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STUDY OF TOLERANCES AND SENSITIVITY TO ERRORS IN THE SPARC HIGH BRIGHTNESS PHOTO-INJECTOR

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Abstract

In order to investigate the stability of the SPARC working point and to predict the most probable values of the projected and slice emittance in realistic conditions a sensitivity study to various type of random errors of the different parameters of the SPARC accelerator was done. The study has been performed through a systematic scan of the main parameters around the operating point by using PARMELA code interfaced to MATLAB. In this note the results of this analysis are presented.

1 INTRODUCTION

The design goal of the SPARC accelerator is to provide a 155 MeV, 10 psec FWHM, 1.1 nC bunch with less than 2 mm mrad for the projected emittance and less than 1 mm mrad for the slice emittance of 50% of slices.
The last optimization of the SPARC working point [1] gives 0.71 mm mrad for the projected emittance and a maximum value of 0.6 mm mrad for the normalized emittance of 9 slices out of 13 for perfect beam conditions with a laser pulse rise time of 1 psec and a beam thermal emittance of 0.34 mm mrad.
In the following the results of a sensitivity study to various type of random errors of the SPARC accelerator are presented.
Starting from the approach described in [2] the study was divided in two steps. In the first step the tolerances of the main tuning parameters were set with the criterium of having a maximum increase of the projected emittance at the undulator entrance of 10% with respect to the nominal case. In the second step the errors were combined in the defined tolerance ranges and a statistical analysis has been performed in order to study the effect of the combination of errors on the projected and slice emittance and on the mismatching at the entrance of the undulator.

2 SETTING OF TOLERANCES

The sensitivity of the projected emittance to errors of individual parameters that can fluctuate during the machine operation was studied by PARMELA code extensive simulations. The parameters that have been considered are relative to the gun region and are reported together their nominal value in table 1.

Table 1

Parameter	**Nominal value**
the gun phase	32°
the beam charge	1.1 nC
the emittance compensation solenoid magnetic field	2.73 Kgauss
the gun electric field amplitude	120 MV/m
the spot radius	1.13 mm
the spot ellipticity (xmax/ymax)	1

A systematic scan of each parameter was done around the optimized working point reference value in order to derive the minimum variation of each parameter giving an increase of emittance of 10% and the results are shown in the plots of figures 1,2,3,4,5,6.

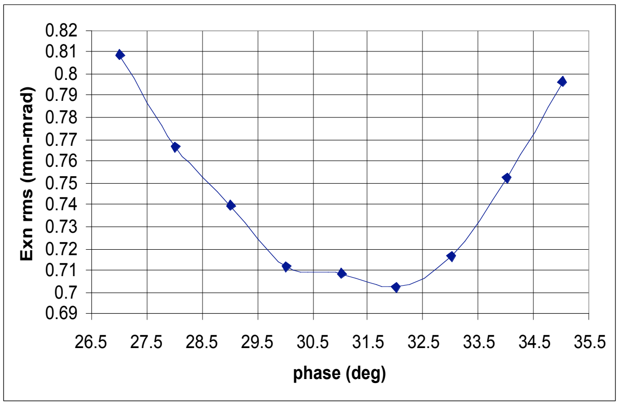


Fig. 1 Emittance vs phase-jitter

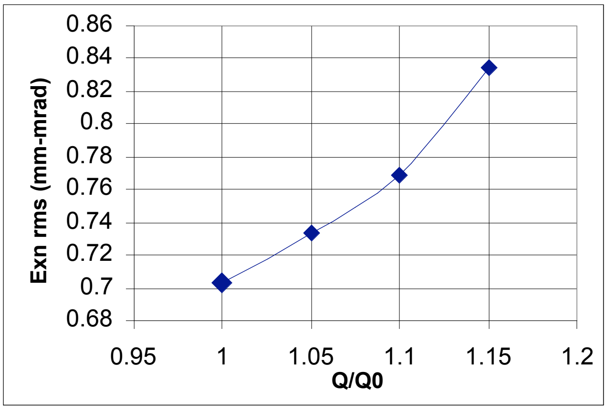


Fig. 2 Emittance vs charge fluctuation ((Q0=1.1 nC)

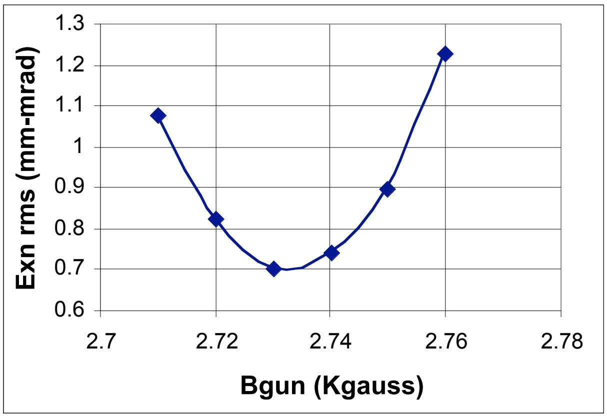


Fig. 3 Emittance vs gun magnetic field

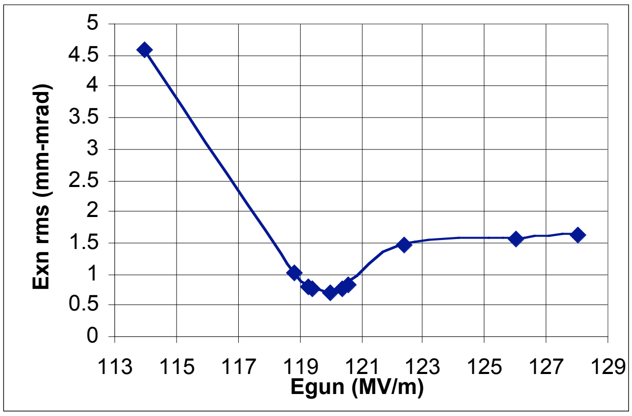


Fig. 4 Emittance vs gun electric field

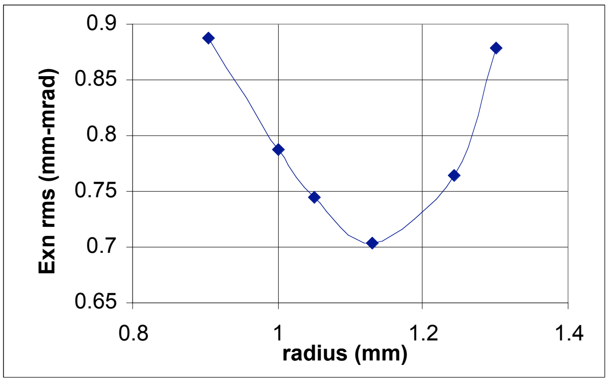


Fig. 5 Emittance vs spot radius

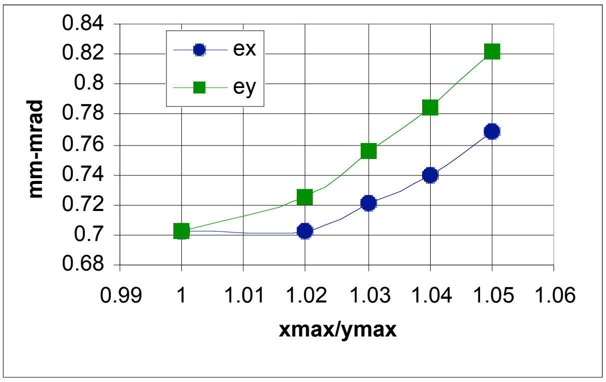


Fig. 6 Emittance vs spot ellipticity (xmax·ymax=R^2)

The resulting tolerances on the different tuning parameters are listed in table 2.

Table 2 Minimum variation of the single parameters value for a 10% emittance increase

Phase jitter	± 3°
Charge fluctuation	+ 10%
Gun magnetic field	± 0.4%
Gun electric field	± 0.5 %
Spot radius dimension	± 10%
Spot ellipticity	3.5% (xmax/ymax=1-1.035)

It can be seen that the most critical parameters are the electric field amplitude and the spot ellipticity.

3 STATISTICAL STUDY OF SENSITIVITY TO COMBINED ERRORS

3.1 Choice of the statistical approach

The statistical study requires the random variation of six parameters within the defined tolerance limits. A strategy was studied in order to minimize the CPU time without loosing information. As sampling approach we choose the technique of the "latin hypercube" [3,4]. This technique is useful when one needs a sample that is random, but is guaranteed to be relatively uniformly distributed over each dimension. It is a constrained Montecarlo sampling scheme that is applied to sample an N-dimensional space at M points. The idea is to partition each design parameter (dimension) into M segments, so that the whole space is partitioned into M^N cells and then choose M cells to contain the

sample points by the following algorithm: the first point is chosen randomly from one of the M^N cells and then all the cells that agree with this point on any of its parameters (that is crossing out all cells in the same row and column) are eliminated leaving $(M-1)^N$ candidates. After one of these is chosen randomly, eliminating new rows and columns and the process goes on until there is only one cell left containing the last sample point. The result of this construction is that each design parameter is tested in every one of its subranges.
This algorithm is implemented in the MATLAB statistical toolbox. So in order to test this technique for our application and to compare it with other approaches we did a MATLAB program which compares different sampling methods by the computation of the standard deviation of six parameters ranging between 0 and 1 on a uniform distribution. The results of this test for three different sampling techniques ("latin hypercube", internal random function, Halton sequences) are shown in figs. 7,8,9 reporting the computed error on standard deviation of a uniform distribution (exact value=1/ 12) in function of the number of runs. It can be observed that for the "latin hypercube" 100 runs are enough to reach an error on standard deviation of only about $5 \cdot 10^{-3}$ for each parameter, while the error of some parameters is around 10% for the pure random number generation and around 2% for the Halton sequences generation.

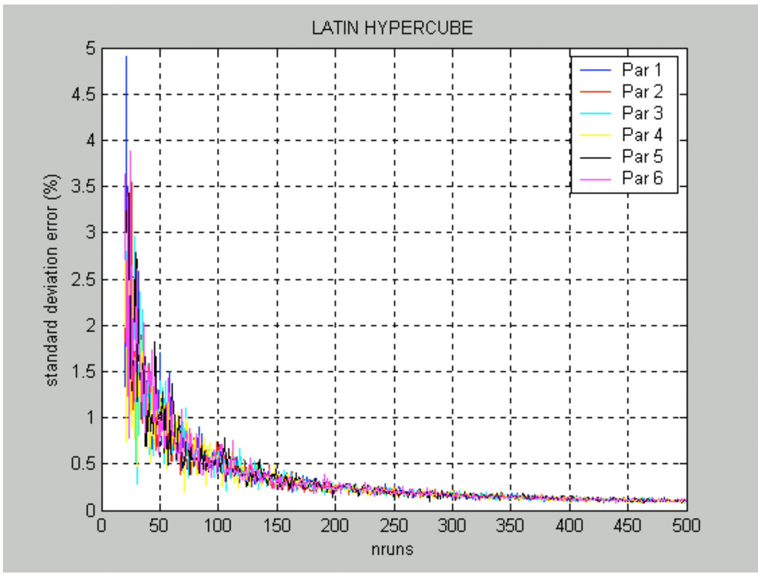


Fig.7 Standard deviation vs number of runs for "latin hypercube" technique

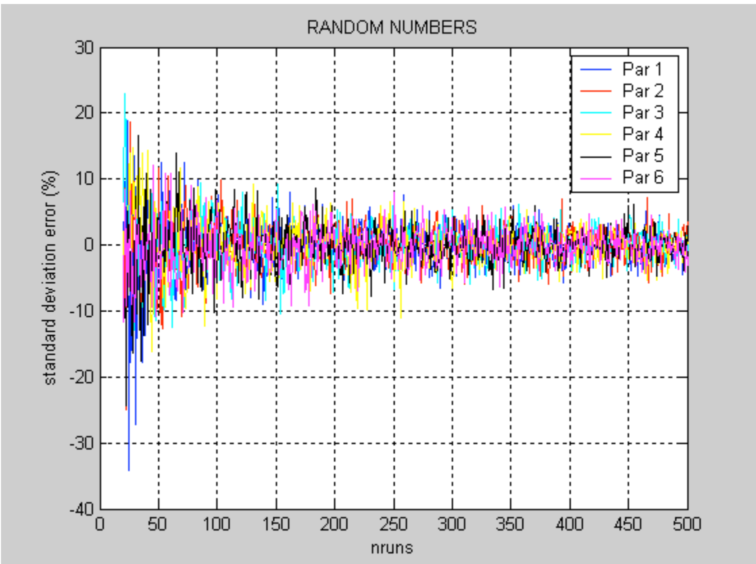


Fig.8 Standard deviation vs number of runs for random generation

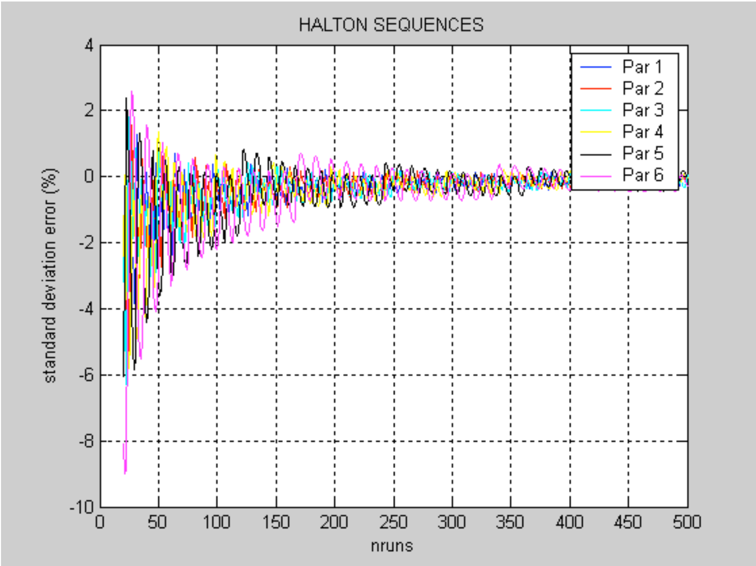


Fig.9. Standard deviation vs number of runs for Halton sequences (base numbers:2,3,5,7,11,13)

Therefore we did a MATLAB-based pre-processor program that accepts in input the limits of variation of the single parameters corresponding to an increase of 10% respect to the minimum emittance and generates a number of PARMELA input files in which the six parameters of our interest are varied randomly in the pre-defined ranges according with the "latin hypercube" technique.

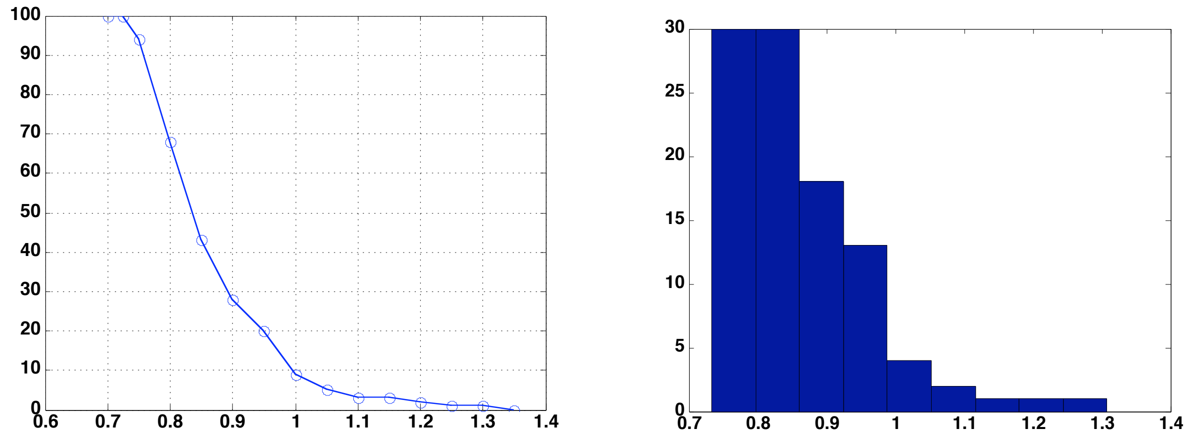
3.2 Effect of combination of errors

One hundred simulations runs were done each with errors set randomly according with the "latin hypercube" sampling technique. The numbers used are uniform distributions with average values and rms widths listed in table 3.

Table 3

Parameter	Average value	RMS value
gun phase	31.5°	1.74°
Charge	1.15 nC	0.032 nC
gun magnetic field amplitude	2733 gauss	5.8 gauss
gun electric field amplitude	119.9 MV/m	0.32 MV/m
spot radius	1.132	0.068 mm
Ellipticity	1	0.02

The interval of errors distribution is around $\pm\ 3\sigma$ around the average value. The small difference between the average values of table 3 and the corresponding nominal values of table 1 is due to the fact that the variation of the beam emittance is not symmetric around the nominal value of the different parameters (that is defined as the parameter value minimizing the emittance) as it can be observed in the figs. 1-6 of the paragraph 2.
PARMELA beam dynamics simulation gave the sensitivity of projected and normalized emittance to such types of errors. The plot in figure 10a gives the probability to obtain an emittance greater or equal than the corresponding value on the abscissa: for example the probability to get a normalized projected emittance ≥ 1 mm mrad is around 10%. The histogram in figure 10b gives the distribution of the emittance values that has an average value of 0.86 mm mrad and a rms value of 0.1 mm mrad. The histogram follows a Poisson distribution indicating the independent nature of the results.



(a) (b)

Fig.10 (a) Probability vs emittance plot over 100 simulations (b) Histogram of the emittance over 100 simulations

About the slice emittance, in the 100 simulations it does not exceed 0.9 mrad for the 9 central slices over 13 slices as it can be seen in fig.11 where two extreme cases obtained from error study simulations results are compared with the computed ideal case.

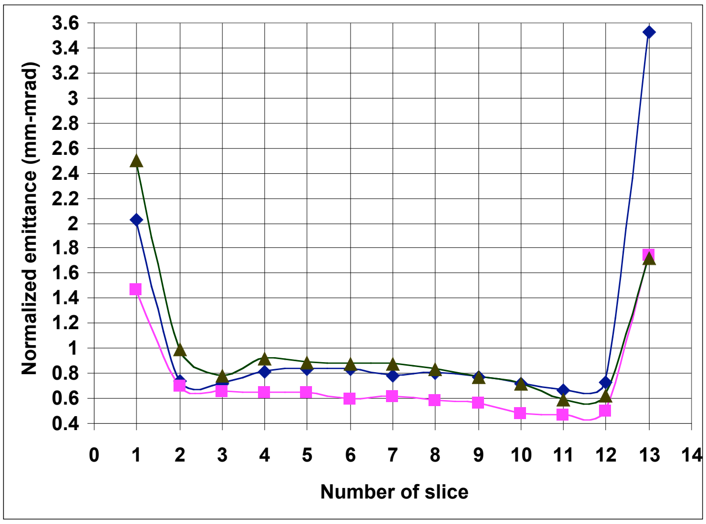
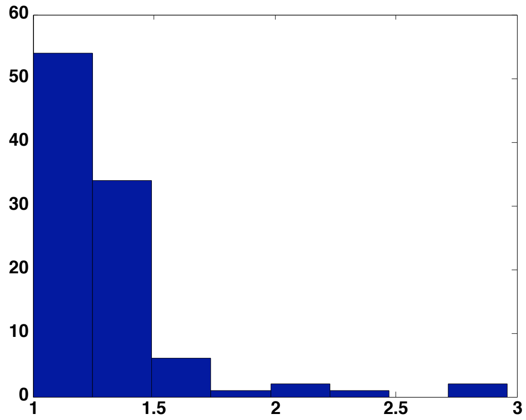
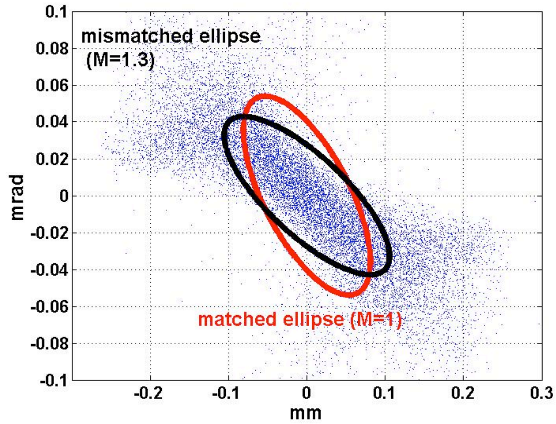


Fig. 11 Interval of variation of slice emittance over 100 simulations

The effect of random errors on the transverse phase space orientation at the entrance of the undulator has also been considered. In fig. 12a the distribution of the mismatching factor defined as $M = \frac{1}{2}\cdot\left(\beta_0\gamma - 2\alpha_0\alpha + \gamma_0\beta\right)$ where β_0, γ_0 are the matched values at the entrance of the undulator is shown: 1.3 and 0.32 are respectively the average and the rms values. The mismatched ellipse corresponding to M=1.3 is compared to the ideal ellipse in fig. 12b.



(a)



(b)

Fig. 12 (a) Histogram of mismatching factor over 100 simulations (b) Comparison of two transverse phase spaces corresponding to M=1 and M=1.3 mismatching factor values

The effect of mismatching on the FEL gain should be evaluated.

4 CONCLUSIONS

On the basis of the present error study it results that, combining multiple errors on the tuning parameters of the gun region, the projected and slice emittance values remain within the limits of the SPARC design (2 mm mrad for the projected emittance and 1 mm mrad for slice emittance).
This analysis has been done for a uniform transverse distribution. For a more detailed tolerance budget the effect of spot non-uniformity is now being studied.

5 REFERENCES

[1] “SPARC injector working point optimisation” SPARC-BD-03/007 note
[2] C. Limborg et al., “New optimization for the LCLS photo-injector “, Proc. EPAC 2002 (Paris), p. 1789
[3] “Numerical Recipes”, cap. 7
[4] “A user guide to LHS:Sandia’s latin hypercube sampling software”, note SAND98-0210