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STATIONARY BEAM LOADING ON DAΦNE CAVITY ACCELERATING MODE

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INTRODUCTION

Due to the very high current value, foreseen for DAΦNE operation - 1.3 Amps/ring into 30 bunches as short-term goal, up to 5.5 Amps/ring into 120 bunches for top luminosity - the beam stability is one of the most challenging topics of the project. In fact, the electromagnetic interaction, between the beam and the vacuum chamber, may lead to a various forms of longitudinal and transverse instability. The beam to the cavity accelerating mode interaction is a potential source of center-of-mass instability because:

A) The longitudinal restoring force of the synchrotron motion does not depends on the voltage, induced by the beam in the cavity, and decreases as the stored current increases. Over a threshold value, the force becomes repulsive and the the beam cannot be stable any more.

B) The accelerating mode of the RF resonator is excited by the sidebands of the bunch synchrotron motion and an additional term, possibly anti-damping, has to be introduced in the synchrotron equation. This term depends on the cavity detuning and therefore on the stored current, again.

Both these effects are known as "Robinson Instability", but they are actually different phenomena and then occurs at different thresholds. Hereafter we will refer to the first one as the "Sands Instability" since we have referred to the M. Sands analysis of the problem.

The aim of this paper is to present some analytical evaluations on this topic based on a simple circuit model including RF source, transmission line, cavity and beam.

The analysis holds only for the stationary regime and it allows to predict which sets of the RF system parameters are compatible with the stability of any beam current.

1) CIRCUIT MODEL

We made our estimations by solving the simple circuit model of Fig. 1. The cavity accelerating mode is sketched as a resonant parallel circuit, while the beam is represented by a current source phasor, at the RF frequency, with an amplitude twice the average stored current. As we consider a 3 cm bunch length, much shorter than the RF wavelength, the approximation is very good.

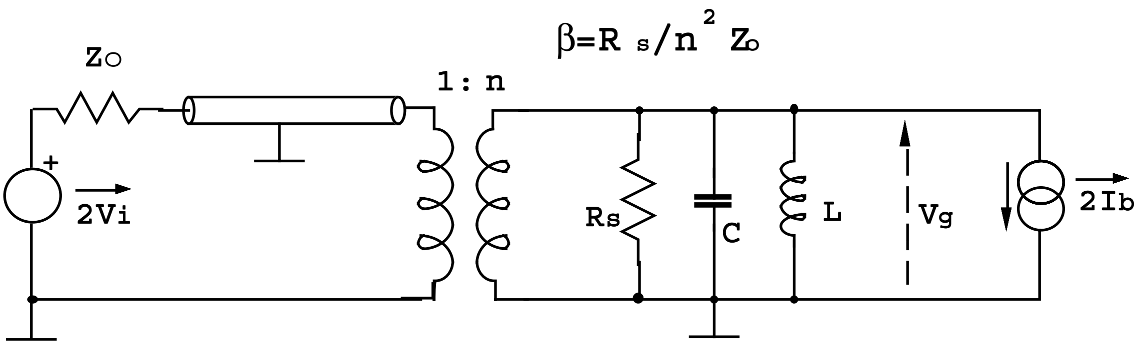

Fig. :

This model is fully equivalent to those reporting the RF source at the gap, but we prefer to use this one because it esplicitally takes into account the coupling coefficient β, a parameter playing an important role in the calculations, and it gives direct information on the incident power demanded to the RF source in the various conditions.

2) SANDS INSTABILITY

A single particle running round a storage ring oscillates around the synchronous phase as effect of a longitudinal restoring force proportional to the derivative of the gap voltage, which is given in this case only by the RF generator.

As the stored current increases, the contribution of the beam loading voltage to the total gap voltage becomes more relevant. In case of coherent synchrotron oscillations (beam center-of-mass oscillations) the equilibrium is still around the synchronous phase but the restoring force depends only on the derivative of the generator voltage as the beam loading voltage does not contribute to it.

Referring to the circuit of Fig. 1, the conclusion is that the beam center-of-mass is stable if ϕ'_s, the phase of the beam current generator to the phase of the voltage induced at the gap by the RF generator alone, is always positive. Otherwise the force is repulsive, and the beam cannot be stable any more.

3) ROBINSON INSTABILITY

The fundamental mode of the resonator is also excited by the sidebands of the synchrotron motion. An additional field results, giving a contribution either positive or negative to the damping term of the synchrotron equation, depending on the cavity tuning angle.

A qualitative explanation of this effect follows. Let us consider a ring working above transition energy and a cavity tuned below the frequency of the bunch harmonic. A particle with a positive energy error has a revolution time longer respect to the synchronous particle and then a lower revolution frequency. As the cavity is tuned toward lower frequencies, the particle dissipates more energy on the real part of the cavity impedance and this accounts for the additional damping term. Of course the situation is just the opposite if the cavity is tuned above the bunch harmonic, or if the ring operates below transition energy.

As DAΦNE operates well above transition energy, the Robinson instability occurs only if the cavity is tuned above the bunch harmonic. So, looking at Fig. 1, we have to consider what happen to the cavity tuning angle under the operation of the automatic tuning system at different values of the stored current.

4) CIRCUIT SOLUTION

We have calculated the main parameters of the Fig. 1 circuit as functions of the stored current to find the RF set-ups compatible to the beam center-of-mass stability.

An important point is the choice of the independent parameters in the calculations. We prefer to use the cavity gap voltage instead of the RF source voltage, and cavity+beam tuning angle instead of the cavity tuning angle itself, because these are the parameters kept constant at any stored current by the operation of the RF servo loop controls.

Here follows a list of the fixed parameters, of the independent variable parameters and of the computed functions together with the computing formulas. We consider different values of some parameters for two cases of machine broadband impedance Z/n (1 or 2 Ohms).

Fixed Parameter List:

		Z/n=1Ω	*Z/n=2Ω*	
ω_{RF}	RF Frequency	368.25		MHz
V_{gn}	Nominal Gap Voltage	130	260	KV
R_s	Cavity Shunt Impedance	2.25		MΩ
Q_0	Cavity Unloaded Quality Factor	30,000		---
V_r	Total Losses per turn	16.3	23.3	KV
ω_s	Synchrotron Frequency (@ V_{gn})	27.3	38.6	KHz
I_{bM}	Max Average Stored Current	5.5	2.75	Amps
N_b	Max Number of Bunches	120	60	---
P_{iM}	Max Available RF Power	150		KW

Independent Variables:

ϕ_{zi}	Cavity+Beam Tuning Angle
β	Coupling Coefficient
V_g	Gap Voltage
I_b	Average Beam Current

Computed Functions:

ϕ_s	Synchronous Phase
ξ	Parameter used in the calculations
Q_L	Cavity Loaded Quality Factor
ϕ_z	Cavity Impedance Phase
ρ	Reflection Coefficient at the Coupling Port (Module)
$\Delta\omega_c$	Cavity Detuning
$\delta^{\pm}$	Cavity Dissonance (@ $\omega_{RF} \pm \omega_s$)
α_R	Robinson Damping Constant
Pi	Incident Power from RF Generator
ϕ'_s	Sands Angle (Bunch to Generator Component of the Gap Voltage Phase Angle)

Computing Formulas:

$$\phi_s = \mathrm{Acos}\,(V_r / V_g)$$

$$\xi = \frac{2}{1+\beta}\,\frac{I_b\,R_s}{V_g}$$

$$Q_L = Q_0 / (1+\beta)$$

$$\phi_z = \mathrm{Arctg}\,[\,\mathrm{tg}\phi_{zi}\ (1 + \xi\,\mathrm{Cos}\phi_s) - \xi\,\mathrm{Sin}\phi_s]$$

$$\Delta\omega_c = -\,\mathrm{tg}\,\phi_z\,\frac{\omega_{RF}}{2\,Q_L}$$

$$\rho = \sqrt{\frac{\left(\dfrac{\dfrac{\beta-1}{\beta+1} - \xi\,\mathrm{Cos}\phi_s}{1+\xi\,\mathrm{Cos}\phi_s}\right)^2 + \mathrm{tg}^2\phi_{zi}}{1+\mathrm{tg}^2\phi_{zi}}}$$

$$\alpha_R = \frac{\xi\,\omega_s}{4}\left[\frac{1}{1+(Q_L\,\delta^-)^2} - \frac{1}{1+(Q_L\,\delta^+)^2}\right]$$

$$\phi'_s = \phi_s - \mathrm{Arctg}\left[\frac{\xi\,\mathrm{Cos}\,\phi_z\ \ \mathrm{Sin}\,(\phi_s - \phi_z)}{1+\xi\,\mathrm{Cos}\,\phi_z\ \ \mathrm{Cos}\,(\phi_s - \phi_z)}\right]$$

$$P_i = \frac{(V_g^2 / 2\,R_s) + V_g\,I_b\,\mathrm{Cos}\,\phi_s}{1-\rho^2}$$

5) THE CASE $Z/n = 1\Omega$

The Figs. 2 to 4 show the results for the case $Z/n = 1\Omega$; here we have plotted the four most significant variables from our point of view, i.e. the cavity detuning $\Delta\omega_c$, the Sands angle ϕ'_s, the Robinson damping constant α_R and the incident power from the RF source P_i.

Fig. 2 shows the results at nominal gap voltage - 130 KV - for a coupling coefficient $\beta = 25$ to match the full beam current and for three different values of the beam+cavity tuning angle, ϕ_{zi} = -20°, 0°, +20°.

If ϕ_{zi} = +20° the beam is antidamped from 0 to 300 mAmps Sands instability occurs beyond 1.8 Amps. The other cases are safe. So positive values of ϕ_{zi} are dangerous for the center-of-mass stability and therefore they do not need to be set.

$\beta = 25$ V_g=130 KV # ϕ_{zi} = -20° ∗ ϕ_{zi} = 0° o ϕ_{zi} = +20°

A: $\Delta\omega_c$ [MHz] vs. I_b [Amps] B: ϕ'_s [Deg] vs. I_b [Amps]

C: α_R [sec^{-1}] vs. I_b [Amps] D: P_i [KW] vs. I_b [Amps]

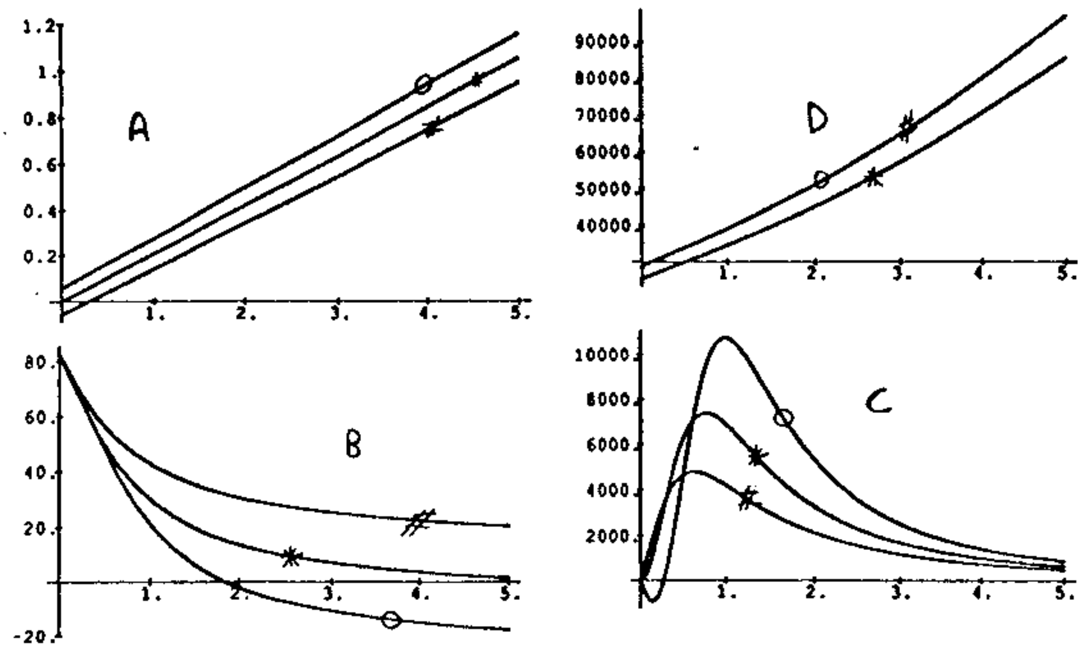

Fig. 2

$\beta = 25$ $\phi_{Zi} = 0°$ # V_g= 90 KV $*$ V_g=130 KV o V_g=300 KV

A: $\Delta\omega_C$ [MHz] vs. I_b [Amps] **B:** ϕ'_s [Deg] vs. I_b [Amps]

C: α_R [sec^{-1}] vs. I_b [Amps] **D:** P_i [KW] vs. I_b [Amps]

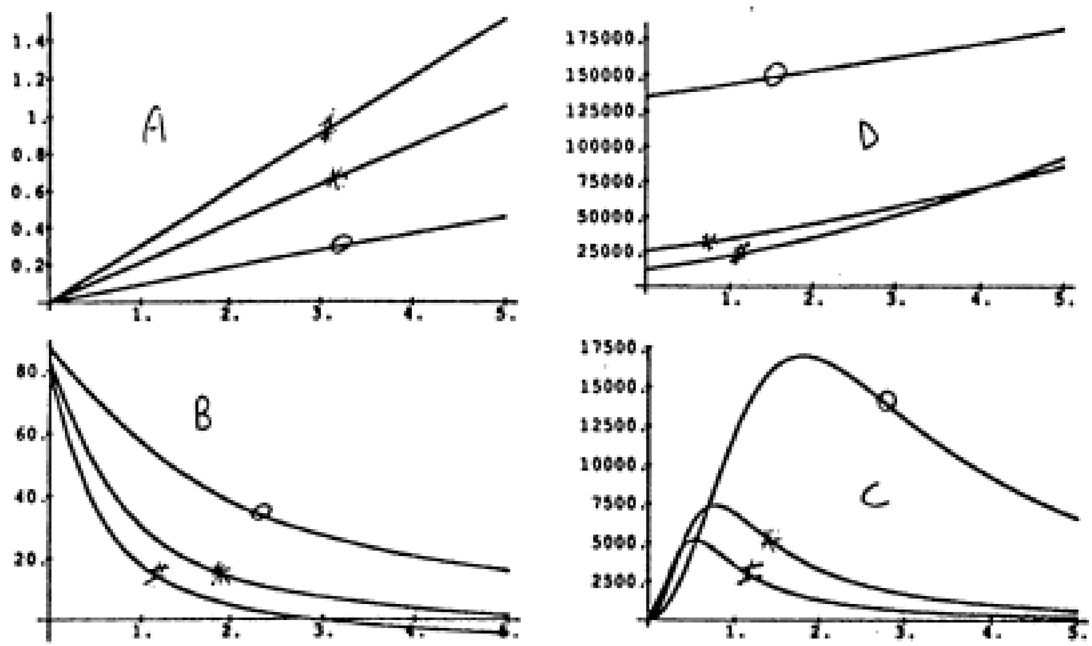

Fig. 3

$\beta = 12$ $\phi_{Zi} = -10°$ # V_g= 90 KV $*$ V_g=130 KV o V_g=300 KV

A: $\Delta\omega_C$ [MHz] vs. I_b [Amps] **B:** ϕ'_s [Deg] vs. I_b [Amps]

C: α_R [sec^{-1}] vs. I_b [Amps] **D:** P_i [KW] vs. I_b [Amps]

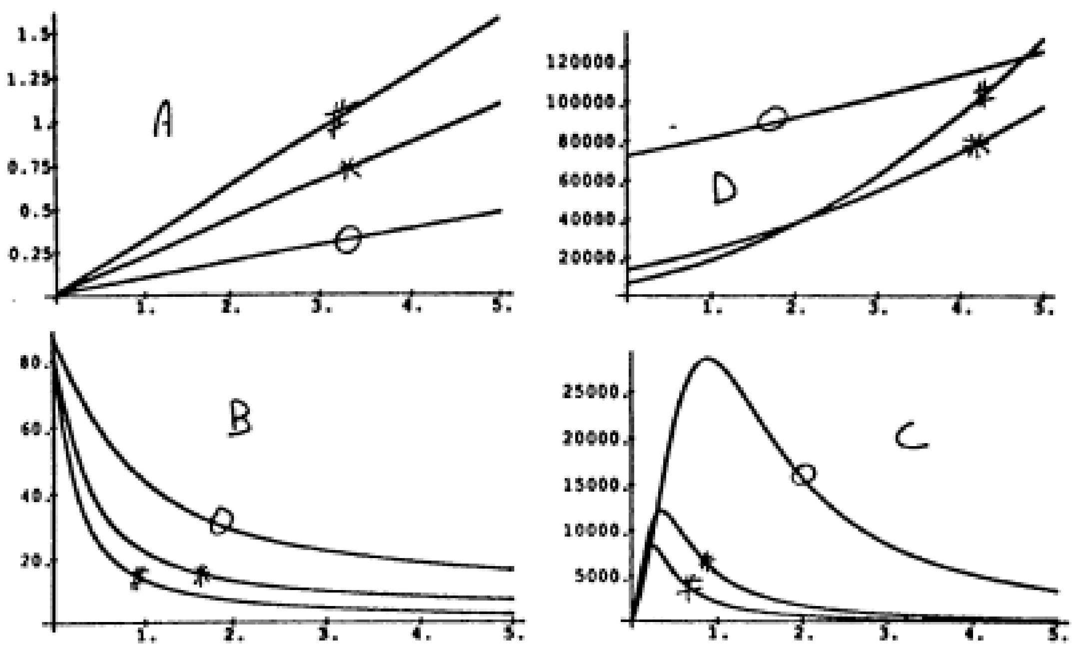

Fig. 4

Fig. 3 Shows the plots at $\beta = 25$ and $\phi_{Zi} = 0°$ for three values of the gap voltage V_g = 90, 130 and 300 KV. In this case the beam is always damped but Sands instability occurs again at V_g = 90 KV for a beam current larger than 3 Amps. Furthermore, the power plot corresponding to the higher gap voltage V_g = 300 KV exceeds the capability of our supply (150 KW).

The plots of Fig. 4 correspond to a possible working point for the case $Z/n = 1\Omega$. A lower coupling coefficient, $\beta = 12$, reduces the power requirements at high gap voltages, and a little negative tuning angle, $\phi_{Zi} = -10°$, acts as a compensation for Sands instability. So the beam is always damped and stable. The power requirements are also inside the range covered by our transmitters.

Some worries remain about the foreseen cavity detuning. As a short term program, storing up to 30 bunches, the maximum cavity detuning is 500 KHz, rising to 1.5 MHz at full current.

6) THE CASE $Z/n = 2\Omega$

The results for the case $Z/n = 2\Omega$ are reported in Figs. 5 to 7 where the same 4 variables are plotted.

Fig. 5 shows again that a positive cavity+beam tuning angle (ϕ_{Zi}=+20°) leads to Sands instability beyond a 800 mAmps current threshold, and cause the beam to be antidamped over a range from 0 to 150 mAmps. at nominal gap voltage. So this situation represents an incorrect working point.

In Fig. 6 we report the results obtained for the "compensate case" (i.e. $\phi_{Zi} = 0°$) at three different gap voltages (V_g = 130, 260, 350 KV). The damping constant remains always positive but a Sands threshold of 1 Amp appears at V_g = 130 KV. It is sufficient to set the tuning angle to a negative value ($\phi_{Zi} = -20°$) to avoid it ; this is shown in Fig. 7 that represents a possible correct working point for the $Z/n = 2\Omega$ case. The coupling coefficient b has been set to 5, a value that correctly matches I_b= 2.5 Amps, the full current foreseen for this case. The beam center-of-mass is fully stable and the power requirements are well inside the range covered by our klystrons.

The cavity detuning needed to match the beam loading is 300 KHz for I_b= 1.3 Amps, the short term goal of the project, and 500 KHz for full current. These values are less demanding respect to the $Z/n = 1\Omega$ case; this is the reason why in this case is possible to set a lower β value and a higher quality factor Q_L results.

7) CONCLUSIONS

This paper shows that there are always some working points of the DAΦNE RF system compatible with any chosen value of the beam current.

A little detuning angle provides stability for a wide gap voltage range and it allows, if necessary, to change a little the coupling coefficient to operate at higher gradients with tolerable power expense.

$\beta = 5$ V_g=260 KV # $\phi_{Zi} = -20°$ * $\phi_{Zi} = 0°$ o $\phi_{Zi} = +20°$

A: $\Delta\omega_C$ [MHz] vs. I_b [Amps] B: ϕ'_s [Deg] vs. I_b [Amps]

C: α_R [sec^{-1}] vs. I_b [Amps] D: P_i [KW] vs. I_b [Amps]

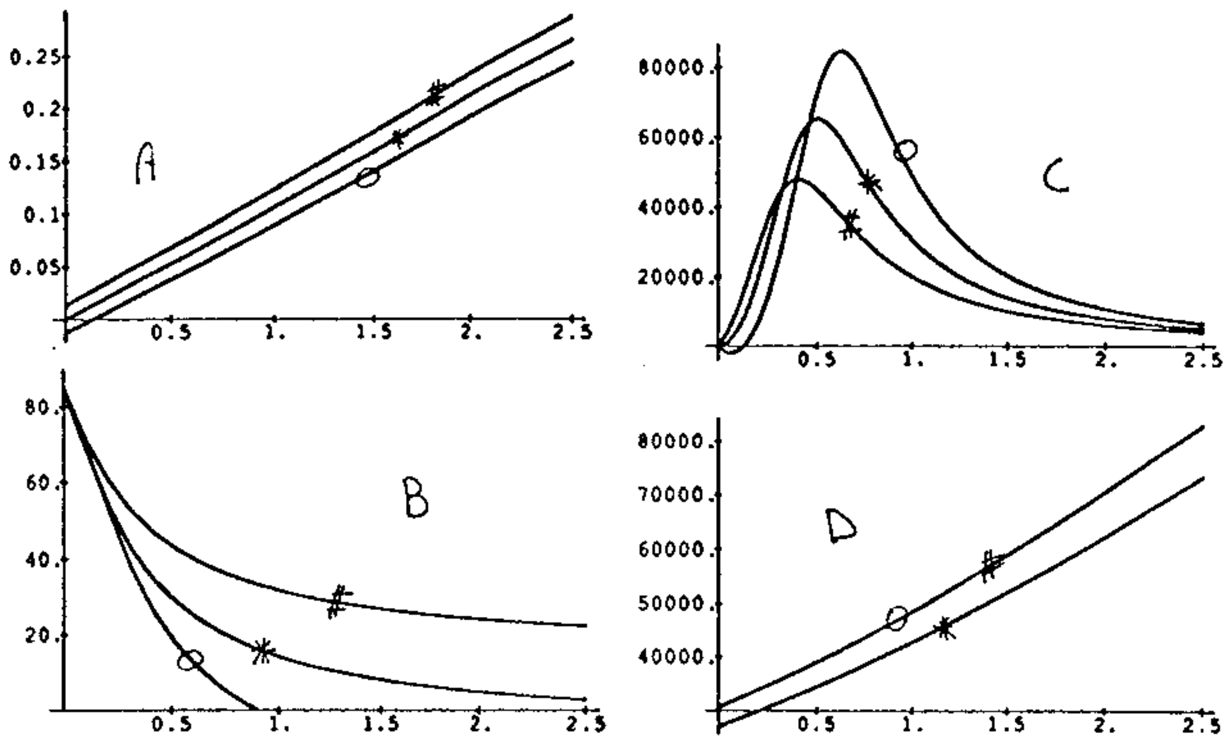

Fig. 5

$\beta = 5$ $\phi_{Zi} = 0°$ # V_g= 130 KV $*$ V_g=260 KV o V_g=350 KV

A: $\Delta\omega_C$ [MHz] vs. I_b [Amps] **B:** ϕ'_s [Deg] vs. I_b [Amps]

C: α_R [sec^{-1}] vs. I_b [Amps] **D:** P_i [KW] vs. I_b [Amps]

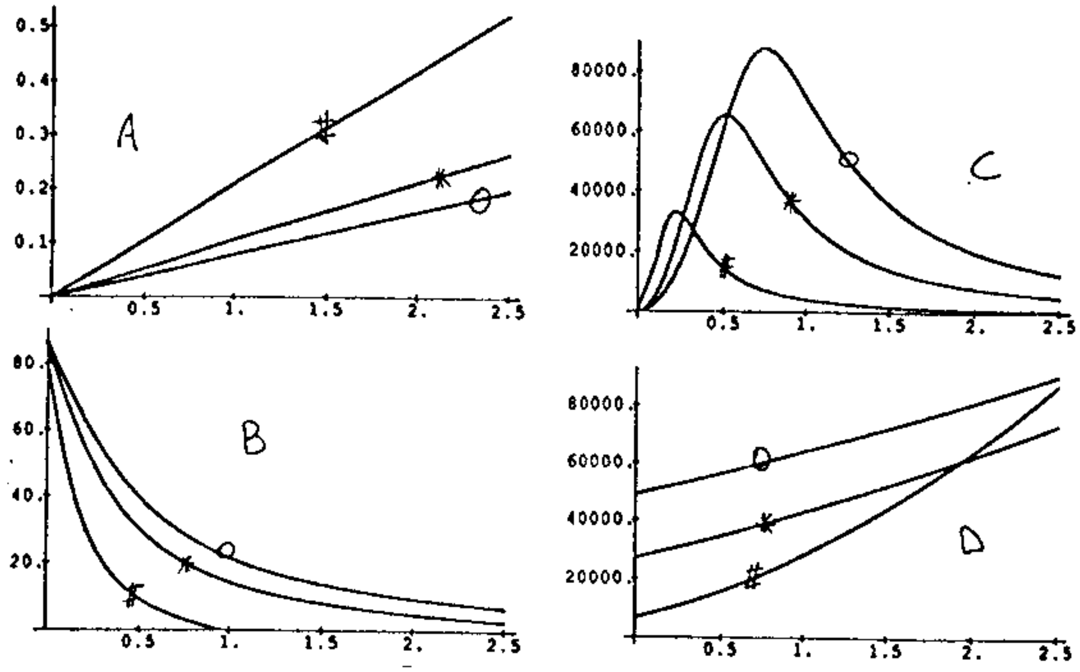

Fig. 6

$\beta = 5$ $\phi_{Zi} = -20°$ # V_g= 130 KV $*$ V_g=260 KV o V_g=350 KV

A: $\Delta\omega_C$ [MHz] vs. I_b [Amps] **B:** ϕ'_s [Deg] vs. I_b [Amps]

C: α_R [sec^{-1}] vs. I_b [Amps] **D:** P_i [KW] vs. I_b [Amps]

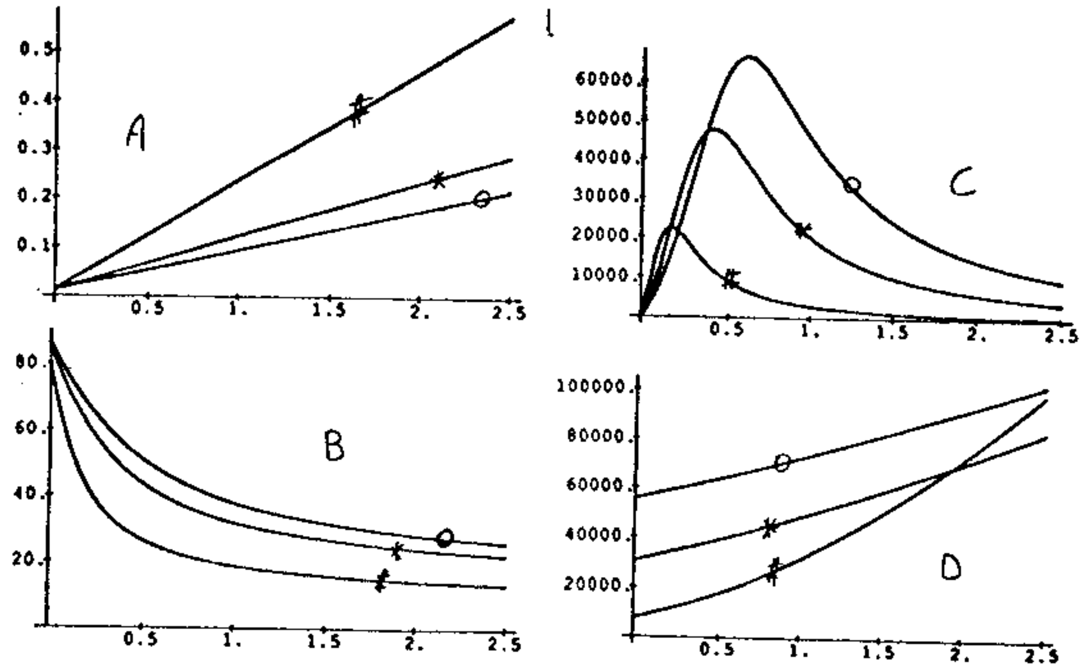

Fig. 7

The beam center-of-mass stability has been proved, providing that the current was adiabatically stored, i.e. at a very slow injection. This is not exactly our case as we hope to inject up to a whole bunch in a single shot. So transient effect must be considered to be sure that stability is guarantied also in a more realistic injection panorama. A work based on a time domain code simulation is now in progress to investigate transient effects.

The amount of cavity detuning to match the beam loading is large compared to most of existing machines; this could be reduced if the cavity will have a quality factor higher than what we have guessed in this calculations (Q_0 = 30,000). However, a careful study of the mechanical stresses on the real cavity structure must be done in order to choose the most convenient way of tuning the fundamental mode.

The study of a fast RF feedback is presently under investigation in order to increase the stability region.

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