



sparc-BD-07/001

# ANALYSIS OF INHOMOGENEITIES EFFECTS ON THE PHOTOCATHODE USING RETAR

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# Abstract

Effects of inhomogeneity at the photocathode are studied using the code RETAR. We model transverse inhomogeneities that can appear due to either laser profile or quantum efficiency at the cathode by shaping the beam particle distribution.

## 1. INTRODUCTION

We describe the use of the code RETAR [1] in the analysis of the effects that transverse inhomogeneities, simulating either the profile of the laser power impinging the photocathode or the quantum efficiency, can have on the particle dynamics. We will present as example the study of the propagation of the electrons extracted from the photocathode and accelerated in the electric and magnetic fields of the SPARC injector [2] up to the first two meters, for two different configurations of in-homogeneities, relevant to a band and to a point structure. The envelope and the emittance of the beam are calculated and their variation is studied as function of the magnitude of the inhomogeneities, following the same line of other works [3, 4].

#### 2. THE CODE RETAR

The code RETAR is a point to point transport code that takes into account the presence of the self-consistent radiation field on each particle. The electrons of the beam are grouped in macro-particles, whose charge is q = -eN, being N the number of electrons represented by each macro-particle. The integration of the relativistic equations of motion for the macroparticles is performed with a forth order RungeKutta method. The coordinates of all macroparticles, together with their velocities and accelerations are stored for all times and are used for calculating the self-consistent fields:

$$\mathbf{E}_{\mathrm{R}}(\mathbf{x},t) = \int d\mathbf{x}' \left\{ \frac{\mathbf{n} \times \left( (\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right)}{c |\mathbf{x} - \mathbf{x}'| (1 - \boldsymbol{\beta} \cdot \mathbf{n})^2} + \frac{\mathbf{n} - \boldsymbol{\beta}}{\gamma^2 |\mathbf{x} - \mathbf{x}'|^2 (1 - \boldsymbol{\beta} \cdot \mathbf{n})^2} \right\} \rho_{\mathrm{ret}}$$
(1a)

$$\mathbf{B}_{\mathrm{R}}(\mathbf{x},t) = \int d\mathbf{x}' \left\{ \frac{\mathbf{n} \times \left( \dot{\boldsymbol{\beta}} \left( 1 - \boldsymbol{\beta} \cdot \mathbf{n} \right) + \boldsymbol{\beta} \left( \dot{\boldsymbol{\beta}} \cdot \mathbf{n} \right) \right)}{c |\mathbf{x} - \mathbf{x}'| \left( 1 - \boldsymbol{\beta} \cdot \mathbf{n} \right)^2} + \frac{\mathbf{n} \times \boldsymbol{\beta}}{\gamma^2 |\mathbf{x} - \mathbf{x}'|^2 \left( 1 - \boldsymbol{\beta} \cdot \mathbf{n} \right)^2} \right\} \rho_{\mathrm{ret}}$$
(1b)

on the mass centre  $\mathbf{x}$  of each macro-particle.

In equations (1),  $\boldsymbol{\beta}(\mathbf{x},t) = \frac{\mathbf{v}(\mathbf{x},t)}{c}$ ,  $\dot{\boldsymbol{\beta}}(t) = \frac{d}{dt}\boldsymbol{\beta}(t)$  and  $\gamma^2 = (1 - \beta^2(t))^{-1}$  must be calculated at  $t = t_{\text{ret}} = t - \frac{1}{c} |\mathbf{x} - \mathbf{x}'|$ ,  $\mathbf{n} = \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}$  and  $\rho_{\text{ret}}$  is the particle density evaluated at the retarded time.

We are interested in the relativistic dynamics of a beam of charged particles in given external

fields  $E_{\rm ext}$  and  $B_{\rm ext}$  , so the equations of motion read as:

$$\frac{d\,\mathbf{r}_j}{dt} = \mathbf{v}_j \tag{2a}$$

$$\frac{d\,\mathbf{p}_j}{dt} = q_j \left( \mathbf{E} + \frac{\mathbf{v}_j}{c} \times \mathbf{B} \right) \tag{2b}$$

where the index j runs over the total number of charges  $q_j$  considered,  $\mathbf{E} = \mathbf{E}_{\text{ext}} + \mathbf{E}_{\text{R}}$  and  $\mathbf{B} = \mathbf{B}_{\text{ext}} + \mathbf{B}_{\text{R}}$  are the total fields experienced by the particle, and  $\mathbf{p}_j = m_j \gamma_j \mathbf{v}_j$ .

#### 3. NUMERICAL EXAMPLES

The initial conditions are given on the photocathode that is placed at z = 0 and the drift zone is about 2 meters. The total charge extracted from the cathode (Q = 1 nC) moves in external fields of the gun that are constituted by the typical electric field of a S-band (2856 MH) cavity with peak intensity  $\mathbf{E}_{\text{max}} = 120 \text{ MV/m}$  and of a solenoid magnetic field of maximum strength  $\mathbf{B}_{\text{max}} = 0.297 \text{ T}$ . The photoinjector parameters are set up for obtaining a structure of double minimum of emittance at 1 and 1.5 meters. The initial bunch shape is a longitudinal and transverse flat top 10 ps long and with an rms radius of 500  $\mu$ m. The RF phase at the bunch head is 34°.

The non uniform electron emissivity induced by inhomogeneities of the laser illumination, or by variations in the quantum efficiency on the photocathode, can affect the position and the value of the minima of the emittance.

#### 3.1. Band inhomogeneity

The first kind of inhomogeneities considered is a circular band structure, simulating the laser transverse intensity pattern obtained durning early run at SPARC [5] and shown in FIG. I. The expression assumed for the transverse particle distribution is:

$$\rho(r) = \frac{N_{\rm mp}}{S} \left\{ 1 - \eta + \frac{\eta}{I} \cos^2\left(2\pi\sqrt{\frac{n_{\rm b}r}{r_{\rm max} - r_{\rm min}}}\right) \right\}$$
(3)

where  $N_{\rm mp}$  is the total number of particles, S id the transverse surface of the beam,  $n_{\rm b}$  is the number of bands that appear in the distribution,  $r_{\rm max}$  and  $r_{\rm min}$  delimit the region of inhomogeneity,  $\eta = N_{\rm in}/N_{\rm mp}$ ,  $N_{\rm in}$  is the number of particles distributed in non uniform way, and  $I = 2\pi \int_{r_{\rm min}}^{r_{\rm max}} dr \ r \ \cos^2\left(2\pi \sqrt{\frac{n_{\rm b}r}{r_{\rm max}-r_{\rm min}}}\right)$ .



FIG. I: Particle distribution for the bands structure: an image of the laser transverse profile at SPARC (*left*) and the simulated distribution (*right*) with 8000 macroparticles 30% of which are inhomogeneity distributed. The inhomogeneity of the laser consists in circular patterns.



The image of the transverse particle density is shown in FIG. I (right side).

FIG. II: Plot of the x transverse rms emittance (left) and of  $\sigma_x$  (right) for the homogeneous beam (*red*) and inhomogeneities of 10% (green), 20% (blue), 30% (orange) and 40% (cyan).

FIG. II gives respectively the rms transverse normalized emittance and the envelope of the beam along z for different values of  $\eta$ .

In FIG. III the values of the emittance minima are reported together with their position (on the right scale) as function of the percentage of non uniformity. The growth of the minima is quadratic, while the positions remain almost constant. The increment in the emittance value for  $\eta = 0.4$  is of about 45%.



FIG. III: In black value and position of the first minimum of emittance, in red value and position of the second minimum.



FIG. IV: Particle distribution for the points structure: an image of laser transverse profile at SPARC (*left*) and the simulated distribution (*right*) with 8000 macroparticles 30% of which are inhomogeneity distributed. The inhomogeneity consists in gaussian dots scattered randomly.

## 3.2. Points inhomogeneity

The second kind of inhomogeneity considered refers to a density constituted by denser spots randomly distributed over a constant base as:

$$\rho(x,y) = \frac{N_{\rm mp}}{S} \left\{ 1 - \eta + \frac{\eta}{n_{\rm p}} \sum_{i=1}^{n_{\rm p}} \frac{1}{\pi\sigma_i} e^{-\frac{(x-x_i)^2 + (y-y_i)^2}{\sigma_i^2}} \right\}$$
(4)

This distribution simulates the laser transverse profile shown in FIG. IV [5], and is represented for a number of spots  $n_{\rm p} = 300$ .

The rms emittances and the envelopes for different values of  $\eta$  are presented in FIG. V.

In FIG. VI the values of the minima of emittance are shown as function of  $\eta$ , together with their positions for this kind of inhomogeneity. The growth of the minima is almost linear with



FIG. V: Plot of the x transverse rms emittance (left) and of  $\sigma_x$  (right) for the homogeneous beam (*red*) and inhomogeneities of 10% (green), 20% (blue), 30% (orange) and 40% (cyan).



FIG. VI: In black value and position of the first minimum of emittance, in red value and position of the second minimum.

the percentage of inhomogeneity, the positions remain almost constant. The increment in the emittance value when  $\eta = 0.4$  is about 27%. A comparison between the two kinds of ripples shows that this second kind of inhomogeneity is less critic.

# Aknowledgments

This work has been partially supported by the EU Commission in the Sixth Framework Program, Contract No. 011935, EUROFEL.

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