DAΦNE TECHNICAL NOTE

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Note: **ME-2**

MECHANICAL ANALYSIS OF THE DAΦNE ACCUMULATOR KICKERS

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1. Introduction

In this note the mechanical structure of the kicker magnet for the DAΦNE accumulator ring is analyzed.

This structure (shown in Fig. 1) includes all the conductors in the vacuum chamber: the four main bars, their two interconnections and the four bus bars. Each main bar is supported by four ceramic columns. The fastening system between columns and bar is realized to allow the axial expansion of the bar.

The conductors system forms two symmetrical electric paths (upper and lower). In the worst working conditions, the paths carry a pulsating current of 1500 A at a frequency of 50 Hz. Each current pulse is composed by a half sinusoid (duration τ = 220 ns) and a rest at 0 A for the remainder of the period (T = 0.02 s). The circulating current produces a magnetic field distribution in the surroundings. The interaction between the currents and the magnetic field produces a distributed force along the conductors. The resulting mechanical load is therefore composed by a train of pulses.

Scope of this work was to investigate the dynamic response of the structure to such a load, with particular attention to the deformation allowed.

2. Methods and results

Most of the work has been done by means of ANSYS finite element code.

A full solution of the problem would require the study of the simultaneous electromagnetic and mechanical transients (quite impractical). However, some partial approach can highlight the dynamic behavior of the system.

Thus, several different models have been employed, with different level of approximation.

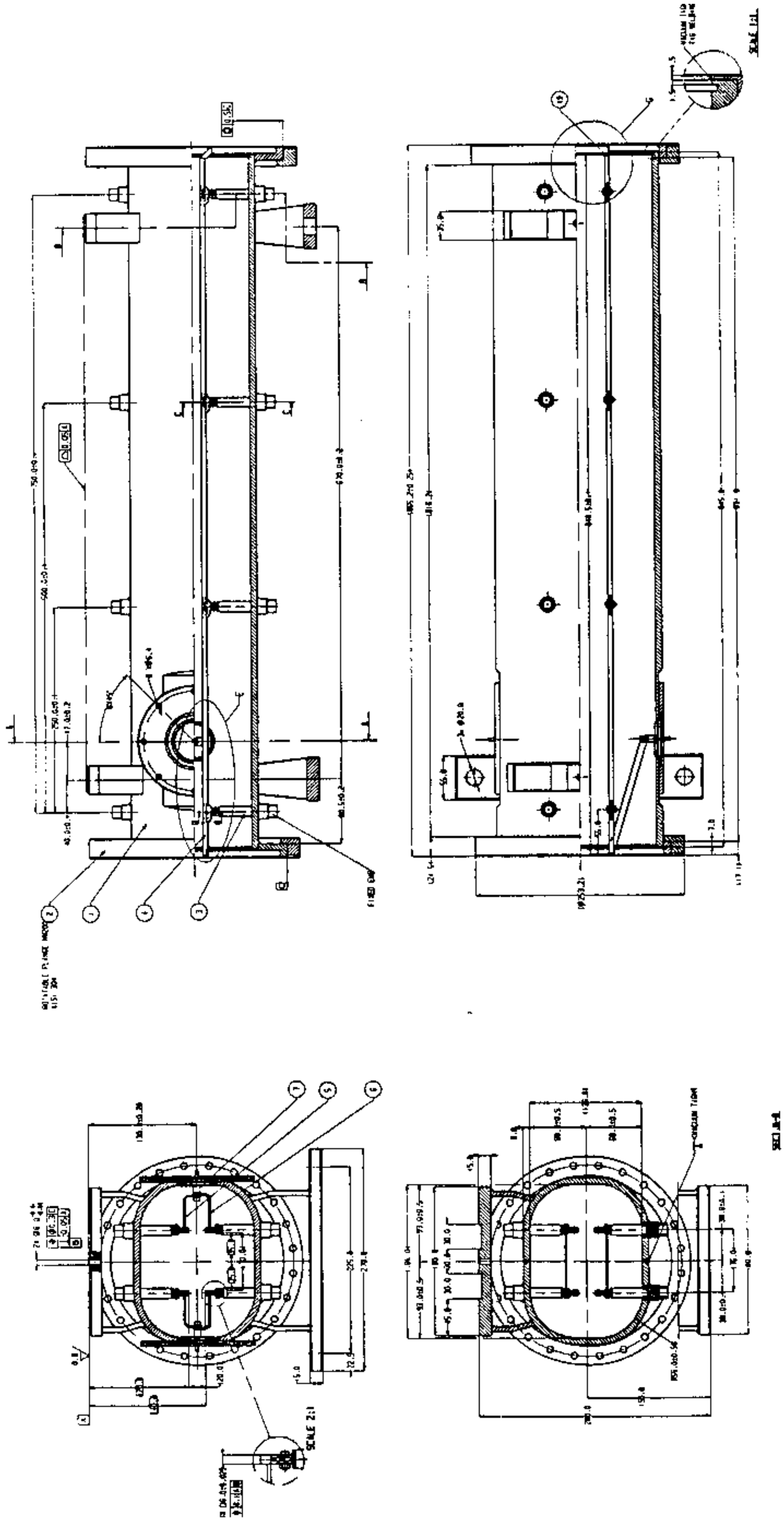

Fig. 1 - Accumulator kicker.

2.1. "Wires / beams" model

Only the four main bars have been included in the model. From the electromagnetic point of view, they have been considered as long wires carrying current (end effects neglected; no interconnections and bus bars: the electromagnetic problem is two-dimensional).

Eddy current and magneto-mechanic effects are neglected as well as the influence of vacuum chamber. From a mechanical point of view, each bar is considered as a beam on four support.

This simple model allows to highlight the dynamic behavior of the system. Furthermore, simple hand calculations can reinforce the computer analysis.

Mechanical load evaluation

Each bar leads a current i(t)

$$i(t) = I \sin\left[\frac{\pi}{\tau}(t - kT)\right] \qquad \text{for} \quad kT < t < kT + t$$

$$i(t) = 0 \qquad \text{for} \quad kT + t < t < (k+1)T$$

with k = 0,1,2,....

Each current originates a circumferential magnetic field given by:

$$B(t) = \frac{\mu_0 i(t)}{2\pi r}$$

where r is the distance from the wire.

This field produces a force F on the other carrying current conductors directed along the line ortogonally joining the conductors themselves:

$$F(t) = |L\mathbf{i} \times \mathbf{B}| = \frac{l \mu_0 i^2(t)}{2\pi r}$$

where r is the distance between two conductors and *l* is the length of the conductors.

Each bar is subject to the sum of the effects of the three other bars; the horizontal and the vertical force components have an amplitude of:

$$|F_v| = 17.89 \text{ N}$$

$$|F_h| = 10.64 \text{ N}$$

corresponding to a distributed force of:

$$|p_v| = 20.80 \text{ N/m}$$

$$|p_h| = 12.37 \text{ N/m}$$

The mechanical load can be expressed by:

$$|\mathbf{F}(t)| = A\sin\left[\frac{\pi}{\tau}(t-kT)\right] \qquad \text{for } kT< t <kT+\tau$$

$$|\mathbf{F}(\mathrm{t})| = 0 \qquad \text{for } kT+\tau < t < (k+1)T$$

with k=0,1,2,....

and

$$A = \frac{l\mu_0 I}{2\pi r}$$

A train of very short pulses with frequency ω is able to excite natural frequencies corresponding to $\omega_n = \omega^*k$, with k = 1,2,... (in that case the pulse is synchronous with the oscillations but happens every k periods).

The same result can be achieved by performing the Fourier analysis of the load, that gives a series of harmonic components with frequency kΩ and comparable amplitude.

The decomposition by Fourier series of the pulse train (see Appendix A) gives:

$$F_v(t) = \sum_{j=1}^{n} A_{vj}\cos(\omega_j t)$$

$$F_h(t) = \sum_{j=1}^{n} A_{hj}\cos(\omega_j t)$$

$$A_{jv} = \frac{|F_v|\omega}{\pi}\left(\frac{1}{\frac{\pi}{\tau} - j\omega} + \frac{1}{\frac{\pi}{\tau} + j\omega}\right)\cos\left(j\omega\frac{\tau}{2}\right)$$

$$A_{jh} = \frac{|F_h|\omega}{\pi}\left(\frac{1}{\frac{\pi}{\tau} - j\omega} + \frac{1}{\frac{\pi}{\tau} + j\omega}\right)\cos\left(j\omega\frac{\tau}{2}\right)$$

By exploiting the calculations for the first 25 components:

$A_{jv} = 0.25056\ 10^{-3}$ N = const

$A_{jh} = 0.14898\ 10^{-3}$ N = const

$\omega_j = j^*50$ $\qquad$ j = 1,...,25

Note that the amplitude of the components is unchanged for a wide range of frequency, because of the small ratio between the duration of a pulse and the period of the train: $\frac{\ }{T} = \frac{\ }{2\pi} = 1.1 \cdot 10 \quad \ll 1$.

Mechanical constraints definition

The behavior of the constraints of the bars (details shown in Fig. 2) has been described by allowing only the expansion along the axis of the bar and the rotation around the transversal horizontal axis centered in the pin. The rigidity of the columns has been supposed high enough to not contribute to the vibration of the bars.

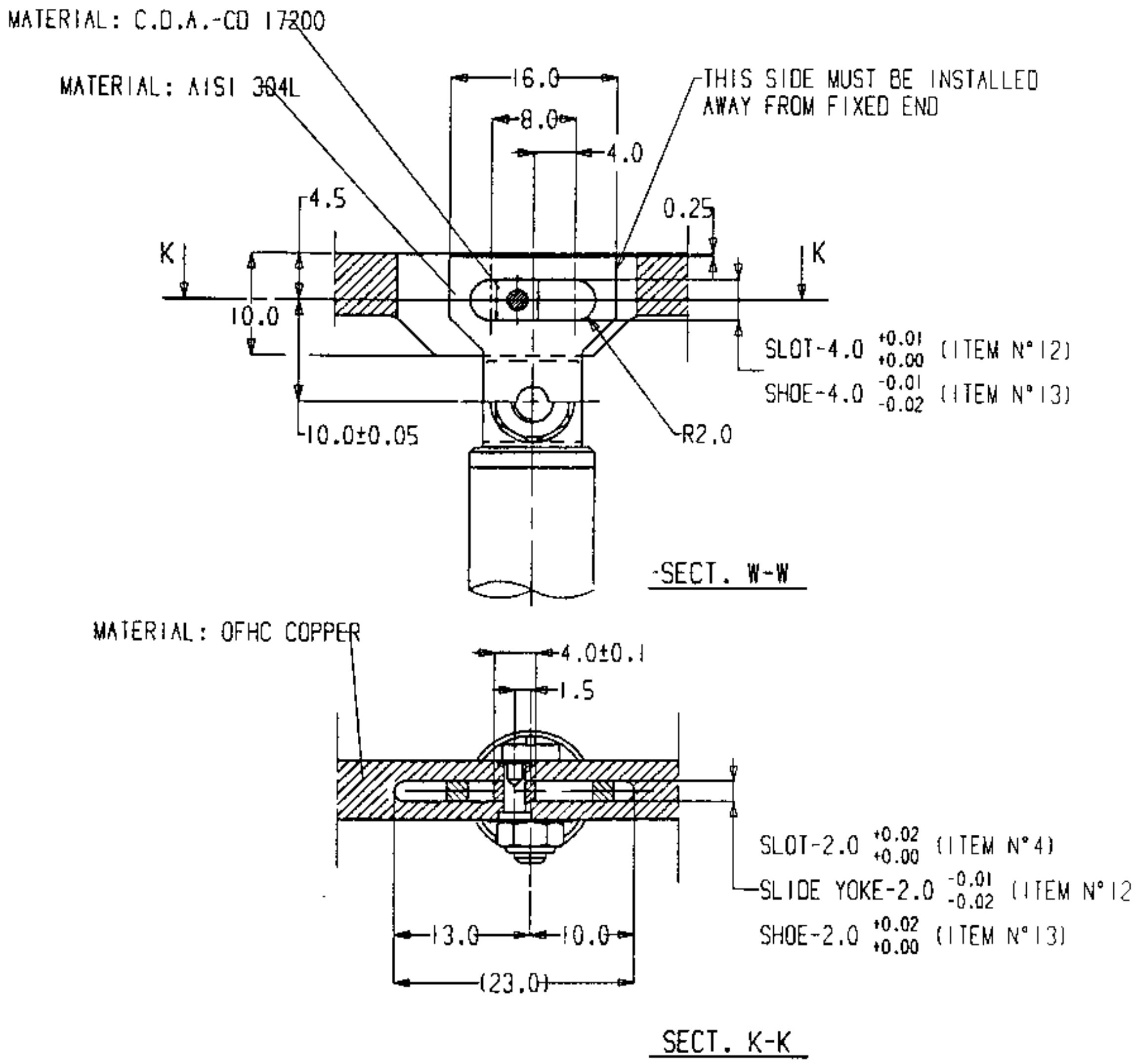

Fig. 2 - Head of support.

Modal analysis

The analysis has been done with ANSYS code as well as by hand calculation.

The structure has been represented by a mono-dimensional beam in the space, using 3D beam elements. Each node has six degrees of freedom, so that the beam can deflect in every direction.

In correspondence of the supports a nodal constraint system simulate the real behavior of the bar as previously described.

The modal analysis gives the following natural frequencies:

mode	frequency (Hz)
1	157.05813
2	196.78916
3	291.64817
4	365.41722
5	365.41722
6	365.41722
7	541.54675
8	578.61679
9	783.96099
10	952.71532

Note that some frequencies correspond to vibrations of the beam in different planes; they do not overlap exactly because of the different behavior of constraints in the two planes and consequently of the different deflection shapes of the bar. That is why the 4^{th}, 5^{th} and 6^{th} frequencies are equal.

Harmonic analysis

As shown above, among the first natural frequencies there is some frequency very close to an exact multiple of the frequency of the train of pulses. In spite of the really small amplitude of the load components, there could be risk of excite dangerous amplifications.

Thus a first harmonic analysis has been performed with ANSYS by excluding any damping effect.

The model is the same that for the modal analysis and has been loaded with sinusoidal distributed forces corresponding to the first 40 components of the Fourier decomposition of the load.

Successive components have been neglected because their frequencies are far from the natural frequencies of the structure.

A composition of the displacements due to all the load components has been performed on selected nodes. The result is almost similar to the vibration excited by the close-to-resonance component alone, given its remarkable dominance in amplitude.

In order to achieve a feeling of the real amplitude of the forced vibrations, the analysis has been repeated with non zero damping effect.

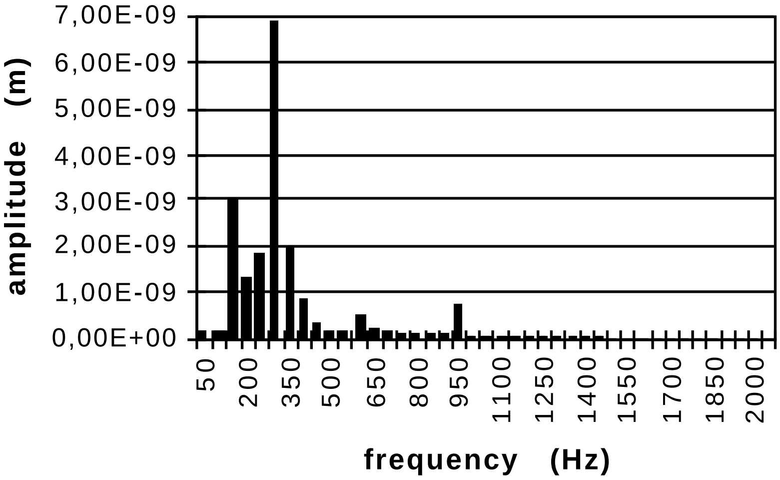

Fig. 3 - Frequency response of the beam.

2.2. "Real geometry" model

To obtain a real distribution of forces along the whole system of conductors, a more accurate finite element model has been considered (Fig. 4). This has allowed a very precise approximation of the behavior of the constraint system, and an evaluation of the mechanical load on the auxiliary conductors (i.e. interconnections and bus bars).

Three-dimensional multi-field brick elements have been employed in three meshes with an inreasing complexity, to verify and improve the computation.

The analysis done with the ANSYS code includes the evaluation of the magnetic field and of the distributed force, as well as the determination of stresses and strains. It has been limited to a static case, considering a constant current load with the same amplitude of the pulsed one. For every model, the input includes just the current source. The displacement in the more refined analysis reaches 0.023E-3 m.

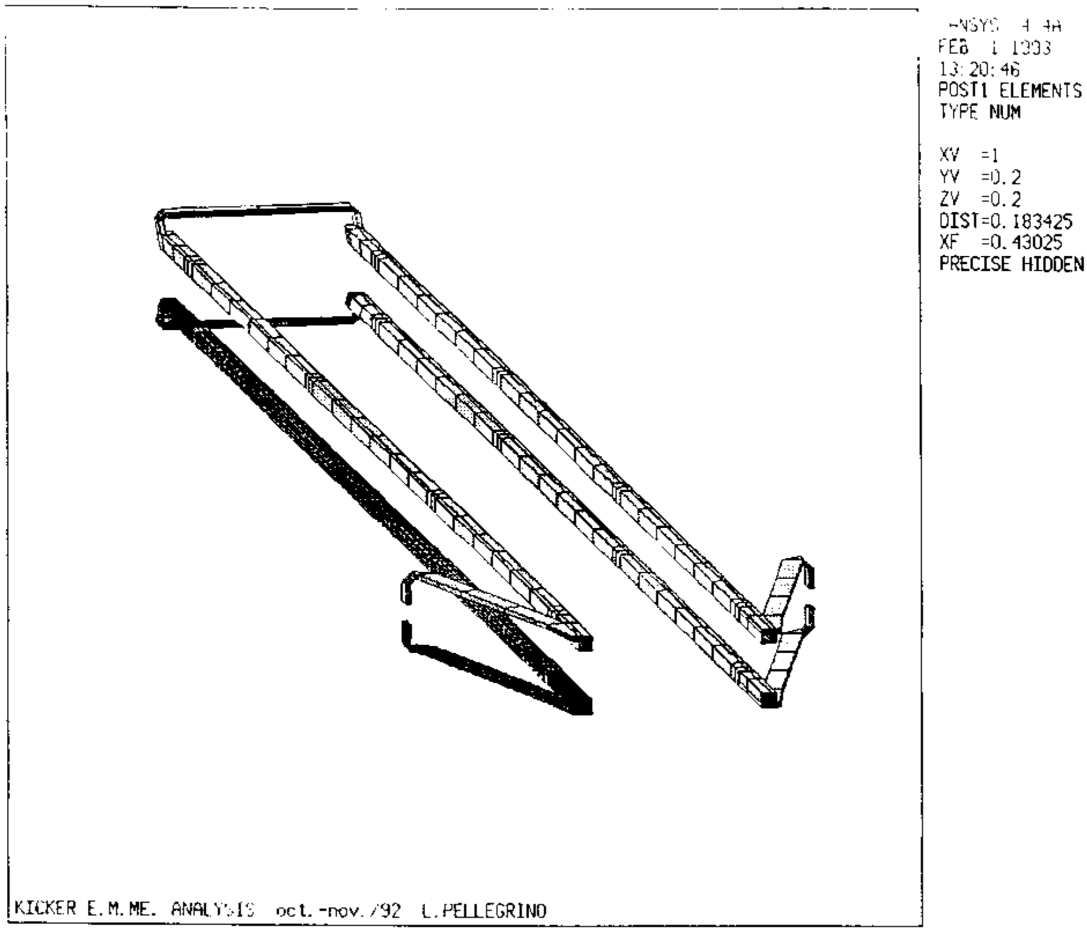

Fig. 4 - Finite element model.

2.3. Partial models "bus bar" and "interconnection"

Two further modal analyses has been done to evaluate the natural frequencies of auxiliary components and to highlight any dangerous situation.

The first model includes only the interconnection and the second only the bus bar, assuming the main bars as fixed. The meshes employed are finer than in the complete model.

No problem results.

3. Discussion

A considerable amplification of the vibrations is always possible. However, a minimum amount of damping should maintain the deformation into an acceptable level.

Furthermore, it seems meaningful that the static deformations should be scaled with the ratio τ/T to obtain the actual amplitude of the forced vibration of each component of the load.

Such a consideration is confirmed by the results of the dynamic analysis on the simplified model ("wires/beam" model).

In addiction, the number of the significant components are limited and the superposition of such components should not give any problem, given the phase difference.

4. Conclusions

A kicker with such characteristics of load and geometry is not critical from a mechanical point of view.

However, a test on a prototype should be advisable.

APPENDIX A

FOURIER SERIES DECOMPOSITION OF A TRAIN OF PULSES WITH SHAPE OF AN HALF SINUSOIDAL WAVE.

The train of pulses is described by:

$$F(t) = A\cos\left[\frac{\pi}{\tau}(t-kT)\right] \qquad \text{for} \quad kT-\frac{\tau}{2} < t < kT+\frac{\tau}{2}$$

$$\text{F(t)=0} \qquad \text{for} \quad kT+\frac{\tau}{2} < t < (k+1)T-\frac{\tau}{2}$$

with k=0,1,2,....

The series of cosines is:

$$F(t) = \sum_{n=1} B_n \cos(n\omega t)$$

Coefficients evaluation:

$$B_0 = \frac{\omega}{\pi}\int_{-\frac{T}{2}}^{\frac{T}{2}} F(t)dt =$$

$$= \frac{\omega}{\pi}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\cos\left(\frac{\pi}{\tau}t\right)dt =$$

$$= \frac{\omega}{\pi}A\frac{\tau}{\pi}\left[\sin\left(\frac{\pi}{\tau}t\right)\right]_{-\frac{\tau}{2}}^{+\frac{\tau}{2}} =$$

$$= \frac{\omega A\tau}{\pi^2}(1+1) = \frac{2\omega A\tau}{\pi^2}$$

$$B_j = \frac{\omega}{\pi}\int_{-\frac{T}{2}}^{\frac{T}{2}} F(t)\cos(j\omega t)dt =$$

$$= \frac{A\omega}{\pi}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \cos\left(\frac{\pi}{\tau}t\right)\cos(j\omega t)dt =$$

$$= \frac{A\omega}{2\pi}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \left[\cos\left(\frac{\pi}{\tau} - j\omega\right)t + \cos\left(\frac{\pi}{\tau} + j\omega\right)t\right]dt =$$

$$= \frac{A\omega}{2\pi}\left[\frac{\sin\left(\frac{\pi}{\tau} - j\omega\right)t}{\left(\frac{\pi}{\tau} - j\omega\right)} + \frac{\sin\left(\frac{\pi}{\tau} + j\omega\right)t}{\left(\frac{\pi}{\tau} + j\omega\right)}\right] =$$

$$= \frac{A\omega}{2\pi}\left[\frac{\sin\left(\frac{\pi}{2} - j\omega\frac{\tau}{2}\right) - \sin\left(-\frac{\pi}{2} + j\omega\frac{\tau}{2}\right)}{\left(\frac{\pi}{\tau} - j\omega\right)} + \frac{\sin\left(\frac{\pi}{2} + j\omega\frac{\tau}{2}\right) - \sin\left(-\frac{\pi}{2} - j\omega\frac{\tau}{2}\right)}{\left(\frac{\pi}{\tau} + j\omega\right)}\right] =$$

$$= \frac{A\omega}{2\pi}\left[\frac{\cos\left(j\omega\frac{\tau}{2}\right) + \cos\left(j\omega\frac{\tau}{2}\right)}{\left(\frac{\pi}{\tau} - j\omega\right)} + \frac{\cos\left(j\omega\frac{\tau}{2}\right) + \cos\left(j\omega\frac{\tau}{2}\right)}{\left(\frac{\pi}{\tau} + j\omega\right)}\right] =$$

$$= \frac{A\omega}{\pi}\cos\left(j\omega\frac{\tau}{2}\right)\left[\frac{1}{\left(\frac{\pi}{\tau} - j\omega\right)} + \frac{1}{\left(\frac{\pi}{\tau} + j\omega\right)}\right] \cong 0$$

In the case of $\tau << T = 2\pi/\omega$ and small j, one has:

$$\frac{1}{\left(\frac{\pi}{\tau} - j\omega\right)} \cong \frac{1}{\left(\frac{\pi}{\tau} - j\omega\right)} \cong \frac{\tau}{\pi}$$

$$\cos\left(j\omega\frac{\tau}{2}\right) \cong 1$$

$$F(t) = A\frac{2\tau\omega}{\pi^2} = A\frac{4\tau}{\pi T}$$

that does not depends on j.