DAΦNE TECHNICAL NOTE

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A CRABBING CAVITY FOR DAΦNE

S. Bartalucci and B. Spataro

The crab crossing scheme was firstly proposed by Bob Palmer[1] (SLAC - BNL). It uses auxiliary RF deflecting cavities to rotate the bunches by an angle equal to the crossing angle so that they can collide head-on.

For DAΦNE, this might be an OPTION for future luminosity upgrading. We show the general design criteria for the optimization of the cavity profile and a full comparison of two possible shapes with the present Cornell design for the B-Factory[2] . In this study no care was taken of the actual cavity location, since it is assumed that the relevant changes to the machine layout will be made possible, as soon as the need for crab crossing is recognized.

The cylinder cavity model

The voltage requirement for crab crossing is given by the Oide - Yokoya formula[3] . In DAΦNE it is ≈ 100 kV only, due to the relatively low operating energy.

Nevertheless it is interesting to look at the general criteria for the optimization of the cavity shape, since our goal is not only to save on RF power but mainly to minimize the HOM content of the cavity itself.

A general expression for the characteristic impedance of the TM_{nml} modes (n, m, l are the number of field variations in ϕ, r, z coordinates) in a cylindrical cavity has been derived as function of the ratio x = a/h (radius/height of the cylinder) :

$$\mathbf{/Q}\,(n,\ m,\ l,\ x=a/h) = \frac{8}{\left(2^{n}\, n!\right)^{2}}\,\frac{1}{\pi \varepsilon c}\,\frac{x\, p_{nm}^{2(n-1)}}{\left(\sqrt{p_{nm}^{2}+(\pi l x)^{2}}\right)^{2n+1}}\ *$$

$$*\left\{\frac{\sin\left[\dfrac{\sqrt{(\pi l)^{2}+\left(\dfrac{p_{nm}}{x}\right)^{2}}-\pi l}{2}\right]}{J_{n+1}(p_{nm})}\right\}^{2}\ \frac{\begin{Bmatrix}1 & l=0\\ 2 & l>0\end{Bmatrix}}{\begin{Bmatrix}2 & n=0\\ 1 & n>0\end{Bmatrix}}$$

where $J_n(p_{nm}) = 0$.

A complete definition of both the longitudinal and the transverse impedance is given in Appendix I. Note that the transverse (dipole) impedance will be marked as R' in the following, while the above formula holds true for any multipole moment of the e.m. fields. This expression is plotted in Fig. 1 for the 2 lowest in frequency monopole modes and for the lowest dipole mode. It is interesting to note that both the TM_{010} and the TM_{110} mode have a broad maximum centered around x = 1 and x = 1.6 respectively, while the TM_{011} mode shows a narrower peak around x = 0.5. This behaviour is reproduced when beam 'holes' are introduced, at least until the mode frequency remains below cutoff. This fact in turn suggests the possibility of finding an optimum shape also for a deflecting cavity, with a minimum shunt impedance for the strongest monopole modes. Indeed, extensive simulations have shown that quite a similar behaviour is reproduced in cavities with regular shape, like bell-shaped or nosecone cavities.

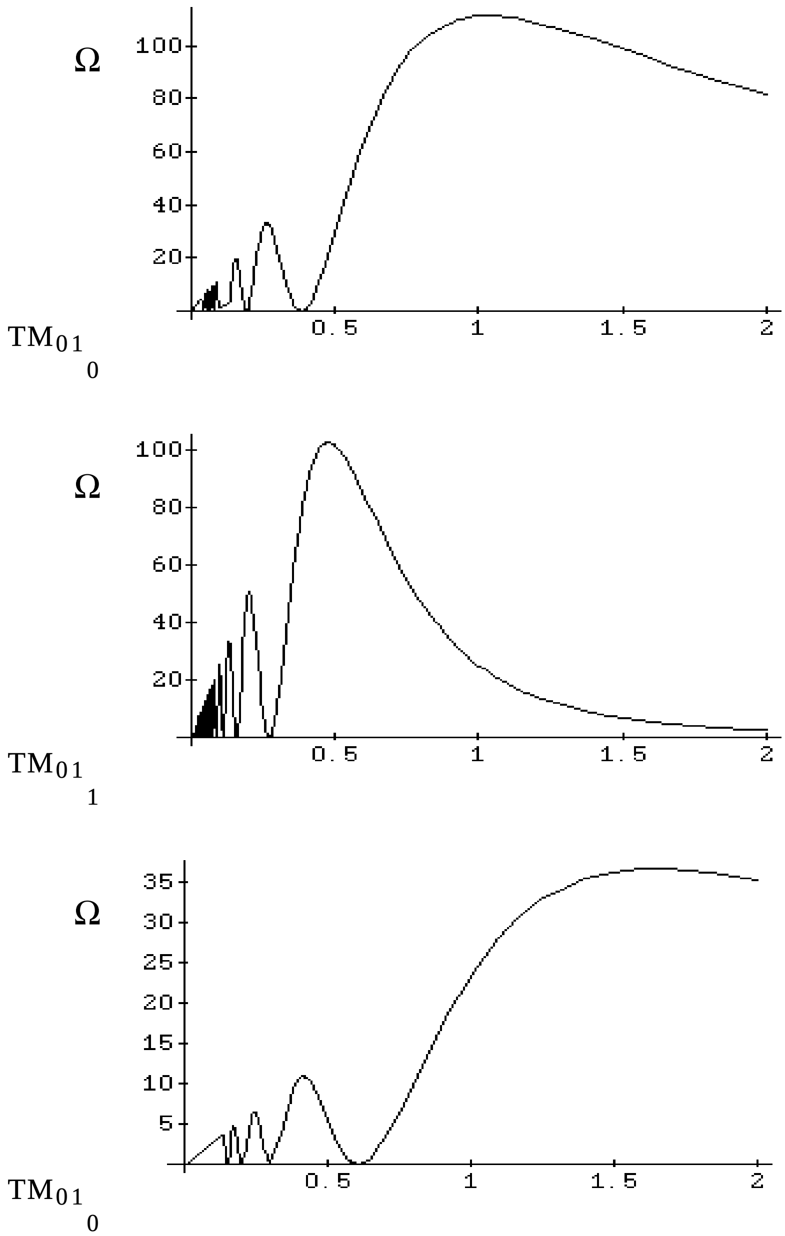


Fig. 1 - R/Q vs. a/h (radius/gap) in a cylinder cavity.

By combining the above expression for the R/Q with the one for the factor of merit Q:

$$\mathbf{Q}(n, m, l, a, h) = \sqrt{\frac{c\,\mu\,\sigma}{2}}\;\frac{\sqrt{a\,p_{nm}}}{1+\frac{a}{h}} \qquad l = 0$$

$$\mathbf{Q}(n, m, l, a, h) = \sqrt{\frac{c\,\mu\,\sigma}{2}}\;\sqrt[4]{p_{nm}^{2}+\left(\pi\, l\,\frac{a}{h}\right)^{2}}\;\frac{\sqrt{a}}{1+\frac{2\,a}{h}} \qquad l > 0$$

we get a general expression for the shunt impedance R_S which is plotted in Fig. 2, still for the same three modes at the DAΦNE RF 368 MHz for the deflecting mode. Again the situation is in favor of looking for a crab cavity with at least a single dangerous HOM besides the deflecting mode.

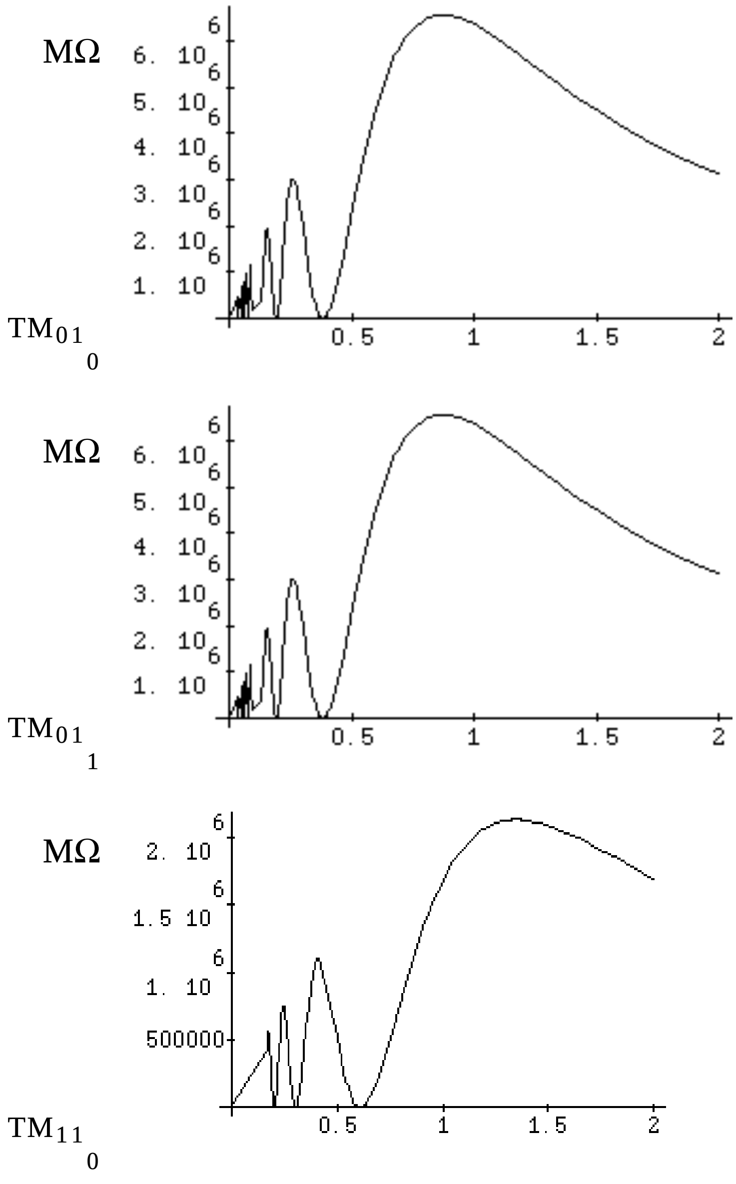


Fig. 2 - R_S vs. a/h in a cylinder cavity at 368 MHz for the TM_{110} mode.

The 'rounded' cell

We started with a typical bell-shaped (in the following named 'rounded') structure, as shown in Fig. 3. We know by this time that we have to include a taper from the cell iris to the beam pipe radius of 4.5 cm, in order to minimize the broadband impedance. The 'radius' is naturally identified as the depth of the bell and the 'gap' as its opening diameter. A number of simulations with URMEL were done by changing the gap and radius in order to have different profiles, each resonating at the nominal frequency of 368 MHz. The results are depicted in Fig. 4. The Q's are decreasing noticeably when h diminishes, while the R/Q's are increased. The global effect for the TM_{110} mode is that the Rs has a broad maximum around a/h = 1.5 and decreases rapidly for a/h < 1.2, just like what happens in the pill-box case.

By inspection of Fig. 4 it is easily seen that the maximum value for the R/Q is more or less equal to its maximum for the pill-box, i.e. 35 Ohm. We cannot expect to go beyond this limit with an 'open' structure like the rounded cavity, which, on the other side, retains some advantages as the parasitic modes are concerned.

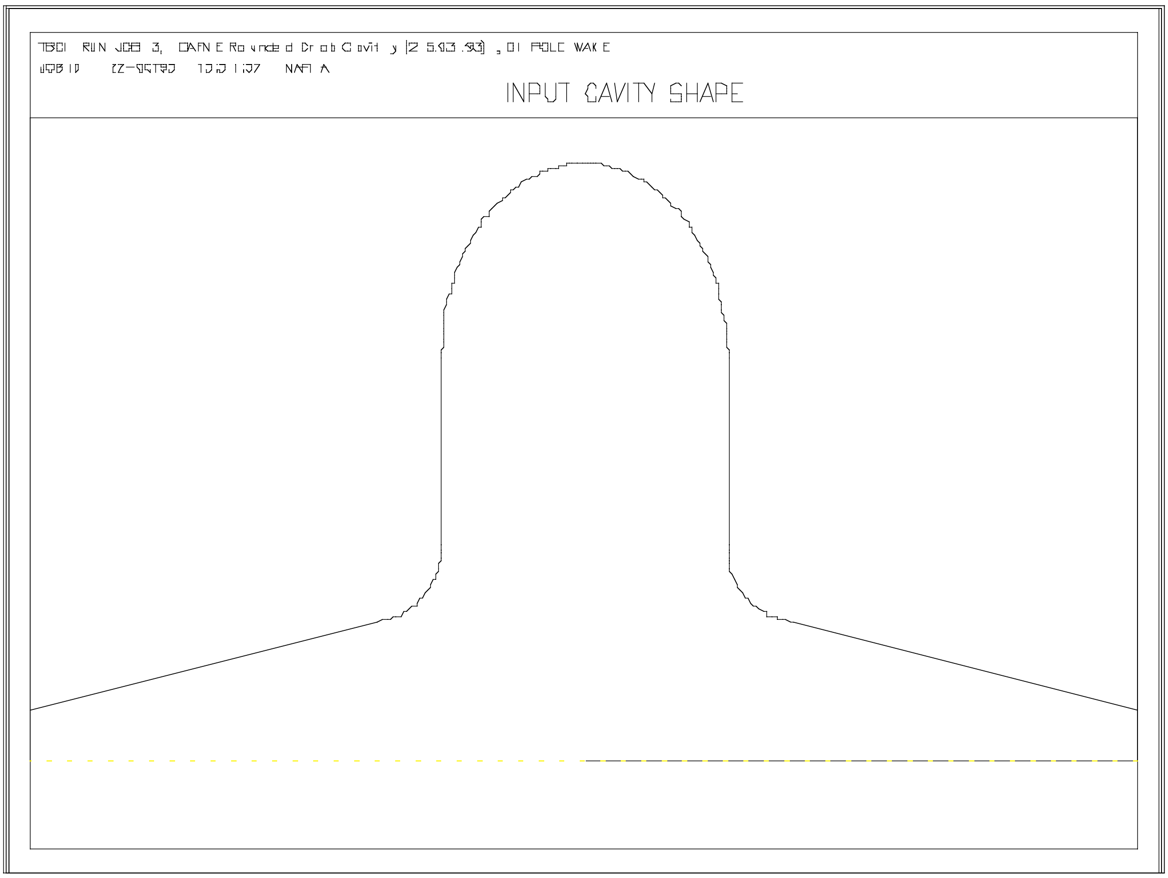
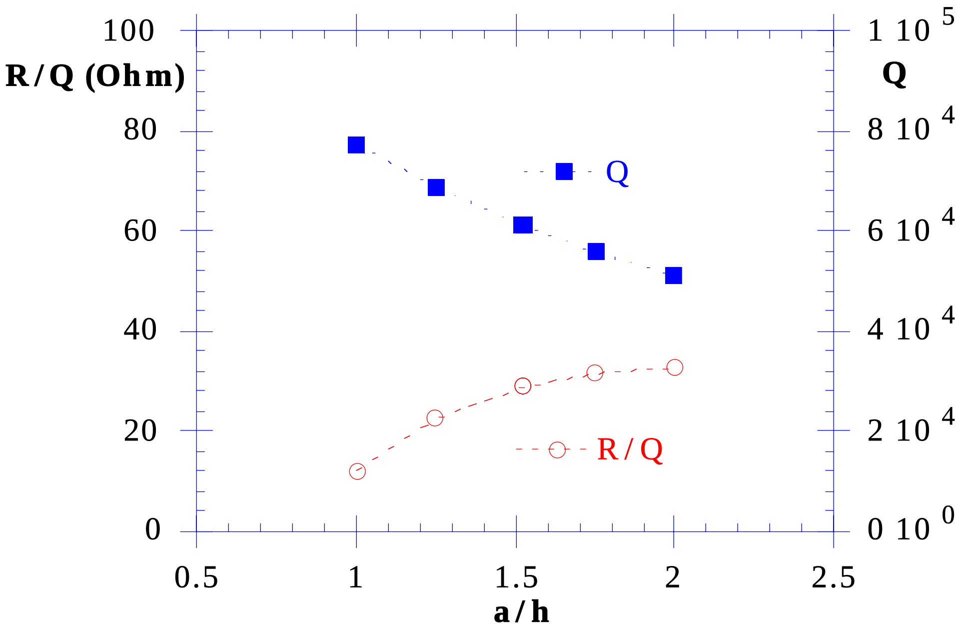


Fig. 3 - The 'rounded' cavity.



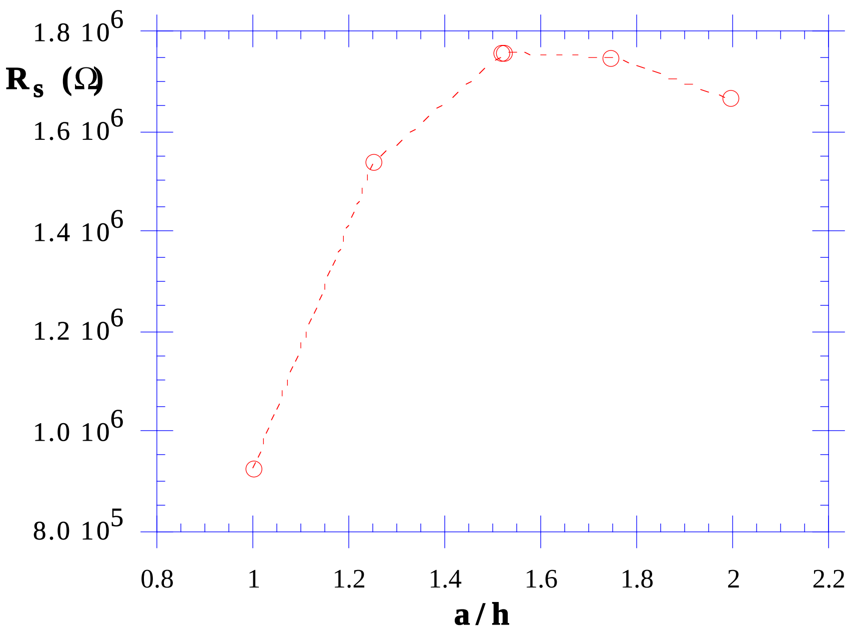


Fig. 4 - R/Q and Q (upper) and R_s (lower) of the rounded cavity vs. a/h.

The 'nosecone' cell

A typical approach to achieve higher values for the R/Q in accelerating cavities is to introduce the characteristic 'nose cones', which help to concentrate the electric field in the beam line zone. The same holds true for deflecting cavities, as Fig. 1 suggests. We started from the rounded structure in Fig. 3 by turning the straight part of the cell profile at a varying angle from the vertical. This is equivalent to introduce the nose cones, evidently. An example is displayed in Fig. 5.

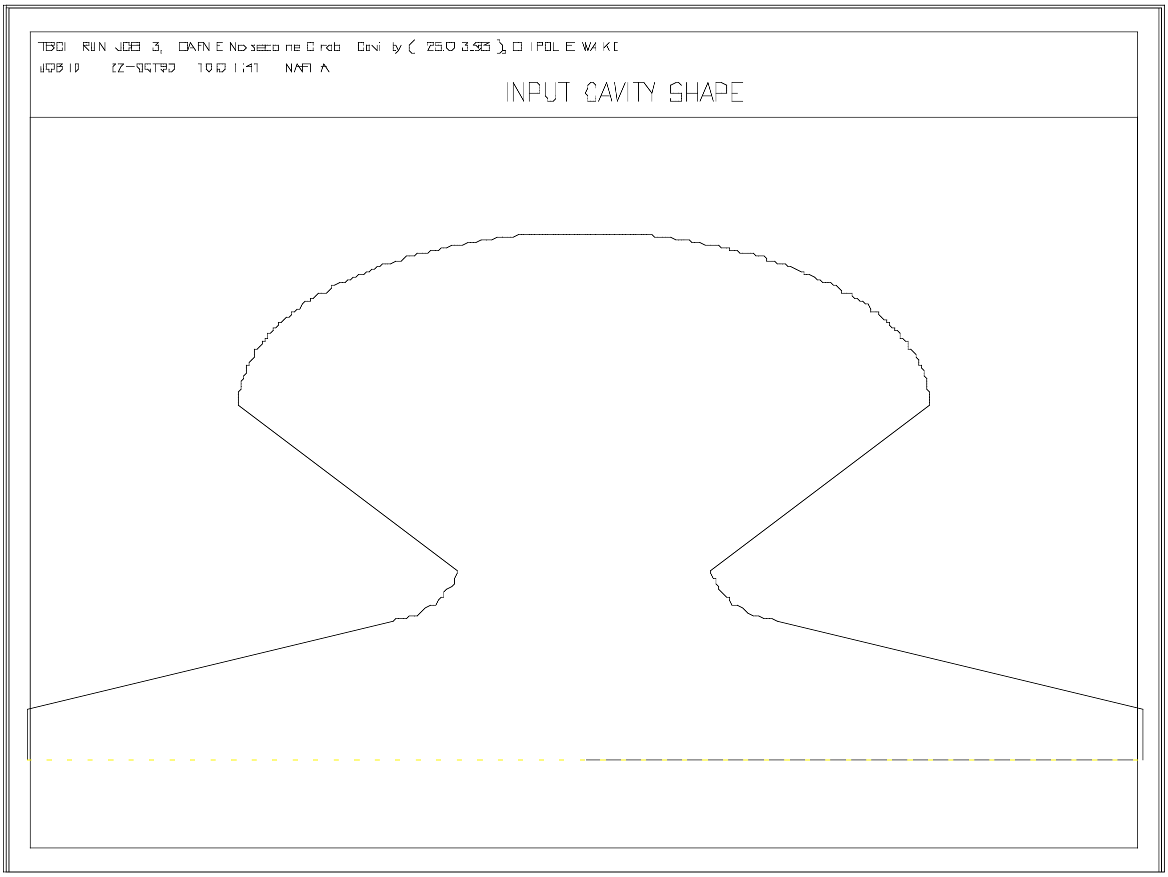


Fig 5 - The 'nosecone' cavity.

Results of this analysis are shown in fig 6. A maximum value of 56 Ω is achieved in this case, although such a cavity has a low Q-value and would certainly be more difficult to build. Anyhow it is clearly demonstrated that a shunt impedance of 3 MΩ can be obtained with a more regular shape, perhaps by optimizing more the Q than the R/Q.

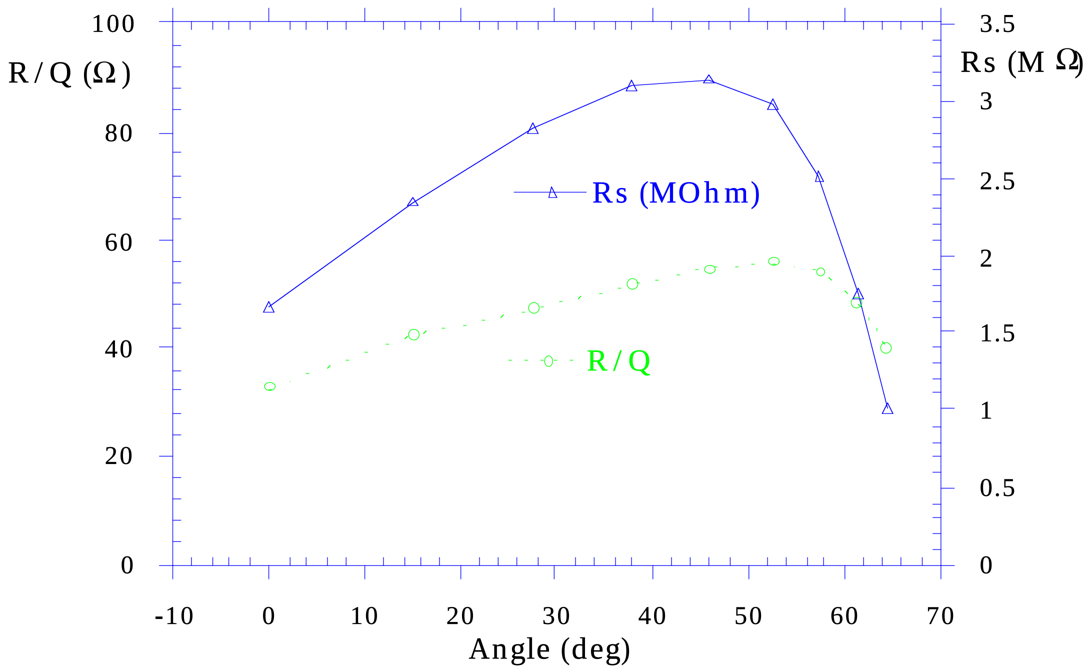


Fig. 6 - The R/Q and R_s of the TM_{110} mode in the nosecone cavity.

It is interesting to look at the two most dangerous higher order modes, the TM_{011} and the TE_{111}(TE_{0ml} modes have higher frequency, unlike TM_{nml} modes). The latter does not couple to the beam in a pill-box, but as soon as beam holes are introduced, it starts developing a small longitudinal component E_z, hence a transverse force according to the Panofsky-Wenzel theorem.

The results are shown in Figs. 7 - 8 for the two modes. There's an angle that makes the R/Q of the TE_{111} mode almost 0, while the one of the TM_{011} mode retains a non-negligible value. When looking at the angle dependence of the frequency, we observe that between 20° and 30° the frequency of both modes is still high enough to allow extraction of the fields by attaching external waveguides to the cavity body, without perturbing the main deflecting mode. Around that angle the R_s of the TM_{110} mode is almost 15% less than its maximum, what cannot be considered a big sacrifice, in view of other substantial advantages.

A possible candidate for a crab cavity with nosecones is shown in Fig. 9. A smooth profile was used to increase the cavity Q.

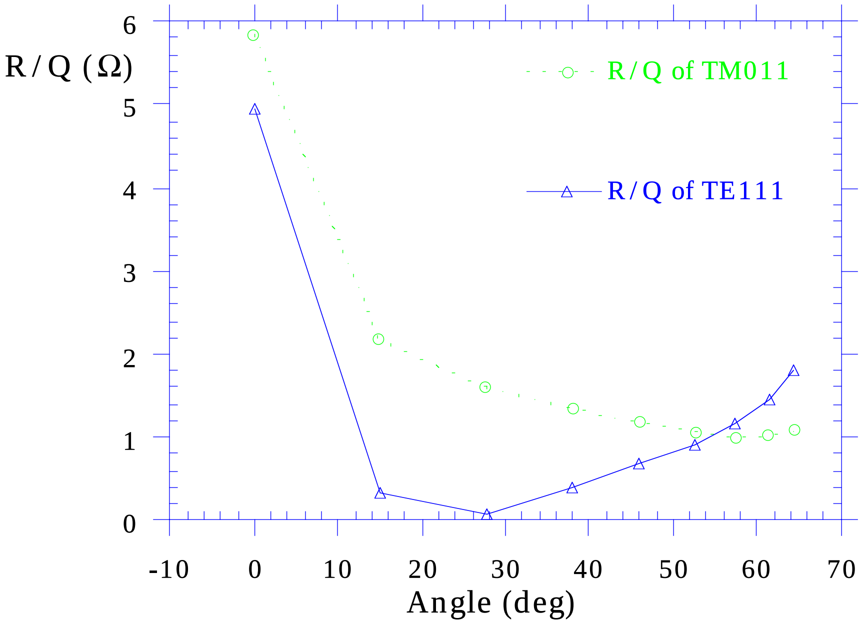


Fig. 7 - R/Q of the TM_{011} and TE_{111} modes vs. angle.

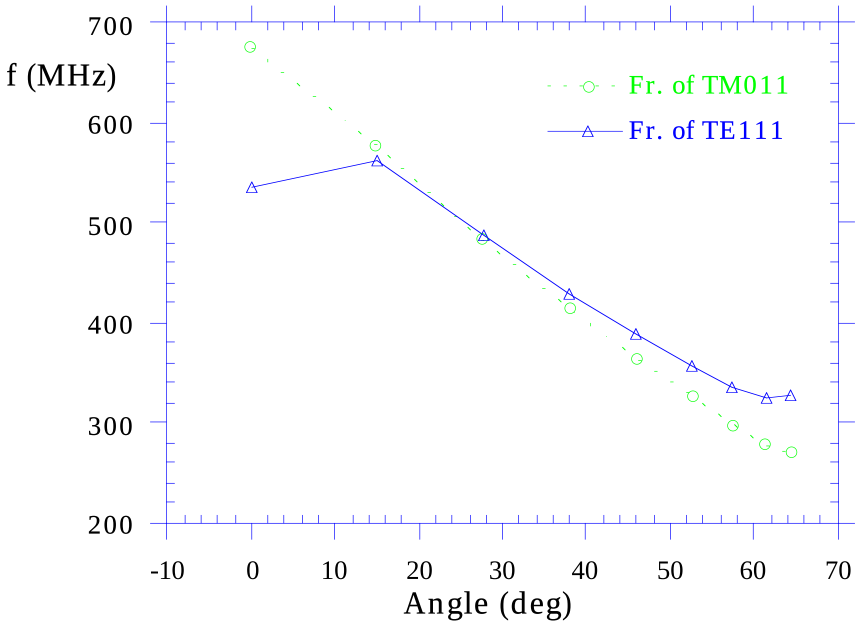


Fig. 8 - Frequency of the TM_{011} and TE_{111} modes vs. angle

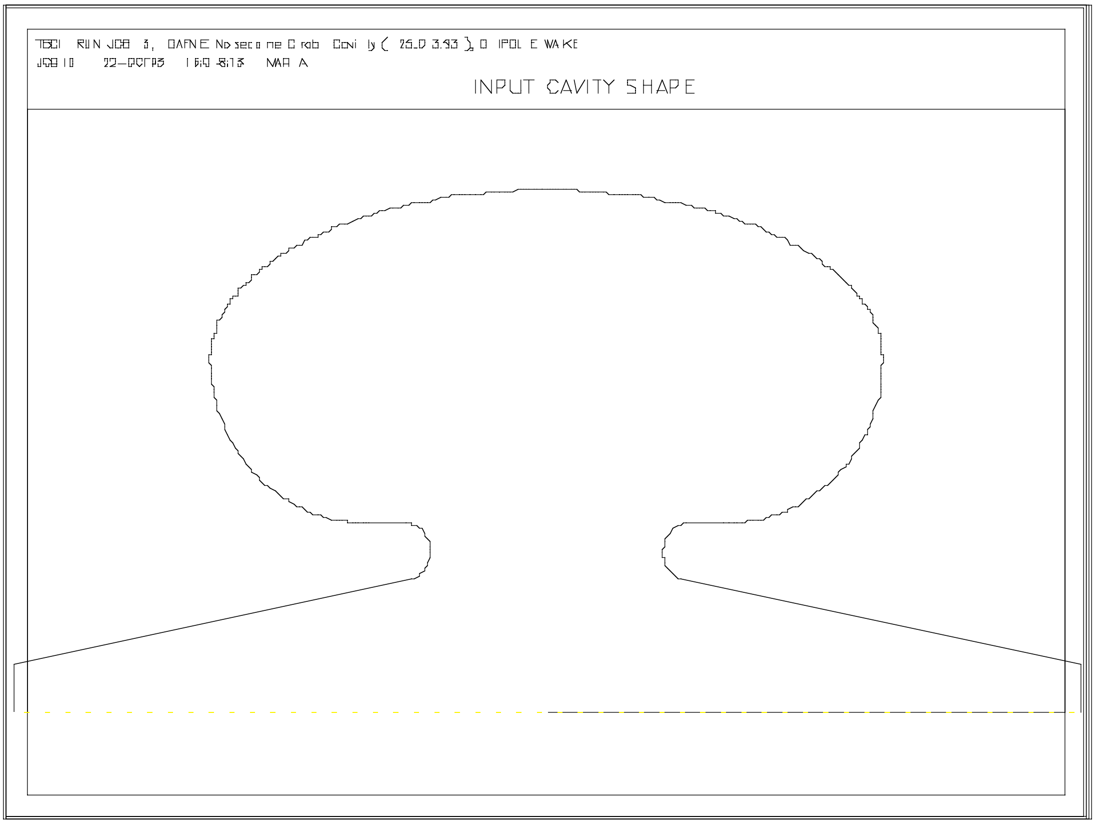


Fig. 9 - The optimized nosecone (or "door-knob") cavity.

To better illuminate the situation, it is useful to compare the two model cavities (Table I), bearing in mind that, although an intermediate choice is always possible, they represent two extreme significant examples of the optimization process. The nosecone (or "door-knob") cavity seems better performing, in particularly when looking at the ratio between the total loss factor to the dipole HOMs and the one of TM_{110} mode. The same holds true for the monopole modes. The main drawback of the nosecone design is the low frequency of some troublesome HOMs, what perhaps makes the rounded design more attractive for DAΦNE, in view of the loose power requirements.

In Table II instead, the two cavities are compared with the design of the 'round-like' superconducting cavity which has been constructed and tested at Cornell University in the framework of their B - Factory Project[2]. The actual cavity will be probably based on a very azimuthally asymmetric design (the so-called 'squashed' design) since the unwanted polarization of the deflecting mode has to be strongly shifted in frequency (and we shall do the same most likely), but the cell profile is clearly of the 'rounded' type and is derived from a 2-D model like ours. Except for the parameters which depend on the beam characteristics and are computed for a deflecting voltage of 2 MV, the other figures of merit, like the R/Q, and the geometry factor $Q*R_{surf}$ seem more favorable to our design. Only, the maximum electric field ratio E_{max} / E_0 is not very good in the nosecone cavity.

Table I - Design parameters for the DAΦNE Crab Cavity: comparison of two typical structures.

	Nosecone	Rounded
TM_{110} mode:		
Frequency (MHz)	368.288	368.288
R'/Q (Ω)	60.7	32.6
Q	60600	51000
R'_s (MΩ)	3.68	1.66
k'_0 (V/pC/m)	1.48	0.84
Dipole Modes:		
k'_t (V/pC/m)	2.04	2.19
k'_t / k'_0	1.38	2.61
TM_{010} mode:		
Frequency (MHz)	179.006	246.673
R/Q (Ω)	118.4	66.8
Q	39600	41300
R_s (MΩ)	4.69	2.76
k_0 (V/pC)	0.066	0.051
Longitudinal Modes:		
k_l (V/pC)	0.127	0.136
k_l/k_0	1.93	2.69
TM_{011} mode:		
Frequency (MHz)	295.557	674.856
R/Q (Ω)	1.4	5.8
Q	37100	46800
R_s (MΩ)	0.053	0.273
TE_{111} mode:		
Frequency (MHz)	331.066	536.192
R'/Q (Ω)	1.46	4.96
Q	47500	54900
R'_s (MΩ)	0.069	0.273

Table II - Final comparison of 'Nosecone' and 'Rounded' cavities for a deflecting voltage of 100 kV with the present 'round' -like SC Cornell Design.

	Nosecone	Rounded	Cornell
Crabbing mode:			
Frequency (MHz)	368.288	368.288	500
R'/Q (Ω)	60.7	32.6	25.6
Q	60600	51000	10^9
Geometry Factor Q^*R_{surf} (Ω)	303	255	218
R'_s (MΩ)	3.68	1.66	-
k'_0 (V/pC/m)	1.48	0.84	-
Stored Energy (Joule)	0.035	0.066	42*
Dissipated Power (kW)	1.4	3.	0.13*
Max. Surf. El. Field (MV/m)	1.66	1.21	25*
Max. Surf. Magn. Field (Oe)	10.4	19.4	520*
Av. El. Field E_0 (MV/m) +	0.245	0.245	6.7*
E_{max} / E_0	6.8	4.9	3.7
H_{max} / E_0 (Oe/(MV/m))	42.5	77.6	78.8
Beam Power @ r=1 mm (kW)	1	1	20*

\+ Average electric field required to obtain the same deflection.
* These values refer to a deflecting voltage of 2 MV.

Open problems

The TM_{010} mode must be treated separately, since it is difficult to damp it without affecting the other modes; perhaps by decoupling it from the beam harmonics by means of an especially dedicated tuner.

The HOMs can be treated with a global damping approach, as for the accelerating cavity, i.e. with waveguides terminated by RF transitions to coaxial cables, which should ensure strong damping on 2 GHz wide frequency band. This can be accomplished through proper shaping of the longitudinal profile (what influences the TM_{011} + TE_{111} frequencies) and of the transverse profile (unwanted polarization of the TM_{110} mode) at a price of a 20% reduction in R/Q, as we have seen.

At last, the cavity contribution to the machine longitudinal impedance has to be estimated, although we can say roughly that, according to a recently proposed broadband model[4], its contribution should not overcome the one of the main RF cavity.

APPENDIX

Definition of the characteristic longitudinal impedance of mode n at $r = r_0$ (tube radius):

$$(R/Q)_n = \frac{1}{2 P_n Q_n} \left| \int_0^L E_{nz}(z, r = r_0)\, e^{j\frac{\omega}{v}z}\, dz \right|^2 \qquad [\Omega]$$

Definition of the characteristic transverse (dipolar) impedance of mode n at $r = r_0$ (tube radius):

$$(R/Q)'_n = \frac{1}{(k_n r_0)^2} \frac{1}{2 P_n Q_n} \left| \int_0^L E_{nz}(z, r = r_0)\, e^{j\frac{\omega}{v}z}\, dz \right|^2 \qquad [\Omega]$$

The definition of the transverse coupling impedance is

$$Z_\perp = j \frac{V_\perp}{I_{b0}\, r_0} \qquad [\Omega\, m^{-1}]$$

where the imaginary unit j indicates that the induced voltage $V_\perp$ is 90° out of phase with the dipole moment of the beam current I_{b0} (like an inductance).

For a given resonant mode n, the following relationship holds between the coupling impedance and the shunt impedance $R'_n = (R'/Q)_n \bullet Q_n$ (see ad example, G. Dôme[5]):

$$Z_\perp(\omega) = \frac{k_n^2}{k} \frac{R'_n}{1 + j\, Q_n \left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega} \right)}$$

References

1) B. Palmer, SLAC-PUB-4707 (1988).

2) K. Akai, et al., "Crab Cavity for the B-Factories", Conference on B-Factories, April 6-10, 1992 SLAC.

3) K. Oide and K. Yokoya, "The crab-crossing scheme for storage ring colliders", Phys. Rev. A40, 315 (1989).

4) S. Bartalucci, L. Palumbo, M. Serio, B. Spataro and M. Zobov, "Broad-band Model Impedance for DAΦNE main rings", to be published in Nucl. Instr. and Meth. in Phys. Res.

5) G. Dôme, "RF Systems: waveguides and cavities", CERN SPS/86-24 (ARF).