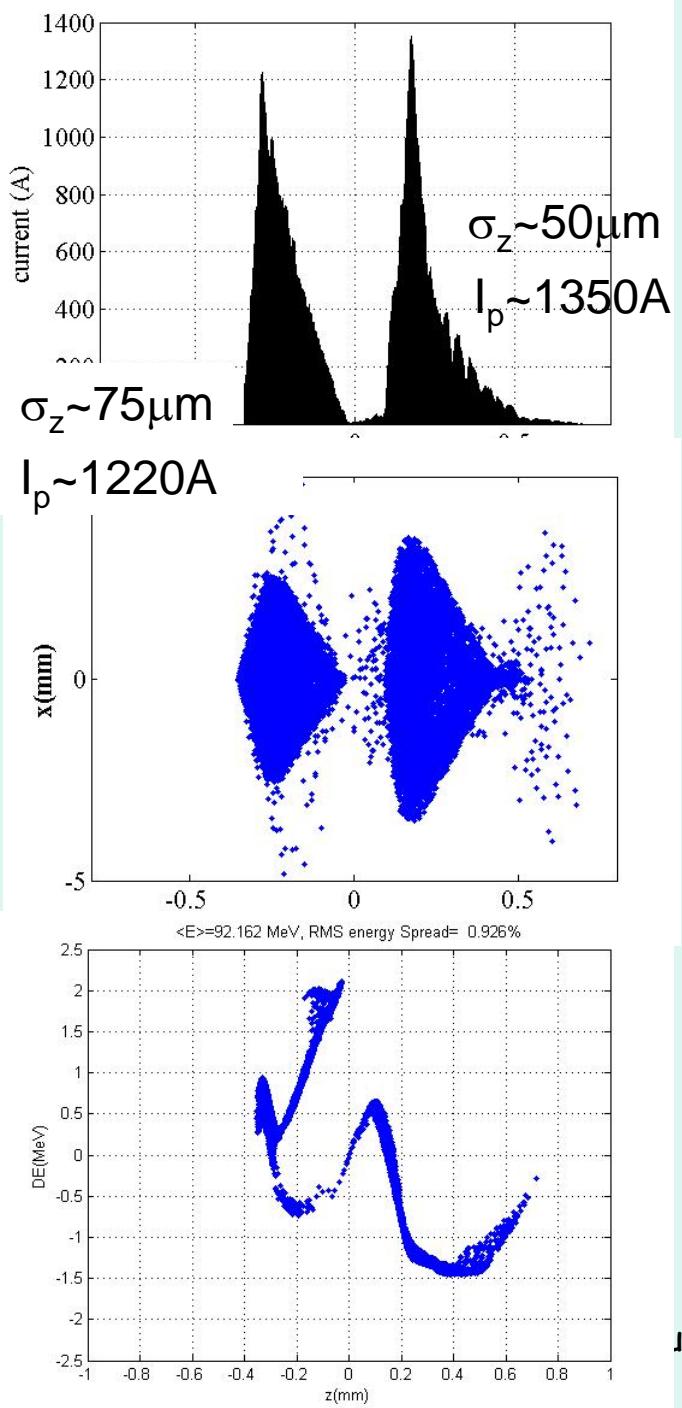
$x-\phi$
spatial configuration



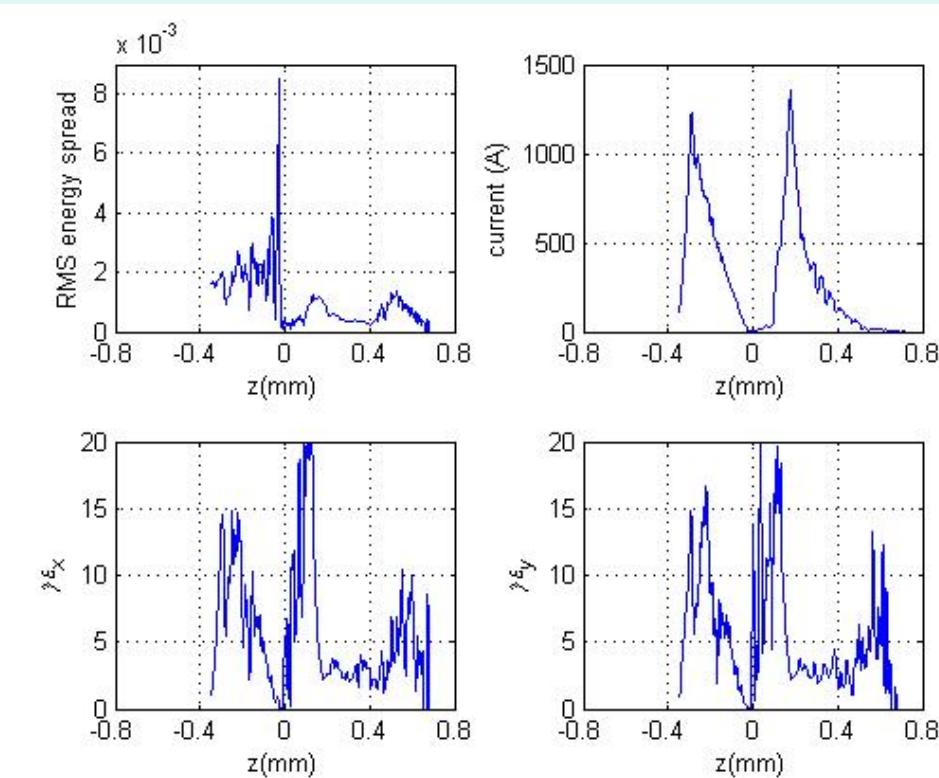
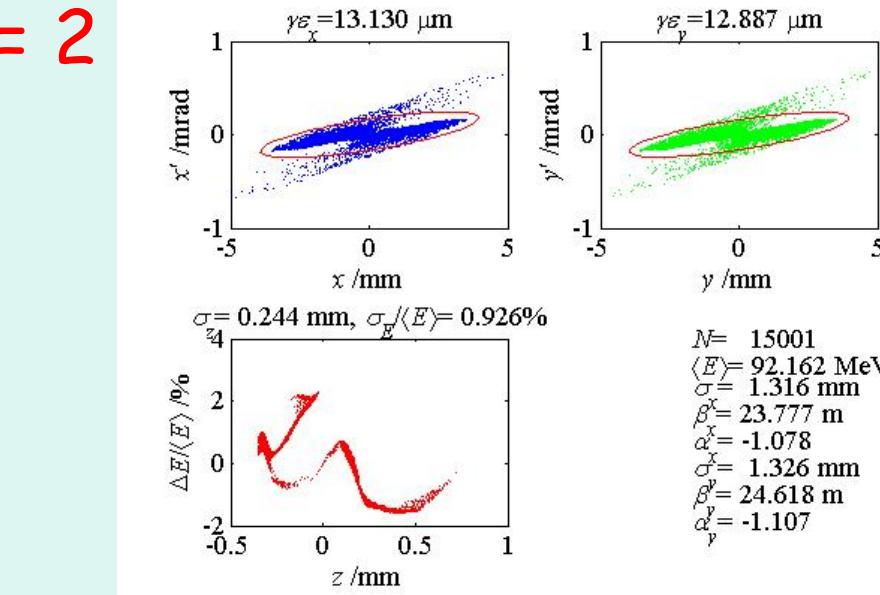
$\Delta E = 0.6$ MeV

cylinders evolve
toward two
simil-ellipsoids

Distance between 2 pulses can be set at any required value:
interesting for pulse probe and in PWFA experiments

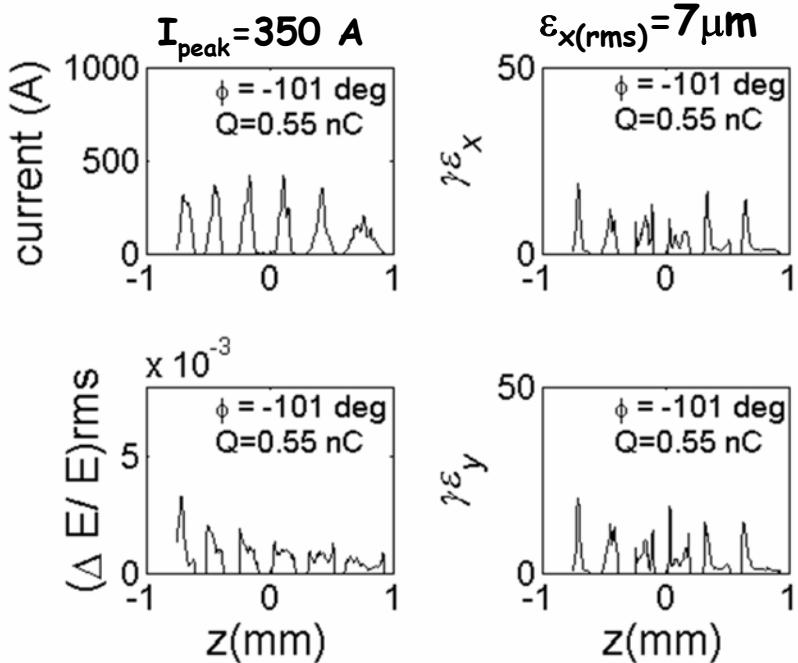


$N = 2$

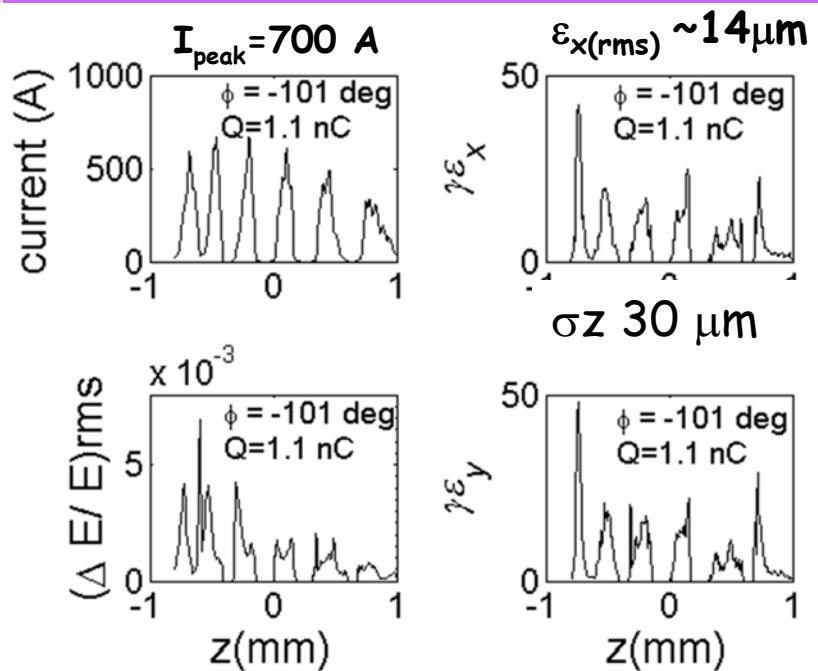


Current and emittance follow the ratio between total charge

Half of nominal charge $Q=0.55\text{nC}$



High nominal charge $Q=1.1\text{nC}$



How to control emittance: there is a trade off between the amplitude modulation and emittance

Emittance is reduced switching on the TW solenoids

Example: $\epsilon_x = 8 \mu\text{m}$ $Q = 1.1\text{nC}$ with $B(G) = 2720 / 1500 / 1600$

but current modulation is lost

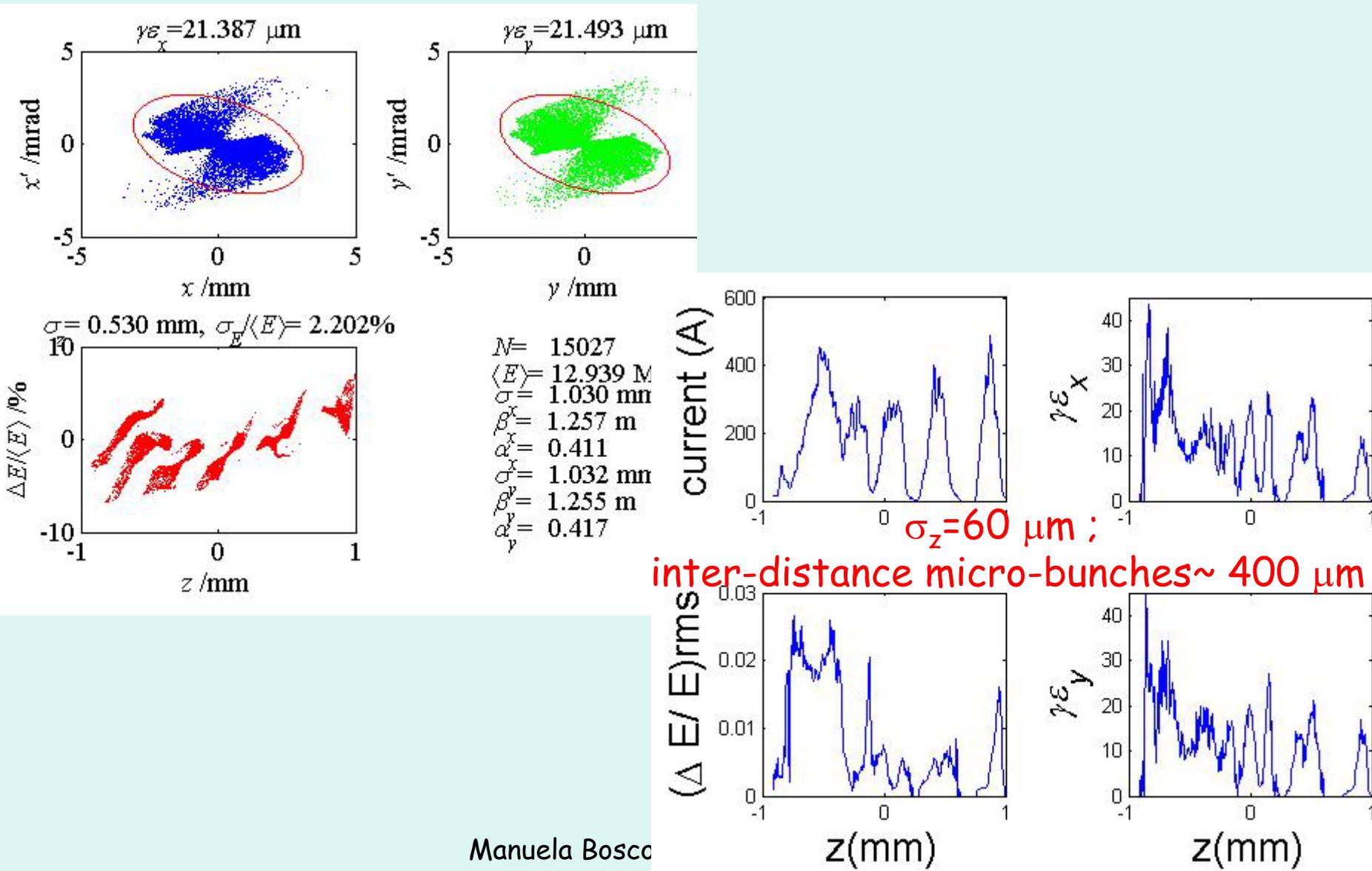
Example of interesting parameter sets

6 peaks 0.2 ps	Exit of 3 TW	Exit of rf compr.	Exit of rf compr
E(MeV)	90	18	13
γ	170	36	25
ε_x rms (μm)	14	17	21
$\Delta E/E$ rms(%)	0.8	6	2
Ipeak (A)	700	1000	450
Inter-distance λr (μm)	200	330	460

Exit from velocity buncher

$\gamma = 25$

Gradient= 16MV/m



Conclusions

The space charge force,
which is considered a destructive force,
in this case is turned into a constructive force:
it provides the turning of the density modulation into
energy modulation.

This energy modulation by a dispersion system is again
transformed into density modulation.

The simulation shows that the system is very efficient.
A comb beam accelerator relies on the capability of the
laser which drives the rf gun to provide target light
profiles by means of a versatile shaping system
inserted in the laser system.

Open Questions

- Control emittance / energy spread
- Fix energy - inter-distance of peaks
- choose set of parameters to study in details and to optimize
- FEL process

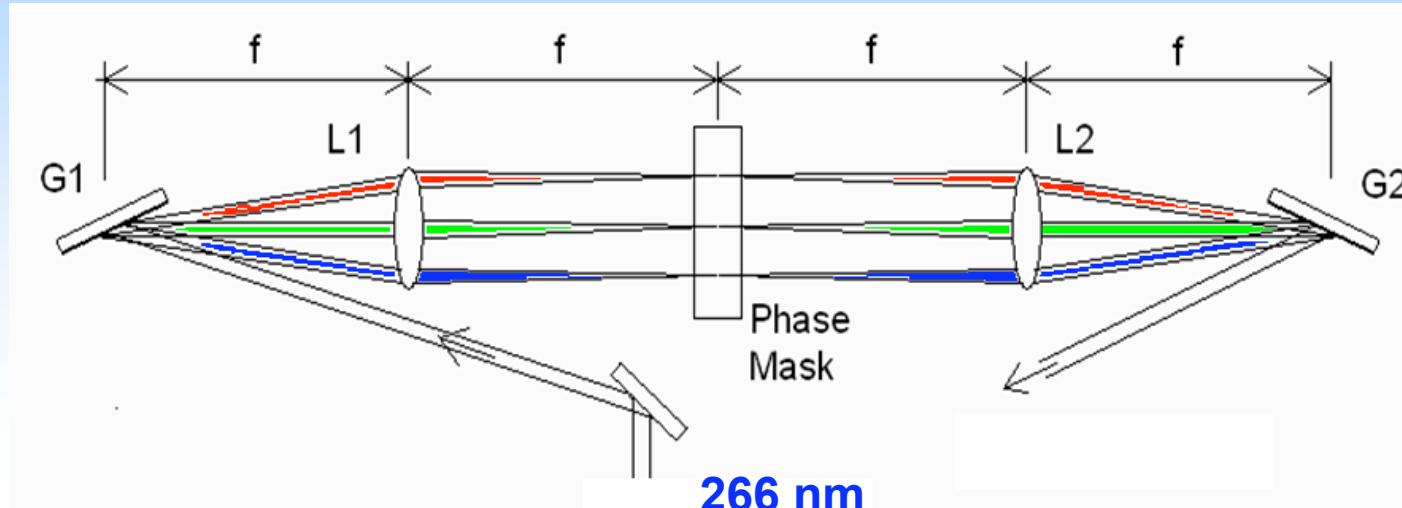
Apparati ottici per la generazione di treni di impulsi laser

Simone Cialdi

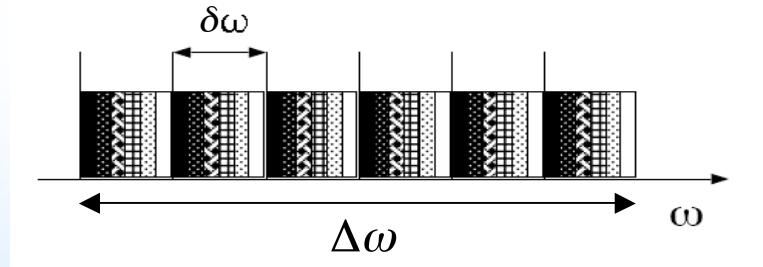
Outline

- 4f + periodic phase modulation
 - Interference
- 4f-stretcher + amplitude modulation
 - Spectrum → Time
- 2f + spatial mask
 - Space → Time
- Beam splitters
 - Delay time
- Interferometer + stretcher
 - Interference → Time
- Considerations

4f + periodic phase modulation



$$I(t) = \left| \int d\omega A(\omega) e^{i\phi_p(\omega)} e^{i\omega t} \right|^2 =$$



$$= \left| \sum_{n=0}^{N-1} \int_0^{\delta\omega} d\Omega A \left(-\frac{\Delta\omega}{2} + n\delta\omega + \Omega \right) e^{i\phi(\Omega)} e^{i\left(-\frac{\Delta\omega}{2} + n\delta\omega + \Omega\right)} \right|^2$$

“microlithographically fabricated phase mask”

J. Opt. Soc. Am. B 5, 1563 (1988)

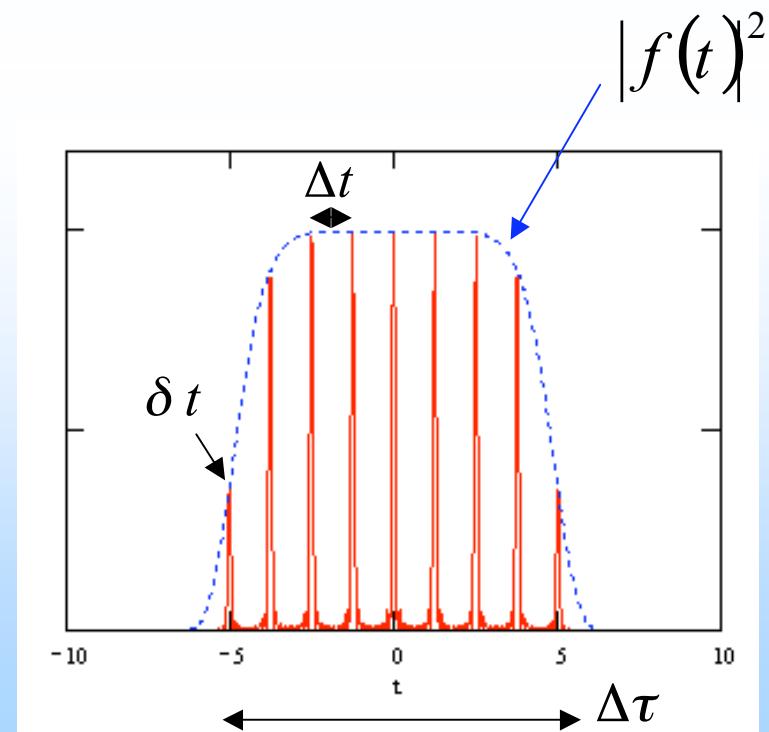
$$= \left| \sum_{n=0}^{N-1} \int_0^{\delta\omega} d\Omega e^{i\phi(\Omega)} e^{i\left(-\frac{\Delta\omega}{2} + n\delta\omega + \Omega\right)} \right|^2 = \left| \sum_{n=0}^{N-1} e^{in\cdot\delta\omega\cdot t} f(t) \right|^2 =$$

$$I(t) = \frac{\sin\left(\frac{N\delta\omega t}{2}\right)^2}{\sin\left(\frac{\delta\omega t}{2}\right)^2} |f(t)|^2$$

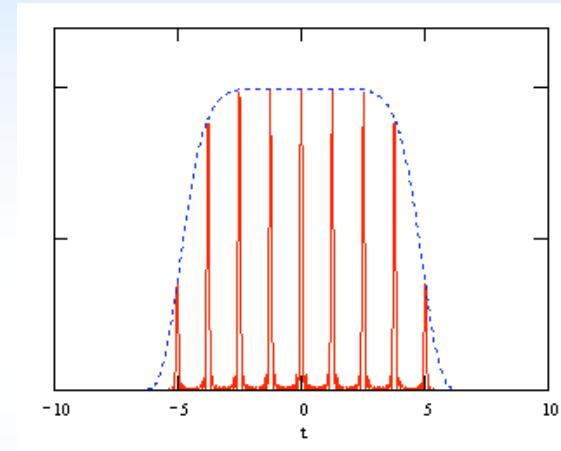
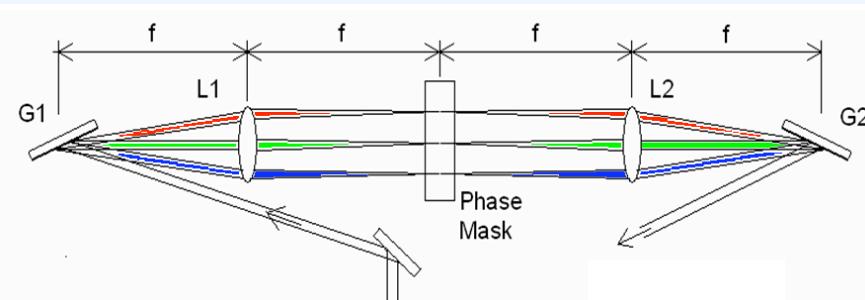
$$\Delta t = \frac{2\pi}{\delta\omega} \quad \delta t = \frac{2\pi}{N\delta\omega} = \frac{2\pi}{\Delta\omega}$$

$$\Delta\tau = \frac{2\pi}{\delta\omega_{pixel}}$$

$f(t) = \int_0^{\delta\omega} d\Omega e^{i\phi(\Omega)} e^{i\Omega t}$



4f + modulazione di fase periodica (pregi e difetti)



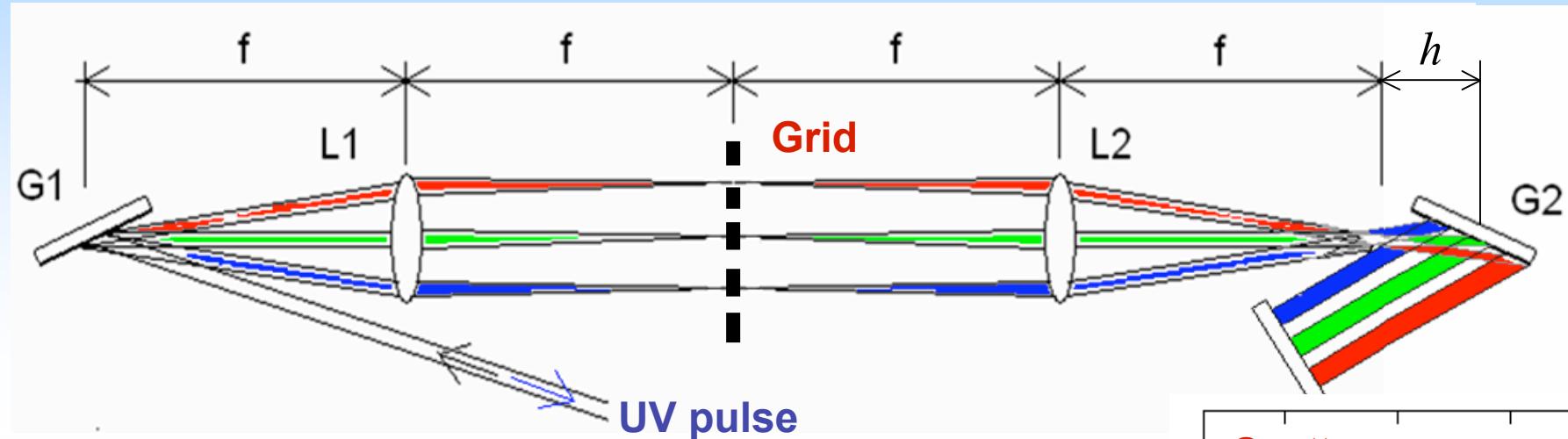
pregi

- Il singolo impulso e' transform- limited
- Il profilo del treno e' libero

difetti

- Tutti gli impulsi stanno alla stessa distanza
- La diagnostica non e' single-shot

4f-stretcher + modulazione di ampiezza

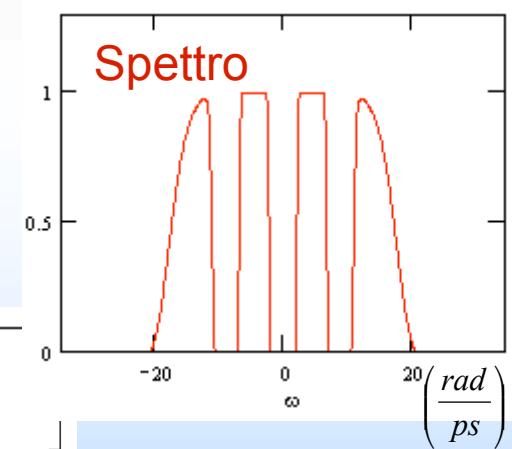
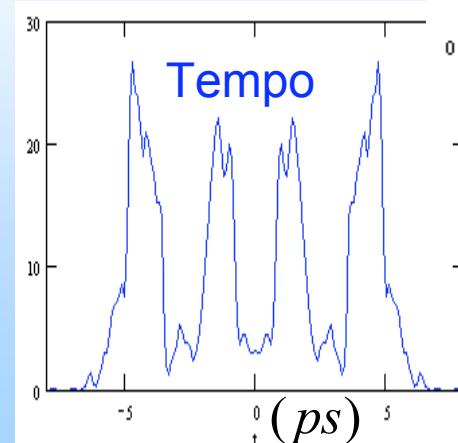


$$I(t) = \left| \int A(\omega) e^{i\frac{1}{2}\beta\omega^2} e^{-i\omega t} d\omega \right|^2 \approx \tilde{I}(\omega(t))$$

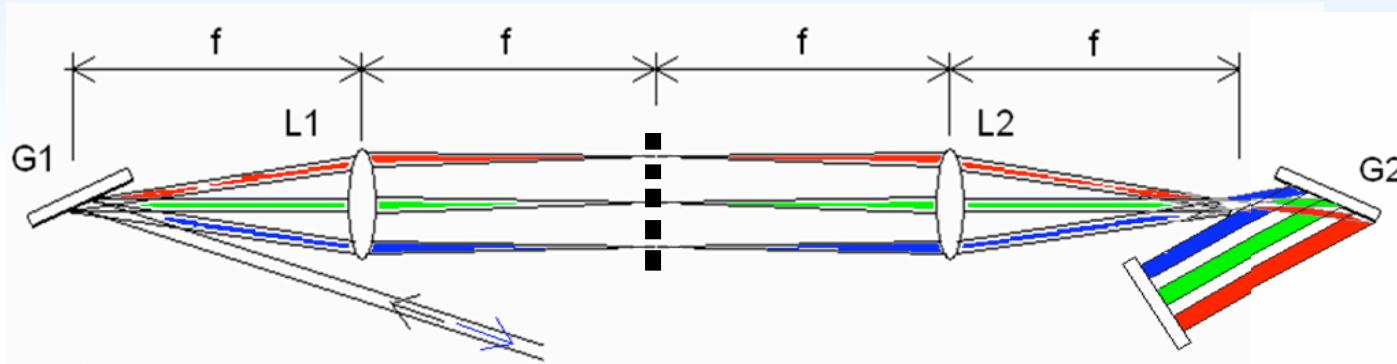
$$\omega(t) = \alpha \cdot t$$

$$\delta t = \alpha \cdot \delta \omega$$

$$\delta t > \frac{1}{\delta \omega}$$



4f-stretcher + modulazione di ampiezza (pregi e difetti)



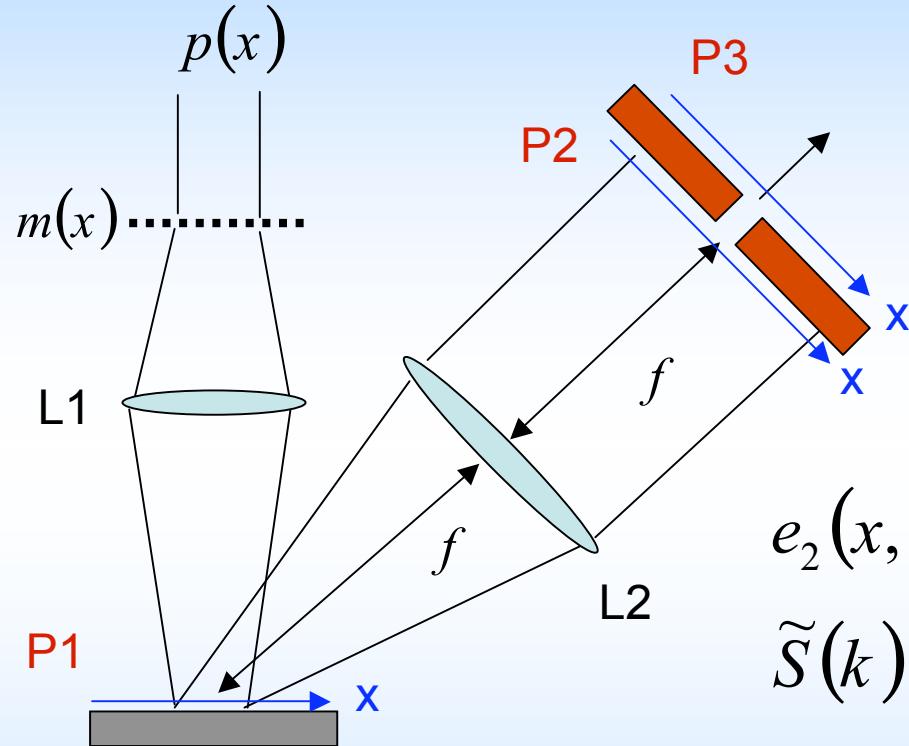
pregi

- La diagnostica e' single-shot
- La realizzazione della maschera e' semplice

difetti

- Il singolo impulso puo' essere minimo 1.5 ps

2f + maschera spaziale



$$e_1(x, t) = \int d\omega \tilde{E}_{in}(\omega) s(x) e^{i\omega \cdot t}$$
$$s(x) = p(x) \cdot m(x)$$

$$e_2(x, t) \propto \int d\omega \tilde{E}_{in}(\omega) \tilde{S}(\beta(x - \alpha\omega)) e^{i\omega \cdot t}$$
$$\tilde{S}(k) = FT(s(x))$$

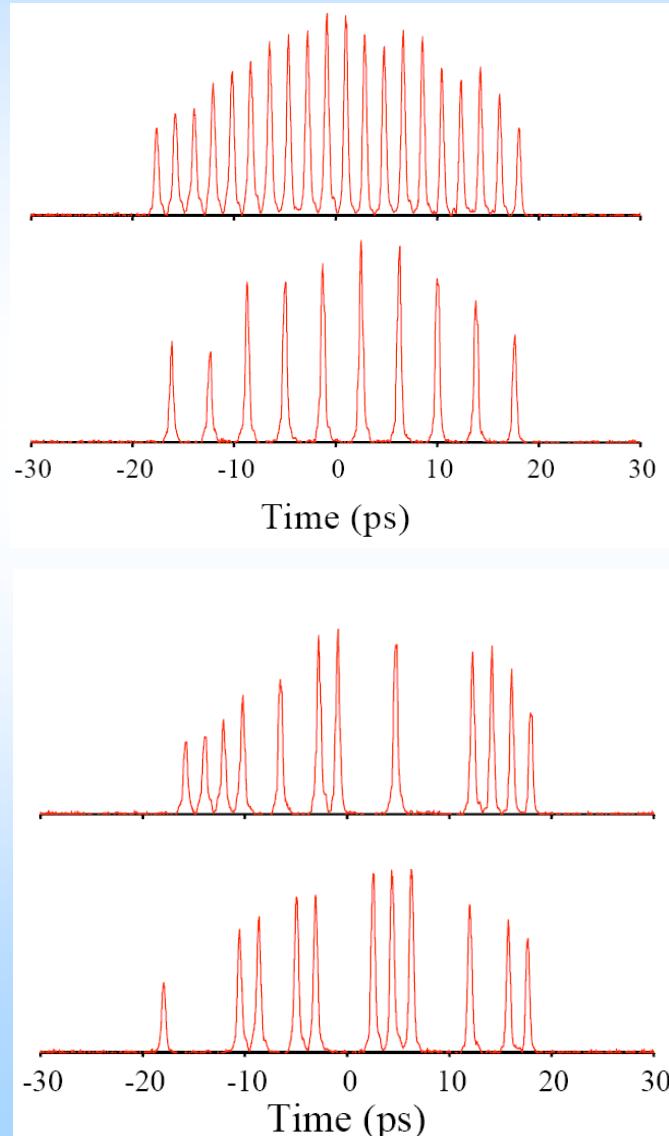
$$k = \beta \cdot x$$
$$\beta = \frac{2\pi}{\lambda f} \frac{\cos(\theta_d)}{\cos(\theta_i)}$$

Scaling factor

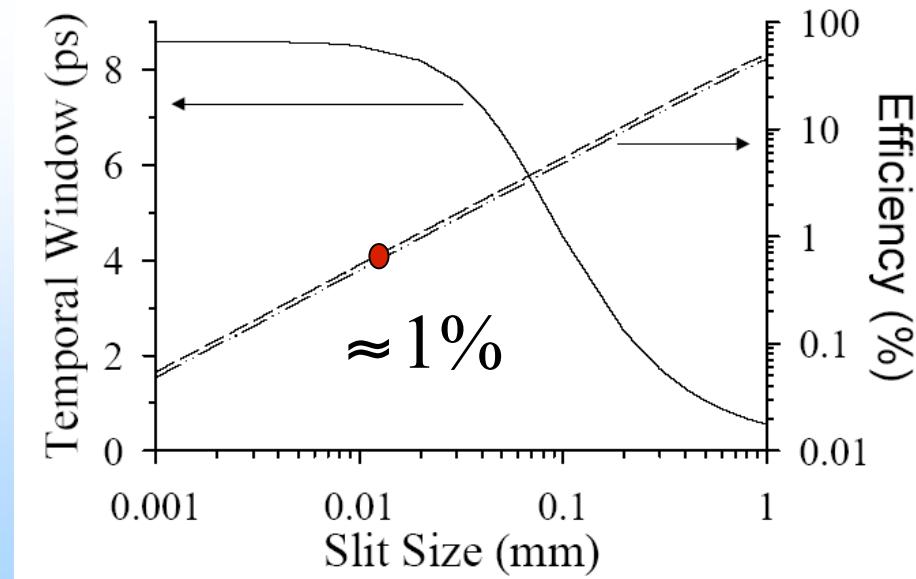
$$\alpha = \frac{\lambda^2 f}{2\pi c d \cos(\theta_d)}$$

Spatial dispersion

$$e_3(t) \propto \int d\omega \tilde{E}_{in}(\omega) \tilde{S}(-\alpha\beta\omega) e^{i\omega t} \propto e_{in}(t) \otimes s\left(-\frac{t}{\alpha\beta}\right) \approx s\left(-\frac{t}{\alpha\beta}\right)$$

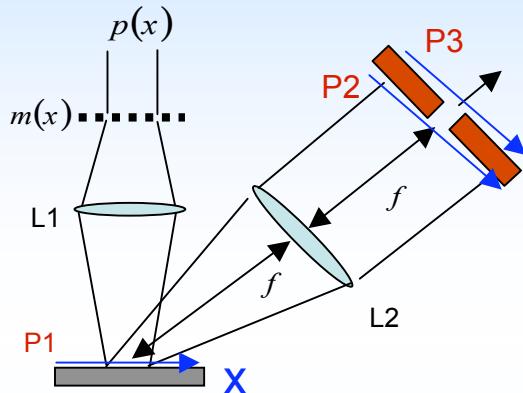


Spazio **Tempo**



2f + maschera spaziale

(pregi e difetti)



$$e_3(t) \approx s\left(-\frac{t}{\alpha\beta}\right)$$

$$s(x) = p(x)m(x)$$

pregi

- La successione degli impulsi e' libera
- La diagnostica e' single shot (CCD)

difetti

- L'apparato deve essere collocato prima dell'amplificatore data la bassa efficienza

Propagazione del treno nell'amplificatore

$$e(t) = e^{-\frac{t^2}{\tau^2}} (1 + \cos(\gamma t)) = e^{-\frac{t^2}{\tau^2}} \left(1 + \frac{1}{2} e^{i\gamma t} + \frac{1}{2} e^{-i\gamma t} \right)$$

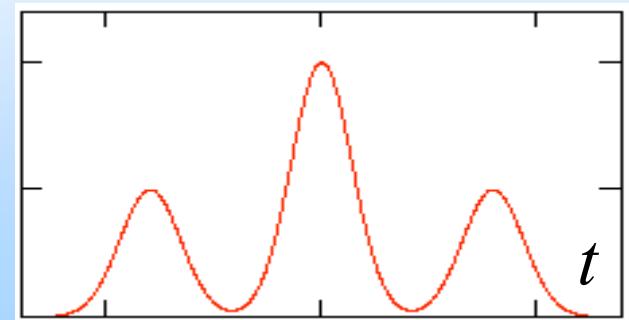
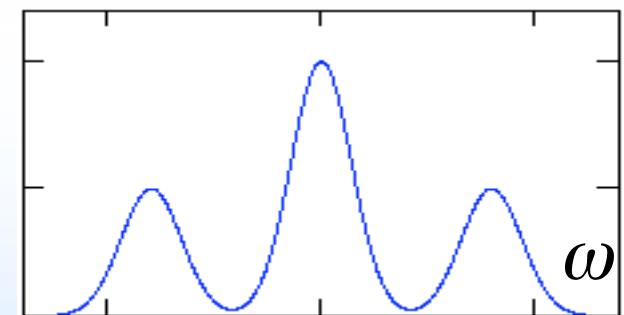
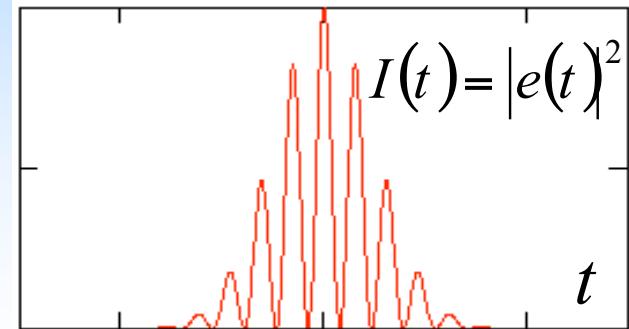
\downarrow
 FT

$$\tilde{e}(\omega) \propto \frac{1}{2} e^{-\frac{\tau^2}{4}(\omega+\gamma)^2} + e^{-\frac{\tau^2}{4}\omega^2} + \frac{1}{2} e^{-\frac{\tau^2}{4}(\omega-\gamma)^2}$$

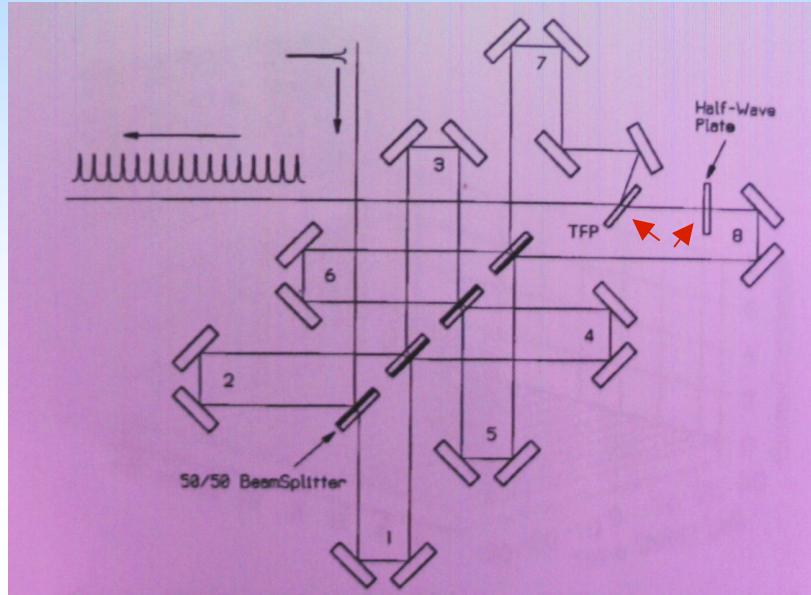
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Stretcher

$$I_{amp}(t) = |\tilde{e}(\omega(t))|^2$$

$$\omega(t) = \beta \cdot t$$



Beam splitters

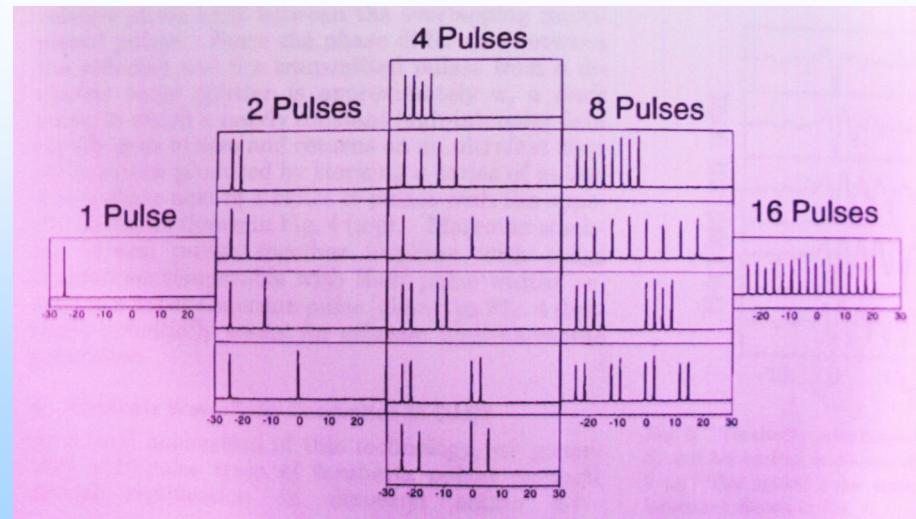


2^n pulse train

$\begin{cases} n \text{ beam splitters} \\ (4n + 2) \text{ mirrors} \\ 2n \text{ linear traslators} \end{cases}$

$$PBS \rightarrow E_{out} \approx E_{in}$$

$$BS \rightarrow E_{out} \approx \frac{1}{2} E_{in}$$

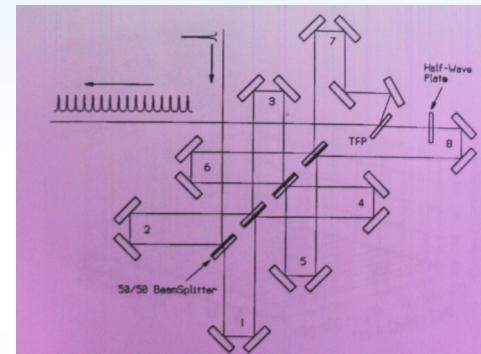


Beam splitters

(pregi e difetti)

$$PBS \rightarrow E_{out} \approx E_{in}$$

$$BS \rightarrow E_{out} \approx \frac{1}{2} E_{in}$$



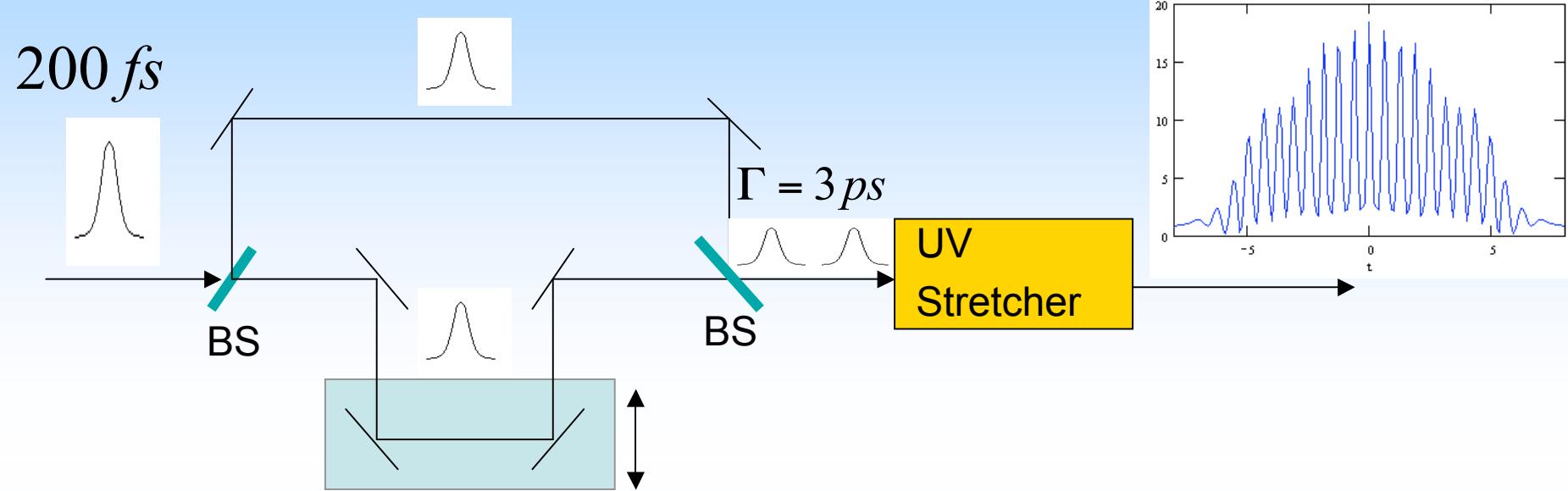
pregi

- La successione degli impulsi e' flessibile
- Alta eff. energetica

difetti

- Costo, allineamento e spazio

Interferometro + stretcher



$$A_{\text{int}}(\omega) = A_1(\omega) + A_2(\omega) = \frac{1}{2} A_{in}(\omega) + \frac{1}{2} A_{in}(\omega) \cdot e^{i\tau \cdot \omega} \quad \text{Interferometro}$$



$$A_s(\omega) = \frac{1}{2} A_{in}(\omega) e^{\frac{i}{2} \beta \omega^2} + \frac{1}{2} A_{in}(\omega) \cdot e^{i\tau \cdot \omega} e^{\frac{i}{2} \beta \omega^2} \quad \text{UV stretcher}$$

$$A_{in}(\omega) = e^{-\frac{\tau^2}{4}\omega^2}$$



$$A_s(\omega) = \frac{1}{2} e^{-\frac{1}{\gamma^2}\omega^2} + \frac{1}{2} e^{-\frac{1}{\gamma^2}\omega^2} e^{i\tau\cdot\omega}$$

$$\frac{1}{\gamma^2} = \frac{\tau^2}{4} - i\frac{1}{2}\beta$$



$$A_s(t) \propto e^{-\frac{\gamma^2}{4}t^2} + e^{-\frac{\gamma^2}{4}(t-\Gamma)^2} = e^{-\frac{\alpha^2}{4}t^2} e^{+i\frac{\eta}{2}t^2} + e^{-\frac{\alpha^2}{4}(t-\Gamma)^2} e^{+i\frac{\eta}{2}(t-\Gamma)^2} \approx e^{-\frac{\alpha^2}{4}t^2} e^{+i\frac{\eta}{2}t^2} (1 + e^{-i\eta\Gamma t})$$



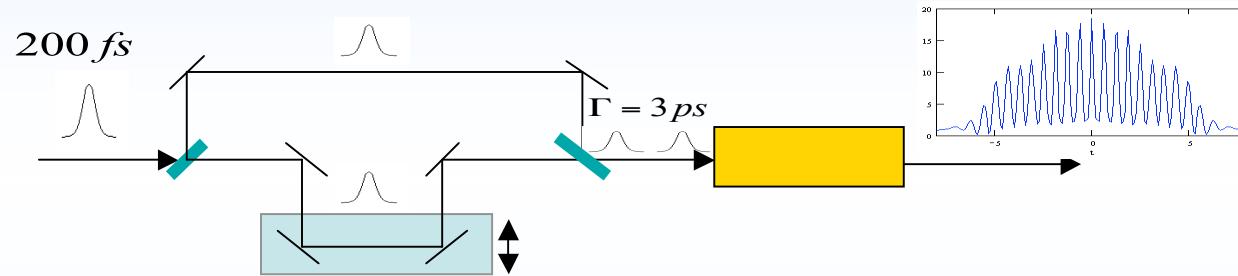
$$\Delta\tau_s \gg \Gamma$$

$$I_s(t) \propto e^{-\frac{\alpha^2}{2}t^2} \cos\left(\frac{\eta\Gamma t}{2}\right)^2$$

$$\alpha^2 = \frac{\tau^2}{\frac{\tau^4}{4} + \beta^2} \quad \eta = \frac{\beta}{\frac{\tau^4}{4} + \beta^2}$$

Interferometro + stretcher

(pregi e difetti)



pregi

- Semplice

difetti

- Modulazione cosinusoidale



Considerazioni finali

Apparato	Impulso Min	Distanza Impulsi	Profilo	λ
4f + PPM	T.L	Uguale per tutti	A piacere	266nm
4f-stretcher + AM	>1.5ps	Limitata	Flessibile	266nm
2f + SM	T.L.	A piacere	A piacere	800nm
BS	T.L.	Flessibile	Limitato	266nm
Interf. +stretcher	T.L.	Uguale per tutti = 2* lunghezza impulso	Flessibile	266nm

**Emissione spontanea da fascio modulato
e
ricaduta sull'azione FEL**

OUTLINE

- Introduzione
- Emissione spontanea da singolo elettrone
- Emissione incoerente da fascio omogeneo di elettroni
- Emissione coerente da fascio modulato
- Un esempio con i parametri di SPARC
- emissione spontanea ed emissione stimolata con un treno di microbunch
- Qualche commento

Introduzione

- IL problema di fare dei FEL con fasci modulati in densità è sempre stata considerata un'opzione molto interessante.

In particolare io e il gruppo di collaboratori di Bari-Lecce abbiamo lavorato su questo problema con la configurazione che abbiamo chiamato TOK transverse Optical Klystron

- Nel FEL il fascio si modula in densità in tutta la fase di letargia che è impegnata la parte maggiore della lunghezza del wiggler. Raggiunta la modulazione del fascio comincia il guadagno esponenziale ed in pochi passo di wiggler questo va in saturazione.
- Noi proponiamo di eliminare la fase di letargia con il sistema presentato da Manuela-Simone. Si capisce subito che vengono risparmiati tantissimi soldi
- Il sistema fascio modulato + wiggler emette una quantità interessante di radiazione spontanea coerente.
- Ma fascio modulato + wiggler naturalmente provocano immediatamente l'azione FEL.

Vogliamo vedere un po' questo problema

1 Le leggi per il calcolo dell'emissione

- La legge che considera *la corrente J*

L'equazione dell'emissione di radiazione da una corrente J è (Jackson)

$$\frac{d^2I}{d\omega d\Omega} = \frac{\omega^2}{4\pi^2 c^3} \left| \int dt \int d^3 \vec{r} [\hat{n} \times (\hat{n} \times \vec{J}) \cdot \exp[i\omega(t - \frac{\hat{n} \cdot \hat{r}}{c})]] \right|^2 \quad (1)$$

- La legge che considera l'emissione dalla *singola carica accelerata*

Il caso di moto non-relativistico ($\beta \ll 1$), la legge di Larmor

$$P_e = \frac{e^2}{6\pi\epsilon_0 c^3} \cdot \dot{v}^2 = \frac{e^2}{6\pi\epsilon_0 c^3} \cdot \ddot{x}^2 \quad (2)$$

$$= \frac{2}{3} r_0 \frac{m_0}{c} \dot{v}^2$$

Il caso generale (any β) ha la formula

$$P_e = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3m^2 c^3} \cdot \left(\frac{dp_\mu}{d\tau} \cdot \frac{dp^\mu}{d\tau} \right) \quad (3)$$

We have to calculate \ddot{x}^2 in undulator case

E' semplice fare i calcoli a partire dalla Eq. di Larmor, quindi seguiamo questo percorso

Assumiamo per comodità di calcolo il caso di undulatore, $K_w \ll 1$
Dobbiamo partire dall'Eq. oraria

$$x = a \sin(k_w z) = a \sin(k_w \beta_{||} ct) \simeq a \sin(k_w \beta ct) \quad (4)$$

Aampiezza di oscillazione

$$a = \left[= \frac{K_w}{\gamma k_w} \right] = \frac{K_w}{\gamma} \frac{\lambda_w}{2\pi} \quad (5)$$

Parametro dell'ondulatore

$$K_w [= a_0] = \frac{e B_w \lambda_w}{2\pi m_0 c} = \frac{e B_w}{m_0 c k_w} \propto B_w \lambda_w \quad (6)$$

2 Calcoli nel sistema in moto con gli elettroni

Il calcolo è immediato.

L'Eq. oraria è

$$x' = a \sin(\omega' t') \quad (7)$$

per il calcolo di \dot{v}^2

$$\ddot{x}^2 = a^2 \omega'^4 \frac{1}{2} = 8\pi^4 a^2 \nu'^4 \quad (8)$$

La potenza dalla formula di Larmor, ($\langle \sin^2 \rangle = 1/2$)

$$P_e = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c^3} \cdot (a \omega'^2)^2 \frac{1}{2} \quad \omega' = c \gamma k_w \quad (9)$$

In definitiva

$$P_e = \frac{1}{3} r_0 m_0 c^3 [\gamma^2 K_w^2 k_w^2] \propto q^2 \gamma^2 K_w^2 k_w^2 \quad (10)$$

The power is a Lorentz invariant quantity.

$$P_e \propto q^2 \dot{v}^2 \propto q^2 a^2 \omega^4 \propto q^2 \gamma^2 K_w^2 k_w^2 \quad a^2 = \left[\frac{K_w^2}{\gamma^2 k_w^2} \right] \quad \omega^4 = [c^4 k_w^4 \gamma^4]$$

Il termine

$$q^2 \gamma^2 K_w^2 k_w^2$$

contiene la fisica del moto dell'elettrone nell'ondulatore dentro nella formula di Larmor.

3 Calcolo nel Lab frame

Abbiamo fatto il conto per esteso nel Lab Frame

$$P_e = \frac{1}{6\pi\epsilon_0} \frac{e^2}{m_0^2 c^3} \dot{p}^2 \Rightarrow \frac{1}{6\pi\epsilon_0} \frac{e^2}{m^2 c^3} \cdot \left(-\frac{\partial p_\mu}{\partial \tau} \cdot \frac{\partial p_\mu}{\partial \tau} \right) \quad (11)$$

where $d\tau = dt/\gamma$ and p_μ is the momentum-energy tetravector of the electron.

In our case of wiggling motion and assuming $K_w < 1$, we get

$$\dot{\vec{\beta}} = \dot{\vec{\beta}}_\perp$$

therefore the equation becomes

$$P_e = \frac{e^2}{6\pi\epsilon_0 c} \gamma^6 \left[\dot{\beta}_\perp^2 - \beta^2 \dot{\beta}_\perp^2 \right]$$

Il risultato finale

$$P_e = \frac{e^2}{6\pi\epsilon_0 c} \gamma^4 \dot{\beta}_\perp^2 = \frac{e^2}{12\pi\epsilon_0 c} \gamma^4 c^2 \frac{K_w^2}{\gamma^2} k_w^2 \quad (12)$$

Con il raggio classico dell'elettrone

$$P_e = \frac{e^2}{12\pi\epsilon_0 c} \gamma^2 c^2 K_w^2 \gamma^2 k_w^2 = \frac{1}{3} r_0 c^3 m_0 \gamma^2 K_w^2 k_w^2 \quad (13)$$

OK

Se non è $K_w \ll 1$

$$P_e = \frac{1}{3} r_0 c^3 m_0 \gamma^2 \frac{K_w^2}{(1 + K_w^2)^2} k_w^2 \quad (14)$$

4 Potenza spontanea emessa da un fascio di elettroni

è data dalla potenza

emessa dal singolo elettrone moltiplicata per il numero di elettroni che oscillano dentro all'ondulatore

$$P_I = P_e \cdot n = P_e \cdot \frac{Q_{oscill}}{e} = \frac{I \tau}{e}$$

Se ho un fascio infinito

$$\tau = \frac{L_w}{\beta_{||} c} \simeq \frac{N_w \lambda_w}{c}$$

e quindi

$$P_I = \frac{4\pi^2}{3} r_0 m_0 \gamma^2 c^2 \frac{K_w^2}{\lambda_w} \cdot \frac{I}{e} N_w \implies \frac{4\pi^2 r_0}{3 e m_0 c^2} E^2(\text{energia}) \frac{K_w^2}{\lambda_w} N_w \cdot I$$

Con la sostituzione

$$m_0 \gamma^2 c^2 = \frac{E^2(\text{energy})}{m_0 c^2}$$

Se ho un impulso di elettroni di lunghezza temporale τ

$$P_{e-bunch} = P_e \cdot \frac{I \tau}{e c} = \frac{1}{3} r_0 c^3 m_0 \gamma^2 K_w^2 k_w^2 \cdot \frac{I \cdot \ell_{e-bunch}}{e}$$

$$P_I \propto e K_w^2 \gamma^2 k_w^2 (I N_w \lambda_w)$$

K_w = ampiezza oscillazione

γ^2 = energia

k_w = frequenza

$I L_w$ = numero di singoli oscillatori incoerenti

5 Energia emessa da un elettrone

$$E_{rad} = \int P_e dt = P_e \cdot \Delta t = P_e \cdot \frac{L_w}{\beta_{||} c}$$

$$E_{rad} = \frac{1}{3} r_0 m_0 c^2 \gamma^2 K_w^2 k_w^2 L_w$$

Energia emessa da un e-beam

Se ci sono N elettroni dell'e-beam che attraversano l'undulatore

$$N = \frac{Q_w(\text{charge})}{2}$$

$$E_{N-electrons-rad} = E_{rad} \cdot N = \frac{1}{3} r_0 m_0 c^2 \gamma^2 K_w^2 k_w^2 L_w \cdot \frac{I\tau}{e}$$

Da qualche parte sono partiti da questa formula per calcolare la emissione spontanea

6 Emissione coerente a partire dalla emissione del singolo elettrone

$$P_e = \frac{1}{3} r_0 m_0 c^3 \gamma^2 K_w^2 k_w^2$$

L'emissione da N_e coerenti/cooperativi elettroni è

$$q \implies Q = e \cdot N_{e-coh} \quad P_{coh} = P_e \cdot N_{e-coh}^2 \quad (15)$$

Il punto è: quanti elettroni sono cooperativi/coerenti?

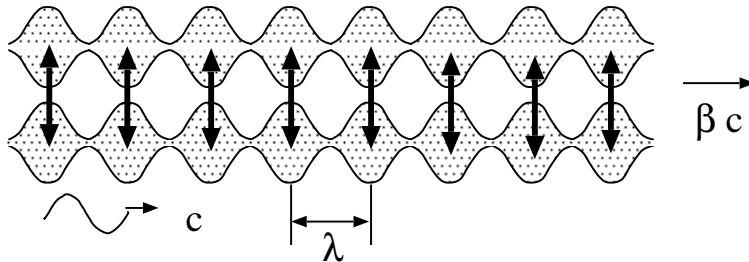
$$N_{e-cooperative} = N_{e-tot} \cdot A_1 = \frac{Q_{tot}}{e} \cdot A_1 = \frac{I \cdot \tau_{eb}}{e} \cdot A_1 \quad (16)$$

Ma

$$I \cdot \tau = I \cdot \frac{\ell_{e-bunch}}{\beta_{||} c} = I \cdot \frac{N_{eb} \cdot \lambda}{\beta_{||} c}$$

e

$$N_{eb} = \text{numero di picchi nel macrobunch} \quad \lambda = \lambda_w / 2\gamma^2$$



Alla fine dei conti

$$P_{coh} = \frac{\pi}{12} \eta_0 K_w^2 \cdot I^2 \left(\frac{N_{eb}}{\gamma} \cdot A_1 \right)^2 \simeq 10^2 \cdot K_w^2 \cdot I^2 \left(\frac{N_{eb}}{\gamma} \cdot A_1 \right)^2 \quad (17)$$

The vacuum impedance $\eta_0 = 377$

$N_{eb} = N_w$ per un fascio più lungo del wiggler

7 Ilario formula del 1980

Partenza la formula del Jakcson

$$\frac{d^2I}{d\omega d\Omega} = \frac{\omega^2}{4\pi^2 c^3} \left| \int dt \int d^3 \vec{r} [\hat{n} \times (\hat{n} \times \vec{J}) \cdot \exp[i\omega(t - \frac{\hat{n} \cdot \hat{r}}{c})]] \right|^2 \quad (18)$$

Il beam di elettroni è modulato in densità

$$\vec{J} = e c \vec{\beta} \rho_0 \sum_{m=0}^{\infty} A_m \cos[m(kz - \omega t)] \quad (19)$$

facciamo uno sviluppo in serie di Fourier della corrente con

$$\rho_0 = \frac{I}{e \cdot c}$$

$$\vec{\beta} = \beta_{\perp} \hat{x} + \beta_{\parallel} \hat{z} = \frac{K_w}{\gamma} \cos(k_w z) \hat{x} + \left[1 - \left(\frac{K_w}{2\gamma} \right)^2 \cos(2k_w z) \right] \hat{z} \quad (20)$$

Dopo paginate di conti molto incasinati, la potenza coerente sull'armonica m-esima per unità di angolo solido

$$\frac{dP_m}{d\Omega} = \left(\frac{dP_{spont}}{d\Omega_{spont}} \right) \frac{1}{4} \frac{\lambda_w}{(2\gamma^2)^2} \frac{1}{m} \frac{I}{e \cdot c} N_w A_m^2 \quad (21)$$

The spontaneous emitted power per unit solid angle in the forward direction by an homogeneneous beam (neglecting the oscillatory z-motion) is

$$\frac{dP_{spont}}{d\Omega_{spont}} = \frac{e^2}{2\epsilon_0} \frac{\gamma^4}{(1 + K_w^2)^2} K_w^2 \frac{1}{\lambda_w} \frac{I}{e} N_w^2 \quad (22)$$

Il calcolo dell'angolo solido da

$$\Delta\Omega \simeq \frac{\pi (1 + K_w^2)}{2\gamma^2 N_w} \quad (23)$$

e quindi

$$P_{spont} = \frac{dP_{spont}}{d\Omega_{spont}} \Delta\Omega \simeq \frac{\pi e}{4\epsilon_0} \frac{K_w^2}{1+K_w^2} \gamma^2 \frac{1}{\lambda_w} I N_w \quad (24)$$

e per la potenza coerente

$$P_{m-coh}(W) = \frac{\pi}{8} \frac{K_w^2}{1+K_w^2} \eta_0 I^2 \left(\frac{N_w}{\gamma} \cdot A_m \right)^2 \frac{1}{m} \quad (25)$$

Per un impulso di elettroni moto corto

N_w va nel numero di microbunches del e-beam $N_e b$

Questa procedura di calcolo permette di trovare

- la distribuzione angolare della radiazione
- la distribuzione in frequenza

Parametri per il nostro conto

energia MeV	5
γ	10
$\lambda_w m$	$2 \cdot 10^{-2}$
$k_w m^{-1}$	$3.14 \cdot 10^2$
N_w	10
current I (catodo) A	100
Q_{tot} C	10^{-9} [1 nC]
K_w	0.1
$\lambda = \lambda_w/2\gamma^2$	$100 \mu m$
A bunching quote	0.5
<hr/>	
P_e W	$2.3 \cdot 10^{-15}$
P_I spontanea	2 mW
P_{coh-1}	22 KW
$P_{coh-ilario}$	33 KW

$$P_e = \frac{1}{3} \cdot 2.82 \cdot 10^{-15} \cdot 0.91 \cdot 10^{-30} \cdot 27 \cdot 10^{24} \cdot 10^2 \cdot 10^{-2} \cdot 10 \cdot 10^4 = 2.3 \cdot 10^{-15} \text{ W} = 2.3 \text{ femto-Watt}$$

$$P_I = 0.07257 \cdot \frac{25 \cdot 10^{-6} \cdot 10^{-2} \cdot 10^2 \cdot 10}{2 \cdot 10^{-2}} \sim 2 \text{ mW}$$

$$P_{coh-1} = P_e \cdot N_{e-coh}^2 = P_e \cdot \left(\frac{Q_{tot}}{e} \cdot A_1 \right)^2 = 2.3 \cdot 10^{-15} \cdot \left(\frac{10^{-9}}{1.6 \cdot 10^{-19}} \cdot 0.5 \right)^2 = \\ 2.3 \cdot 10^{-15} \cdot (0.31 \cdot 10^{10})^2 = 22 \text{ kW}$$

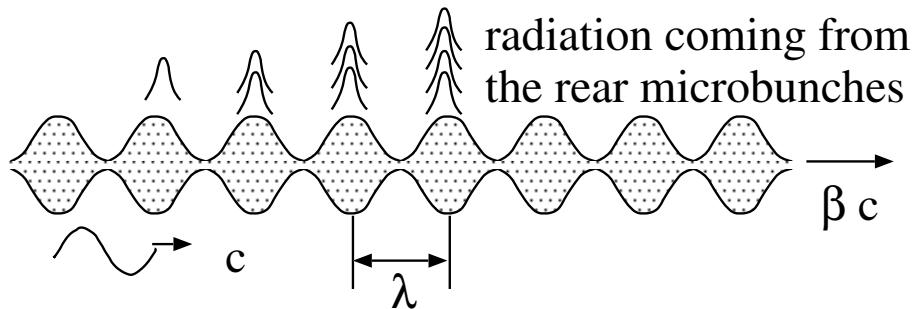
8 Qualche considerazione

- la emissione spontanea coerente da un fascio modulato in intensità ha un funzionamento indipendente dallo
 - spread di energia
 - dalla emittanza
 - dal detuning del FEL

Non c'è, a prima vista, una condizione di risonanza da soddisfare, ma c'è una condizione di fase da soddisfare.

- Invece, la situazione è più complicata perché la radiazione emessa dai microbunch di coda slappa in avanti e va ad agire sui pacchetti che sono più avanti secondo l'equazione

$$\frac{dmc^2\gamma}{dz} = eE_0 \frac{K_w}{2\gamma} \cos[(k + k_w)z - \omega t]$$



Questa è l'azione FEL di emissione stimolata.

Quindi è necessario mettere un wiggler taperato opportunamente per mantenere la risonanza.

- Noi non abbiamo qui la fisica del FEL SASE classico: in questo il fascio di elettroni si auto-organizza nello spezzone lungo quanto la lunghezza di cooperazione.

Nel nostro abbiamo un fascio già organizzato e per questo emette una radiazione buona per essere amplificata (per stimolare la emissione) e quindi è più utile pensare al tapering.

9 Nota sul Thomson scattering

It is interesting to write down the spontaneous emitted power in the form

$$P_e = \frac{1}{3} r_0 \frac{e^2}{m_0} c B_w \gamma^2 \quad (26)$$

and introducing the Thomson cross section for a photon diffusion by an electron

$$\sigma = \frac{8\pi}{3} r_o^2 \quad [= 0.665 \text{ barn}] \quad (1 \text{ barn} = 10^{-24} \text{ cm}^2) \quad (27)$$

and

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

the spontaneous power emitted by an electron in crossing an undulator

$$P_e = \frac{1}{2} \sigma c \frac{B_w^2}{\mu_0} \gamma^2 \quad (28)$$

The term

$$\frac{B_w^2}{2 \mu_0}$$

is the energy density of the magnetic field and γ^2 is the relativistic compression of that energy (one γ factor because of the electron sees the energy compressed and the second γ factor because of the electron does a second Doppler compression. Therefore P_e represents the scattered energy per unit time whose volume is Doppler compressed.

10 Thomson scattering con fascio modulato

Adesso la pseudoradiazione del wiggler vede dei centri diffusori ordinati in pacchetti risonanti con la pseudoradiazione: i microbunch fanno da specchi di Bragg che avrà una sua efficienza.

Simulazioni FEL per laser comb

V. Petrillo

- Due possibili scenari
- Fasci a bassa energia ($\gamma \approx 10$), lunghezza d'onda di radiazione submillimetrica($\lambda \approx 100$ micron), modulazione di densità sulla lunghezza d'onda
- Fasci ad alta energia($\gamma > 100$), lunghezza d'onda ottica, modulazione sulla lunghezza di cooperazione

Simulazioni fatte con codice FEL

- Equazioni 1d (per cominciare)

$$\left(\frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) A = \langle b \rangle$$

Aampiezza lenta del potenziale

$$\left(\frac{\partial}{\partial z} + \frac{1}{v} \frac{\partial}{\partial t} \right) p_j = -(A e^{i\theta_j} + cc)$$

Fattore di bunching

$$\langle b \rangle = \frac{1}{N_{L_M}} \sum_{L_M} e^{-i\vartheta_j}$$

Lunghezza di media (non necessariamente λ)

$$\left(\frac{\partial}{\partial z} + \frac{1}{v} \frac{\partial}{\partial t} \right) \theta_j = p_j$$

Modulazione sulla lunghezza d'onda

- Un singolo pacchetto con $l_b \approx \lambda$
- $\gamma = 10$, $\lambda_w = 2\text{cm}$, $\lambda = 200 \text{ micron}$, $I = 200 \text{ A}$, $r_b = 200 \text{ micron}$.
- Due pacchetti di dimensione minore della lunghezza d'onda al variare della distanza
- Dieci pacchetti
- Discussione sui problemi del modello

Dato che $I_b \approx \lambda$ il bunching iniziale è vicino ad 1.

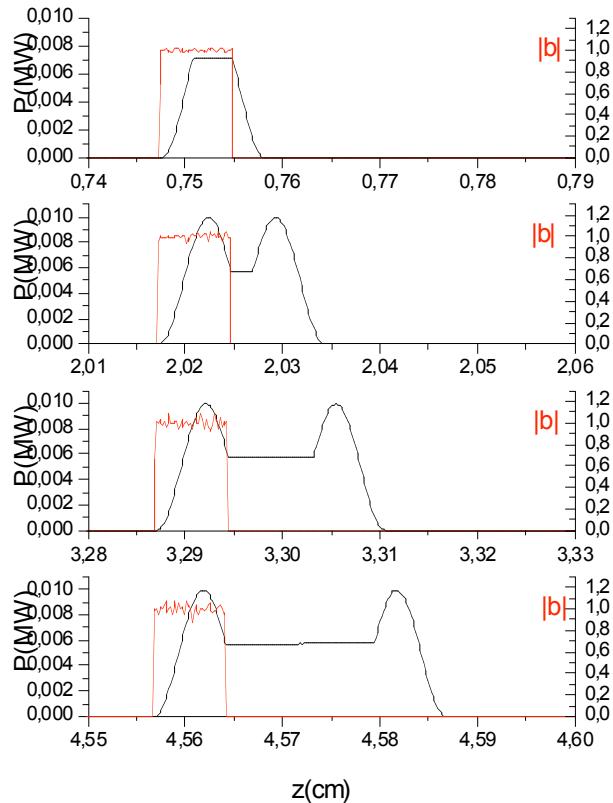
Singolo pacchetto prebunched : emissione spontanea **coerente** (superradianza debole)

$$P < P_{beam} \rho$$

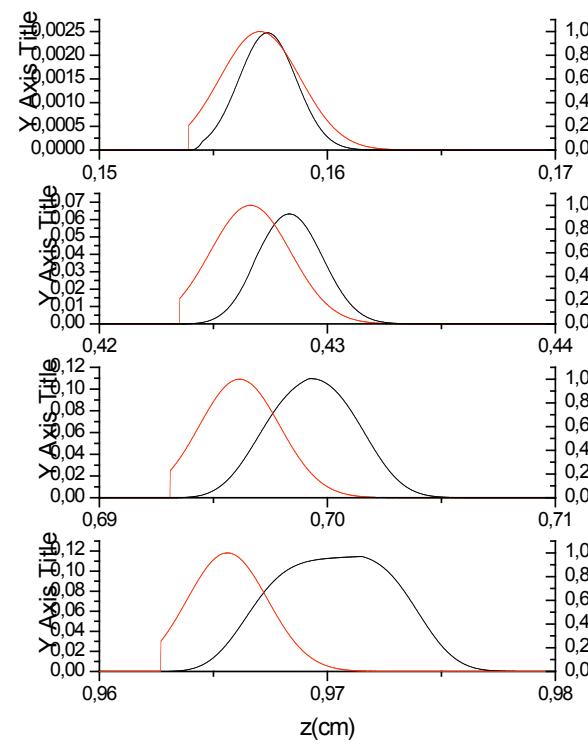
Differenza rispetto alla emissione spontanea **incoerente** (Thomson Scattering):
Invece di sommare le intensità si sommano i campi e si tiene conto dei termini di interferenza. Si tiene conto della deformazione (longitudinale) della traiettoria dovuta alla radiazione.

Simulazioni fatte con codice FEL 1d.

Fascio rettangolare

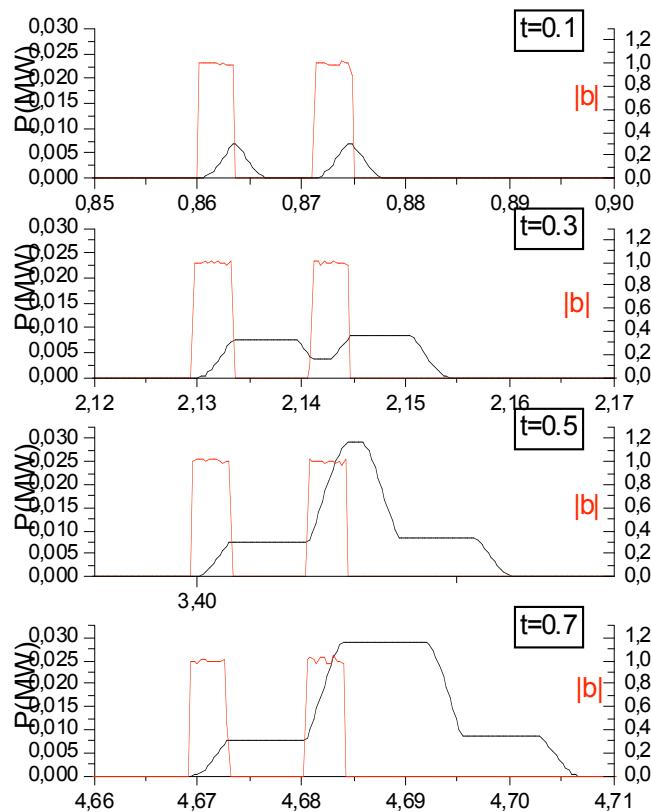


Fascio Gaussiano



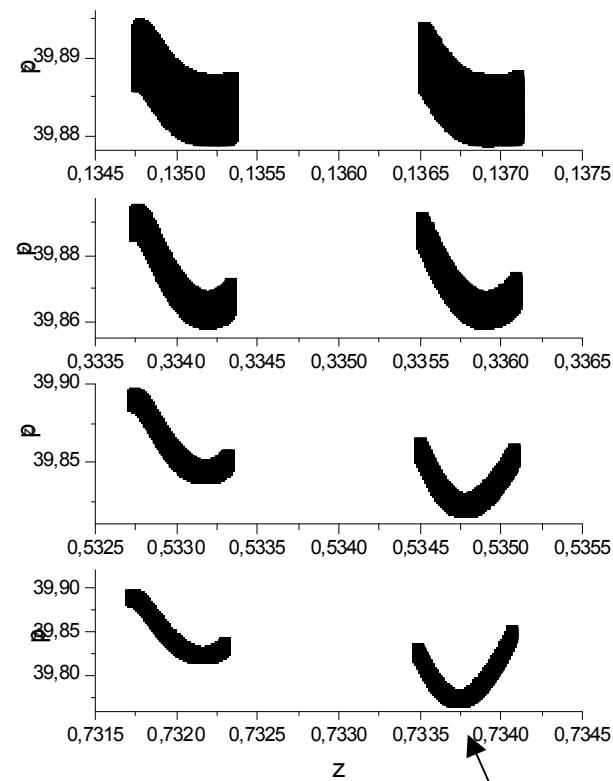
P

Due pacchetti in fase



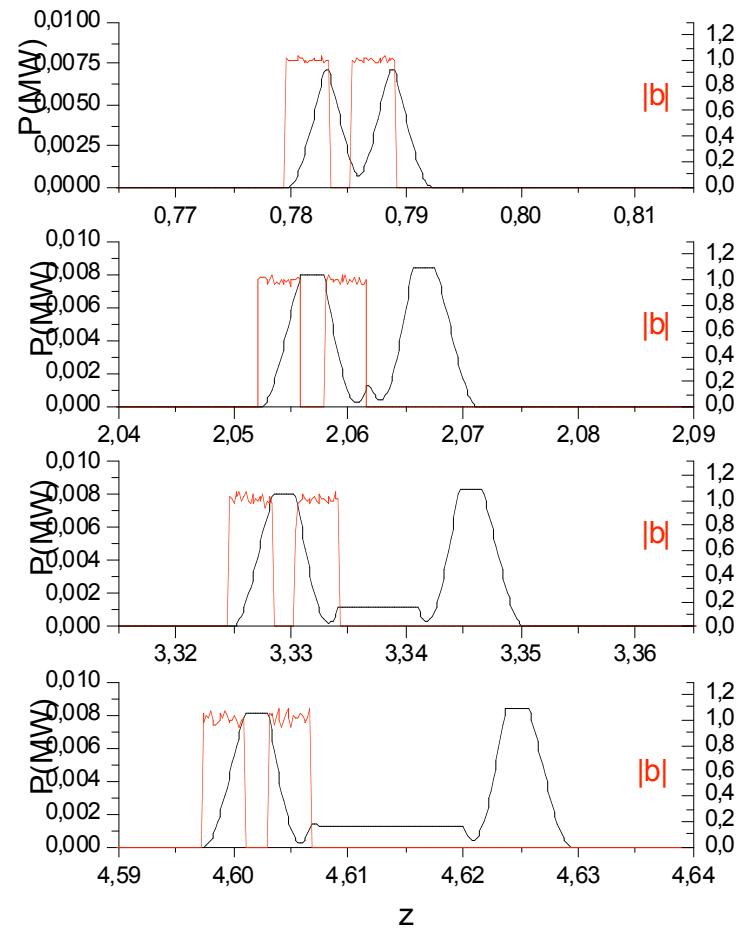
$$|\mathbf{E}|^2 = |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\text{Re}(\mathbf{E}_1^* \mathbf{E}_2)$$

Termine incoerente

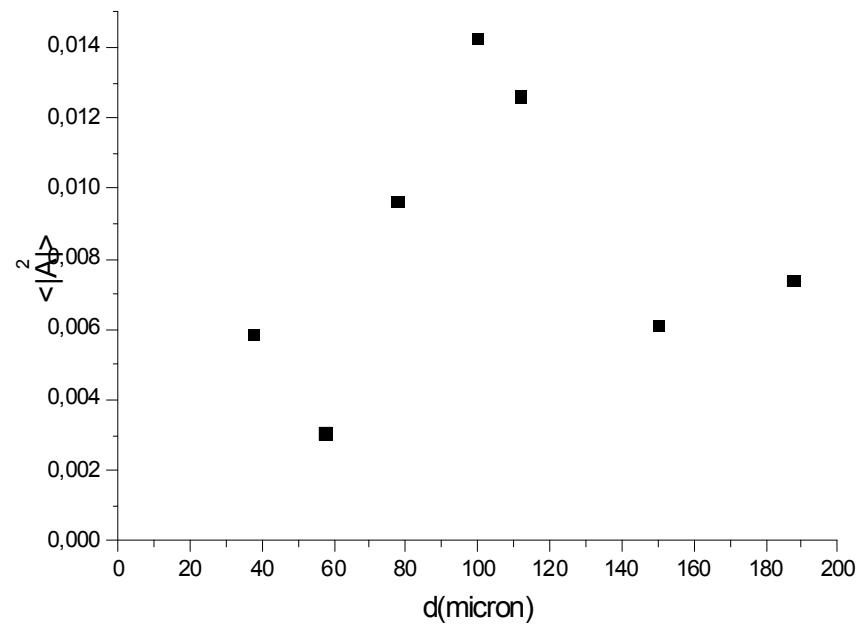
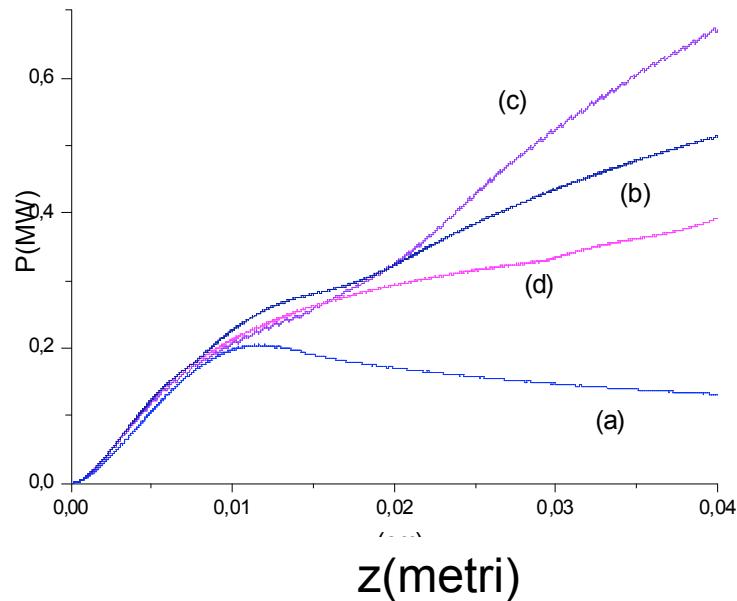


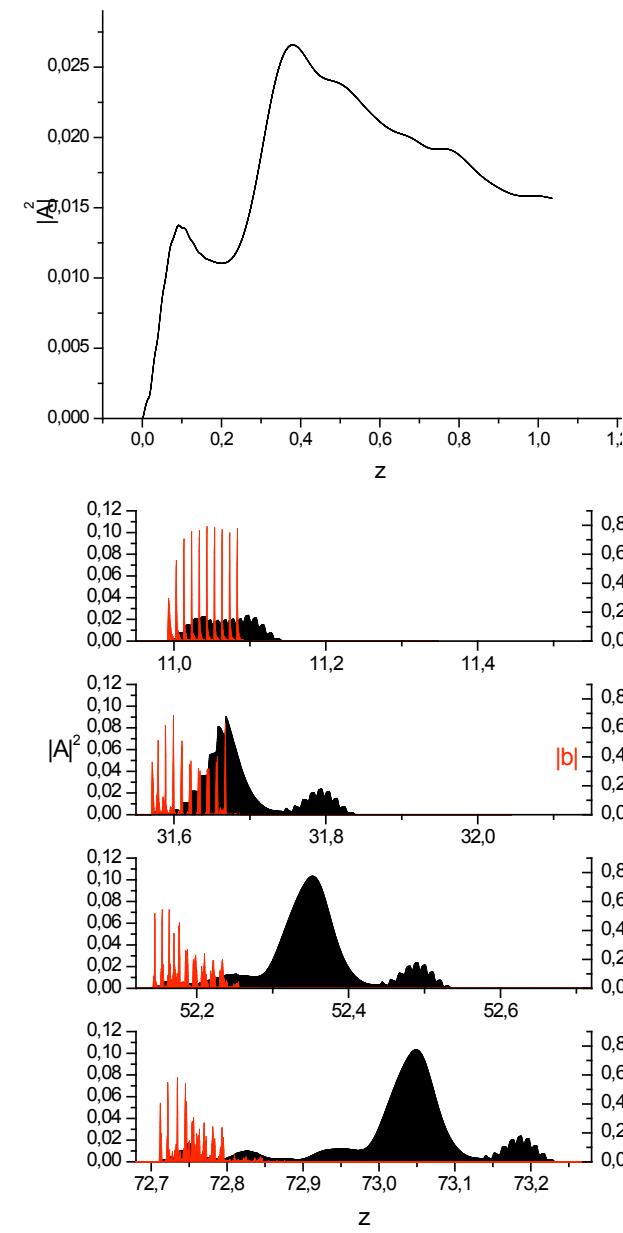
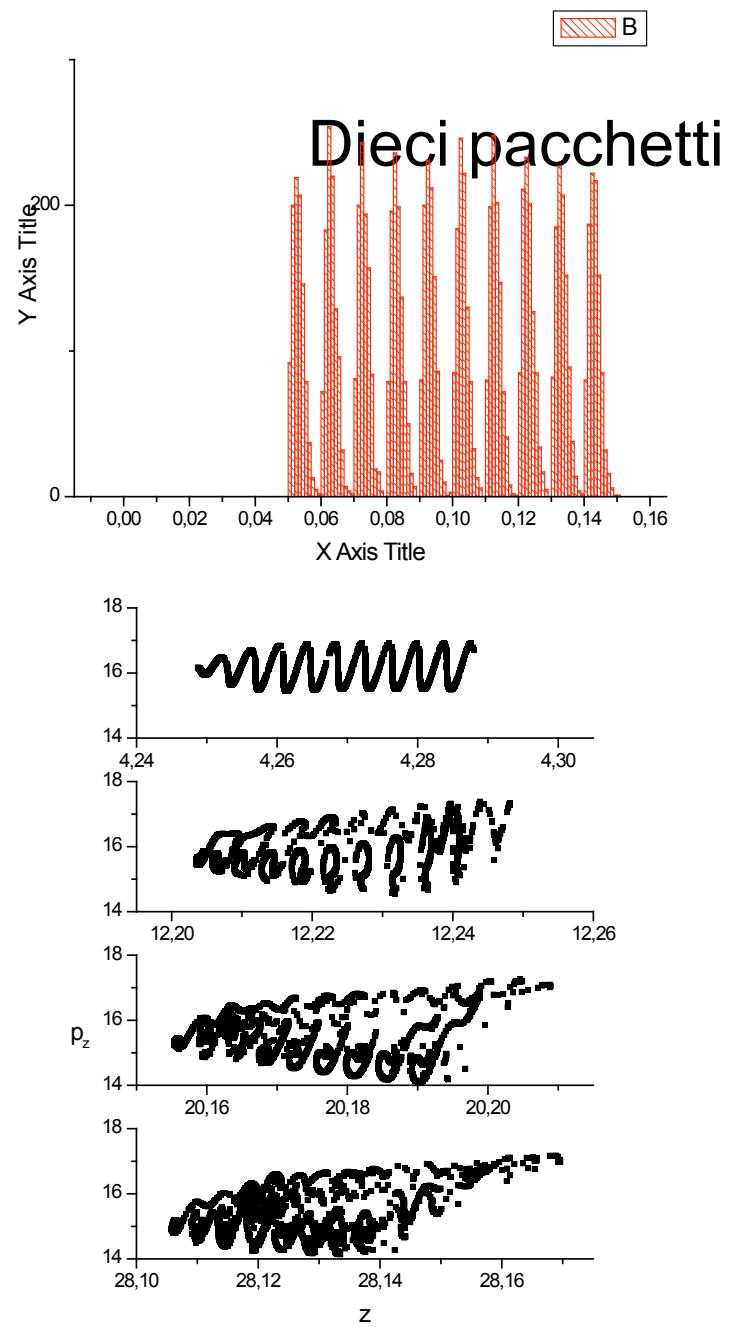
Effetto del moto longitudinale
nell'onda ponderomotrice

Due pacchetti in controfase



Radiazione al variare della distanza tra i pacchetti





- Problemi del modello

I parametri violano la Svea!!

Si possono scrivere e integrare equazioni FEL
NON-SVEA

Si può utilizzare il codice RETAR

Gli effetti trasversi sono importanti!!

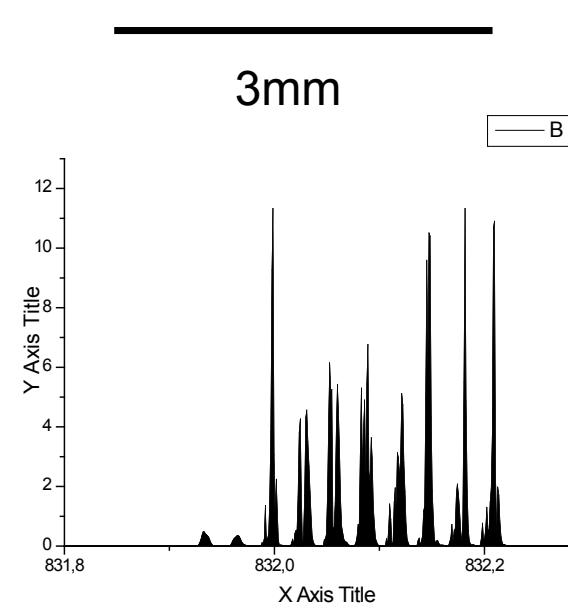
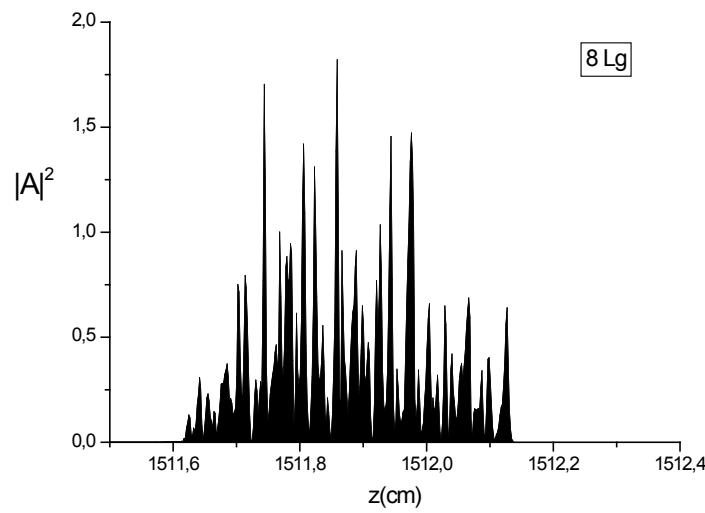
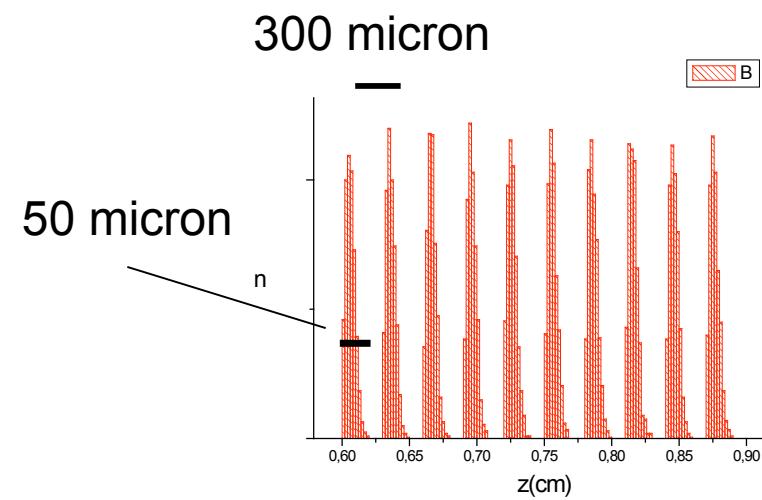
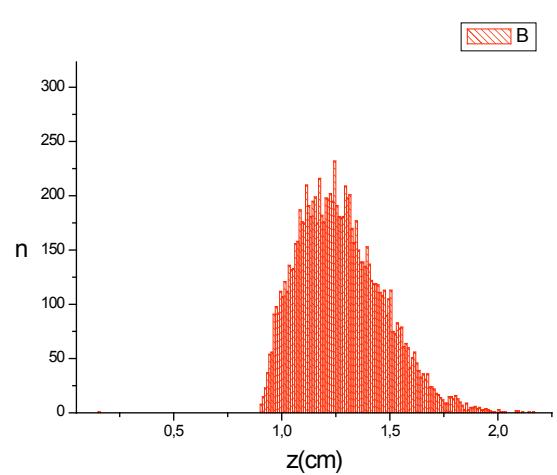
Soprattutto la diffrazione della radiazione

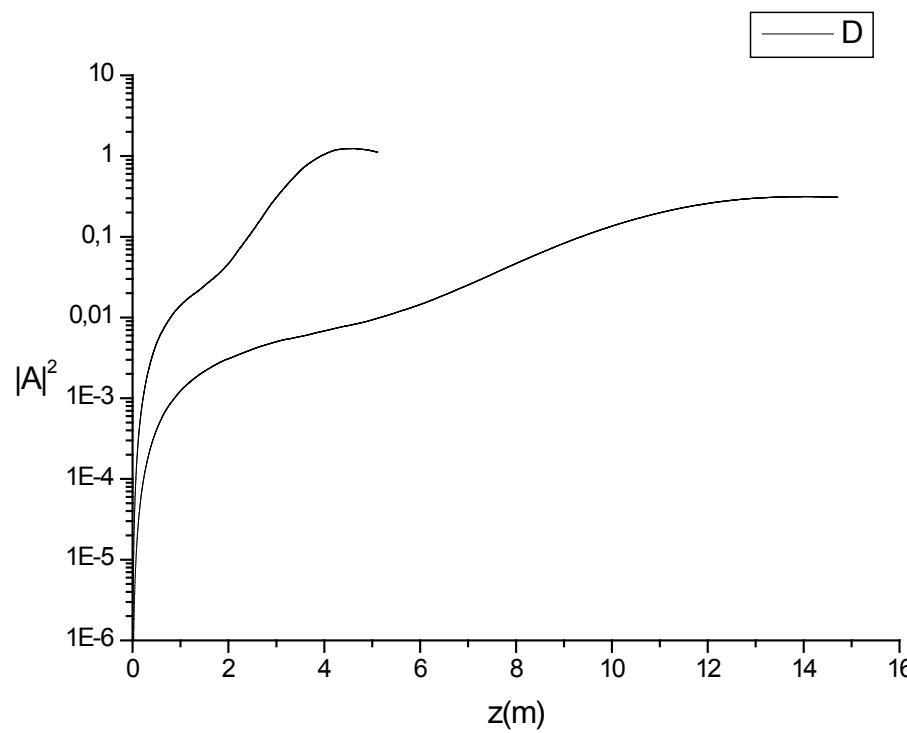
Stime di Ming Xie, codici tridimensionali

- $\rho=0,015$ $L_g=61 \text{ mm}$ $L_R=5 \text{ mm}$

Per mantenere il più possibile i pacchetti in risonanza
si può taperare

SPARC case aw0=1.4 γ =310, $\rho=3.5 \times 10^{-3}$, r=1.13 mm, lb=3 mm, $\lambda w=2.8$ cm





$\lambda=0.69\text{micron}$ $\text{dens}=9.94\text{E}+012$ $\rho=3.56\text{E}-003$ $\gamma=170.$ $p_0=280.4$
 $L_g=44.63\text{ cm}$ $L_c=15.44\text{micron}$ $N_{\text{spike}}=10.30$
 $D_{\text{spike}}=97.05\text{micron}$ $q=5.769\text{E}-006$

