DAФNE TECHNICAL NOTE
INFN - LNF, Accelerator Division

Note: L-25

# ACCUMULATOR MODELLING 

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In order to describe the accumulator lattice a model is generally used where the dipole is represented by an ideal sector magnet with field index $n$ and a parameter $b$, the fringe length, to take into account the effect of the fringing field on the vertical focusing, and the quadrupoles are described by a rectangular model. In this note, after a review of the measurements, a study of the accumulator lattice aimed at reproducing the measured tune values is presented. The optical functions plot and lattice parameters are presented in Appendix 1. Appendix 2 contains the parameter list of the accumulator.

## 1. TUNE MEASUREMENTS

A set of tune measurements has been performed during the last accumulator shifts (third weekend of December): in particular we will use those with a fixed variation of the current ( $\pm 1$ or $2 \%$ ) in one quadrupole family at a time, with a beam current of 1 mA , for which the ion induced tune shift and spread are negligible and there were no synchrotron sidebands. After varying the current in each quadrupole family, the tunes with the nominal settings were measured again and about the same values were found:

$$
\begin{array}{lll}
v_{x}{ }^{\text {meas }}=3.147 & v_{y}{ }^{\text {meas }}=1.1605 & \text { sextupoles on } \\
v_{x^{\text {meas }}}=\mathbf{3 . 1 5 2} & v_{y^{\text {meas }}}=1.168 & \text { sextupoles off }
\end{array}
$$

All the measurements were performed with sextupoles on and off. In the following Tables we summarise the results. We call QF 1 and SF the first quadrupole and sextupole from the electron injection septum in the clockwise direction. The quadrupole strengths were computed assuming for the beam energy the value corresponding to the dipole current ( 594 A ), that is: $\mathrm{E}=.5113 \mathrm{GeV}$, $\mathrm{V}_{\mathrm{RF}}=120 \mathrm{KV}$ and $\mathrm{F}_{\mathrm{RF}}=368.39 \mathrm{MHz}$. The last column of Table II refers to the tunes measured again after setting the quadrupole family to the nominal set.

We recall here the constants used [1] for computing the quadrupole and sextupole strengths:

$$
\begin{array}{ll}
\mathbf{K}_{\mathbf{i}}^{2}\left(\mathrm{~m}^{-2}\right)=\mathbf{I}_{\mathbf{i}}(\mathbf{A}) * \mathbf{C}_{1} / \mathbf{E}(\mathbf{G e V}) & \text { quadrupole }\left(\mathbf{C}_{\mathbf{1}}=9.1286^{*} 10^{-3}\right) \\
\mathbf{K}_{\mathbf{i}}^{2}\left(\mathrm{~m}^{-2}\right)=\mathbf{I}_{\mathbf{i}}(\mathbf{A}) * \mathbf{C}_{2} / \mathbf{E}(\mathbf{G e V}) & \text { quadrupole }\left(\mathbf{C}_{2}=9.1430^{*} 10^{-3}\right) \\
\mathbf{K}_{\mathbf{i}}^{2}\left(\mathbf{m}^{-2}\right)=\mathbf{I}_{\mathbf{i}}(\mathbf{A}) * \mathbf{C}_{3} / \mathbf{E}(\mathbf{G e V}) & \text { quadrupole }\left(\mathbf{C}_{3}=9.1466^{*} 10^{-3}\right)
\end{array}
$$

$$
\underset{\mathbf{K} .2\left(\mathbf{m}^{-2}\right)}{\mathbf{K}_{\mathbf{i}}{ }^{2}\left(\mathbf{I}_{\mathbf{- 2}}(\mathbf{A}) * \mathbf{C}_{\text {SF }} / \mathbf{E}(\mathbf{G e V}) \quad \text { sextupole } \mathbf{S F}\left(\mathbf{C}_{\text {SF }}=2.0302 * 10^{-2}\right)\right.}
$$

$$
\mathbf{K}_{\mathbf{i}}^{2}\left(\mathbf{m}^{-2}\right)=\mathbf{I}_{\mathbf{i}}(\mathbf{A}) * \mathbf{C}_{\text {SD }} / \mathbf{E}(\mathbf{G e V})
$$

sextupole $\mathrm{SD}\left(\mathrm{C}_{\mathrm{SD}}=2.0191 * 10^{-2}\right)$

Table I - Quadrupole and Sextupole nominal settings (sign is arbitrary)

|  | SET VALUE (A) | K $^{\mathbf{2}}\left(\mathbf{m}^{\mathbf{- 2}}\right)$ |
| :---: | :---: | :---: |
| QF1 | 250. | 4.463165 |
| QD | 272. | -4.86358 |
| QF2 | 248. | 4.43619 |
| SF | 183. | -7.27 |
| SD | 150. | 5.93 |
| CHVA1001 | +1. (only H) | - |
| CHVA1002 | -3. (only H ) | - |

Table II - Tune measurements vs. $\Delta \mathrm{I}$, with sextupoles on.

|  | $v_{\mathbf{x}}$ |  | $v_{\mathbf{y}}$ | $v_{\mathbf{x}}$ | $v_{\mathbf{y}}$ | $v_{\mathbf{x}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | SET $+\mathbf{1} \%$ |  | SET $-\mathbf{1} \%$ |  | SET |  |
| QF1 | .1630 | .1428 | .1318 | .1776 | .1471 | .1605 |
| QD | .1465 | .2151 | .1482 | .1037 | .1471 | .1581 |
| QF2 | .1611 | .1404 | .1337 | .1751 | .1471 | .1586 |

There is a difference of $\approx 1.9 / 2.4 \times 10^{-3}$ on the vertical tune with respect to the nominal one, after setting the power supplies to the nominal values.

Table III - Tune measurements vs. $\Delta \mathrm{I}$, with sextupoles off.

|  | $v_{\mathbf{x}}$ | $v_{\mathbf{y}}$ | $v_{\mathbf{x}}$ | $v_{\mathbf{y}}$ |
| :--- | :---: | :---: | :---: | :---: |
|  | SET $+\mathbf{2} \%$ |  | SET -2 \% |  |
| QF1 | .1824 | .1330 | .1202 | .2033 |
| QD | .1495 | .1806 | .1544 | .055 |
|  | SET $+\mathbf{1} \%$ |  | SET $-\mathbf{1} \%$ |  |
| QF2 | .1672 | .1489 | .1374 | .1843 |

From the values in Table II we can compute $\Delta \mathrm{K}_{\mathrm{i}}$ and the average $\beta_{\mathrm{x}}$ and $\beta_{\mathrm{y}}$ values at the quadrupoles, as listed in Table IV:

$$
\beta_{\mathrm{i}}=\frac{4 \pi \Delta \nu_{\mathrm{i}}}{\Delta \mathrm{~K}_{\mathrm{i}}}
$$

$\Delta v_{i}$ are the differences between the tunes measured with the excitation current changed by $\pm 1 \%$ (sextupoles on) and those measured at the nominal current setting.

Table IV - Computed average $\beta$ values at quadrupole vs. $\Delta \mathrm{I}$ (sextupoles on)

|  | $\Delta \mathbf{I}(\boldsymbol{\%})$ | $\Delta v_{\mathbf{x}}$ | $\Delta v_{\mathbf{y}}$ | $\beta_{\mathbf{x}}(\mathbf{m})$ | $\beta_{\mathbf{y}}(\mathbf{m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| QF1 | +1 | +.016 | -.0172 | 4.5 | 4.8 |
|  | -1 | -.0152 | +.0176 | 4.3 | 4.95 |
| QD | +1 | -.0005 | +.0551 | .13 | 14.3 |
|  | -1 | +.0012 | -.0563 | .31 | 14.6 |
|  | +1 | +.0141 | -.0196 | 4. | 5.6 |
|  | -1 | -.0133 | +.0151 | 3.8 | 4.3 |

## 2. CHROMATICITY MEASUREMENT

A chromaticity measurement has been performed with sextupoles on, for $v_{x}=3.147, v_{y}=1.160$, by moving the RF frequency by a .005 MHz step. Table V summarises the results. $\mathrm{F}_{\mathrm{x}}$ and $\mathrm{F}_{\mathrm{y}}$ are the measured horizontal and vertical resonance frequencies.

Table V - Measured tunes vs. RF frequency (sextupoles on)

| $\mathbf{F}_{\mathbf{R F}}(\mathbf{M H z})$ | $\mathbf{F}_{\mathbf{x}}(\mathbf{M H z})$ | $\mathbf{F}_{\mathbf{y}}(\mathbf{M H z})$ | $\Delta v_{\mathbf{x}}$ | $\Delta v_{\mathbf{y}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 368.385 | 148.71 | 148.839 | .1472 | .1612 |
| 368.39 | 148.71 | 148.834 | .1470 | .1605 |
| 368.395 | 148.71 | 148.834 | .1468 | .1603 |

From these measurements we get:

$$
C_{x}=+0.6, \quad C_{y}=+3.6
$$

assuming a momentum compaction value of .041 . While the horizontal tune is linear around the "central" frequency, the vertical one is not, probably due to the large frequency interval measured. A measurement on a larger frequency range but with smaller frequency steps is needed to better measure the dependence of the tune on the beam energy.

The contribution to the chromaticity due to the lumped sextupoles only has been computed with the lattice functions listed in Table VI:

Table VI - Lattice functions at sextupoles

|  | $\beta_{\mathbf{x}}(\mathbf{m})$ | $\beta_{\mathbf{y}}(\mathbf{m})$ | $\mathbf{D}_{\mathbf{x}}(\mathbf{m})$ | $\mathrm{K}\left(\mathbf{m}^{-2}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{S F}$ | 4.4 | 4.1 | .73 | -7.27 |
| $\mathbf{S D}$ | 1.25 | 9.05 | .78 | +5.93 |

From these values we get:

$$
C_{x}=+5.6, \quad C_{y}=+6.4
$$

## 3. TUNE SHIFT DUE TO SEXTUPOLES

The tune measurements with and without sextupoles showed a tune shift:

$$
\Delta v_{x}=-0.005, \Delta v_{y}=-0.0075
$$

From orbit measurements performed in previous shifts (Nov. 96) we could compute the tune-shift induced by an off-axis orbit in the sextupoles. It turned out that the effect of such a displacement is negligible. Moreover in the present runs the vertical orbit was almost corrected to zero by moving mechanically the position of the magnetic centre of two quadrupoles (QUAA2002 by -.3 mm , QUAA4002 by +.1 mm ).

If we assume that the measured tune shifts are due to a displaced RF frequency with respect to the "central" one, we can estimete its new value. With the previously computed values of the sextupole contributions to the chromaticity, assuming $\alpha_{c}=.041$, we get:

$$
\Delta \mathrm{p} / \mathrm{p} \approx 1 \times 10^{-3}, \quad \Delta \mathrm{~F} \approx-15 \mathrm{kHz}, \quad \mathrm{~F}_{\mathrm{RF}}=368.375 \mathrm{MHz}
$$

## 4. DISPERSION MEASUREMENTS

Dispersion measurements were also performed, by reading the horizontal orbit at two stripline monitors located at the injection straights, for two different RF frequencies, with a 1 mA beam. Table VII shows the data and results. The dispersion was computed assuming the model described in the following section. The last two measurements refer to two different quadrupole settings giving the same tune values but different computed dispersion at the injection straight.

Table VII - Dispersion measurements

| Monitor | $v_{\mathbf{x}}{ }^{\text {meas }}$ | $v_{\mathbf{y}}{ }^{\text {meas }}$ | $\mathbf{F}_{\mathbf{R F}}(\mathbf{M H z})$ | $\left\langle\mathbf{x}_{\mathbf{0 c}}\right\rangle(\mathbf{m m})$ | $\alpha_{\mathbf{c}}$ | $\mathbf{D}_{\mathbf{x}}{ }^{\mathbf{c a l}}(\mathbf{m})$ | $\mathbf{D}_{\mathbf{x}}{ }^{\text {meas }}(\mathbf{m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BPSA4001 | 3.147 | 1.16 | 368.34 | -0.8 | .041 | 0.13 | 0.131 |
| BPSA4001 |  |  | 368.44 | -1.67 |  |  |  |
| BPSA3001 | 3.147 | 1.16 | 368.34 | +0.85 | .041 | 0.13 | 0.185 |
| BPSA3001 |  |  | 368.44 | +2.075 |  |  |  |
| BPSA3001 | 3.10 | 1.15 | 368.34 | -1.41 | .044 | 0.13 | 0.248 |
| BPSA3001 |  |  | 368.44 | -2.94 |  |  |  |
| BPSA3001 | 3.10 | 1.15 | 368.34 | +1.27 | .044 | 0.0 | 0.153 |
| BPSA3001 |  |  | 368.44 | +0.33 |  |  |  |

## 5. TUNES FIT

The vertical tune can be easily fitted by adjusting a parameter $b$, which represents the fringing field length. We use a dipole ( $3 \times 3$ ) transfer matrix in rectangular approximation for the horizontal plane:

$$
\mathbf{M}_{\mathbf{x}}=\left(\begin{array}{ccc}
\cos k_{x} s & \sin k_{x} s / k_{x} & \left(1-\cos k_{x} s\right) / \rho k_{x}^{2} \\
-k_{x} \sin k_{x} s & \cos k_{x} s & \sin k_{x} s / \rho k_{x} \\
0 & 0 & 1
\end{array}\right)
$$

where: $\mathrm{k}_{\mathrm{x}}=\frac{\sqrt{1-\mathrm{n}}}{\rho}$, $\mathrm{n}=$ field index, $\rho=$ bending radius, while for the vertical plane we added a thin lens to simulate the effect of the fringing:

$$
\mathbf{M}_{\mathbf{y}}=\left(\begin{array}{cc}
1 & 0 \\
\frac{2 b}{6 \rho^{2}} & 1
\end{array}\right)\left(\begin{array}{cc}
\cos \mathrm{k}_{\mathrm{y}} \mathrm{~s} & \sin \mathrm{k}_{\mathrm{y}} \mathrm{~s} / \mathrm{k}_{\mathrm{y}} \\
-\mathrm{k}_{\mathrm{y}} \sin \mathrm{k}_{\mathrm{y}} \mathrm{~s} & \cos \mathrm{k}_{\mathrm{y}} \mathrm{~s}
\end{array}\right)\left(\begin{array}{cc}
1 & 0 \\
\frac{2 b}{6 \rho^{2}} & 1
\end{array}\right)
$$

where $\mathrm{k}_{\mathrm{y}}=\frac{\sqrt{\mathrm{n}}}{\rho}$.

The dipole magnetic length is assumed $\mathrm{L}_{\mathrm{B}}=.864 \mathrm{~m}$, the nominal bending radius is $\rho=1.10008$ m . From the dipole field measurements [2] a field index $n=.47$ at the magnet center was obtained, while its average value is $n \approx .42$.

For semplicity, being the three quadrupolar constants very similar, we assume in the following for the three families the same constant $\mathrm{C}_{1}$, relative to the QF 1 family.

If we choose as free parameters the field index $n$ and the fringing length $b$, keeping the quadrupole constant $\mathrm{C}_{1}$ fixed to its measured value for the QF1 family, to match the tunes 3.147, 1.1605 we get:

$$
n=.452186 \quad b=.116119
$$

Another possibility is to fix the field index and let vary the quadrupole constant $\mathrm{C}_{1}$.
In order to find the most suitable combination of free parameters to fit the tunes, we used all the tune measurements with sextupoles on, which are summarised in Table VIII.

Table VIII - Quadrupole settings and tune measurements @ 368.39 MHz , .511 GeV (sext. on).

|  | QF1 | QD | QF2 | $v_{\mathbf{x}}{ }^{\text {meas }}$ | $v_{\mathbf{y}}{ }^{\text {meas }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{I}(\mathbf{A})$ | 249.4 | 269.7 | 249. | 3.1501 | 1.1074 |
| $\mathbf{I}(\mathbf{A})$ | 243.9 | 264.1 | 237.8 | 3.059 | 1.1111 |
| I (A) | 246.5 | 264.8 | 243.5 | 3.1037 | 1.0665 |
| I (A) | 247. | 269.1 | 243.3 | 3.1038 | 1.1501 |
| I (A) | 250. | 272. | 248. | 3.147 | 1.1605 |
| I (A) | 252.5 | 272 | 248. | 3.1630 | 1.1428 |
| I (A) | 247.5 | 272 | 248. | 3.1318 | 1.1776 |
| $\mathbf{I}(A)$ | 250. | 274.7 | 248. | 3.1465 | 1.2154 |
| I (A) | 250. | 269.3 | 248. | 3.1482 | 1.1037 |
| I (A) | 250. | 272. | 250.5 | 3.1611 | 1.1404 |
| I (A) | 250 | 272 | 245.5 | 3.1337 | 1.1751 |

We used two couples of different parameters: ( $n, b$ ), leaving $C_{1}=9.1286 \times 10^{-3}$ (measured value), CASE 1, and ( $\left.\mathrm{C}_{1}, b\right)$, leaving $n=.47$ (measured value), CASE 2 . We then computed the average value of the two couple of parameters, and used them to check the computed tunes. The average values were:

$$
\begin{array}{lll}
\text { CASE 1: } & <\boldsymbol{n}\rangle=. \mathbf{4 5 0 4 9 5}, \quad<\boldsymbol{b}\rangle=. \mathbf{1 1 2 5 3} & \mathrm{C}_{\mathbf{1}}=\mathbf{9 . 1 2 8 6} \times 10^{-\mathbf{3}} \\
\text { CASE 2: } & \left.\left.<\mathrm{C}_{1}\right\rangle=9.2338 \times 10^{-3},<b\right\rangle=.158008 & n=.47
\end{array}
$$

Table IX shows the differences in tunes found in the two cases. It is clear that the best couple of parameters is represented by CASE 1.

The largest discrepancy is for tunes close to the integer, in particular the vertical tune $\left(\Delta v_{y}=.009\right)$. As a check we fit the measurements setting the constants for the three quadrupole families at their measured values $\left(\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}\right)$ and performing the average again for CASE 1 . We obtain:

$$
\begin{gathered}
\langle n>=.451945,\langle b\rangle=.11833, \\
C_{1}=9.1286 \times 10^{-3}, \quad C_{2}=9.1430 \times 10^{-3}, \quad C_{3}=9.1466 \times 10^{-3}
\end{gathered}
$$

In Table X a comparison of the results is presented.

Table IX - Fit and data deviations on all the measurements

|  |  | CASE 1 |  | CASE 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $v_{\mathbf{x}}{ }^{\mathbf{m e a s}}$ | $\nu_{\mathbf{y}}{ }^{\mathbf{m e a s}}$ | $\Delta \mathrm{v}_{\mathbf{x}}$ | $\Delta \nu_{\mathbf{y}}$ | $\Delta \mathrm{v}_{\mathbf{x}}$ | $\Delta \mathrm{v}_{\mathbf{y}}$ |
| $\mathbf{1}$ | 3.1501 | 1.1074 | -.001 | +.0025 | -.001 | +.004 |
| $\mathbf{2}$ | 3.059 | 1.1111 | +.004 | +.005 | +.006 | +.011 |
| $\mathbf{3}$ | 3.1037 | 1.0665 | -.0011 | +.0085 | +.0007 | +.016 |
| $\mathbf{4}$ | 3.1038 | 1.1501 | -.0009 | -.002 | +.0005 | -.0006 |
| $\mathbf{5}$ | 3.147 | 1.1605 | -.0012 | -.0004 | -.001 | -.003 |
| $\mathbf{6}$ | 3.1630 | 1.1428 | -.001 | -.0006 | -.002 | -.001 |
| $\mathbf{7}$ | 3.1318 | 1.1776 | +.00003 | -.002 | +.0003 | -.005 |
| $\mathbf{8}$ | 3.1465 | 1.2154 | -.0007 | -.0087 | -.001 | -.015 |
| $\mathbf{9}$ | 3.1482 | 1.1037 | -.001 | +.005 | -.001 | +.006 |
| $\mathbf{1 0}$ | 3.1611 | 1.1404 | -.002 | -.002 | -.003 | -.003 |
| $\mathbf{1 1}$ | 3.1337 | 1.1751 | +.0006 | -.005 | +.001 | -.008 |

Table X - Comparison between fitted data

|  |  | $\mathbf{C}_{\mathbf{1}}=\mathbf{C}_{\mathbf{2}}=\mathbf{C}_{\mathbf{3}}$ |  | $\mathbf{C}_{\mathbf{1}} \neq \mathbf{C}_{\mathbf{2}} \neq \mathbf{C}_{\mathbf{3}}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\nu_{\mathbf{x}}{ }^{\mathbf{m e a s}}$ | $\nu_{\mathbf{y}}{ }^{\mathbf{m e a s}}$ | $\Delta \mathrm{v}_{\mathbf{x}}$ | $\Delta \mathrm{v}_{\mathbf{y}}$ | $\Delta \mathrm{v}_{\mathbf{x}}$ | $\Delta \nu_{\mathbf{y}}$ |
| $\mathbf{1}$ | 3.1501 | 1.1074 | -.001 | +.0025 | -.001 | +.003 |
| $\mathbf{2}$ | 3.059 | 1.1111 | +.004 | +.005 | +.004 | +.006 |
| $\mathbf{3}$ | 3.1037 | 1.0665 | -.0011 | +.0085 | +.0011 | +.010 |
| $\mathbf{4}$ | 3.1038 | 1.1501 | -.0009 | -.002 | +.001 | -.002 |
| $\mathbf{5}$ | 3.147 | 1.1605 | -.0012 | -.0004 | -.0013 | -.0007 |
| $\mathbf{6}$ | 3.1630 | 1.1428 | -.001 | +.0006 | -.0013 | +.0004 |
| $\mathbf{7}$ | 3.1318 | 1.1776 | +.00003 | -.002 | +.0002 | -.002 |
| $\mathbf{8}$ | 3.1465 | 1.2154 | -.0007 | -.0087 | -.0008 | -.0096 |
| $\mathbf{9}$ | 3.1482 | 1.1037 | -.001 | +.005 | -.0011 | +.005 |
| $\mathbf{1 0}$ | 3.1611 | 1.1404 | -.002 | -.002 | -.002 | -.002 |
| $\mathbf{1 1}$ | 3.1337 | 1.1751 | +.0006 | -.005 | +.0005 | -.005 |

The difference is negligible. Excluding the two measurements with the larger deviation (n. 3 and 8 ) and computing the average again we don't get substantial improvements. The difference between computed and measured tune is plotted in Figs. 1 and 2 for the horizontal and vertical one.


Fig. 1 - Horizontal tune deviations from fit


Fig. 2 - Vertical tune deviations from fit

To improve the fit a least square method was also applied, using as free parameters $n, b$ and $\mathrm{C}_{1}$. The results were not better than those obtained with the previous average.

A list of the lattice parameters, with a plot of the optical functions of one sector, is presented, for the nominal set 3.147, 1.1605, in Appendix 1. A copy of the accumulator parameter list is given in Appendix 2.

## 6. CONCLUSIONS

The proposed model is satisfactory if we use as free parameters the field index and the fringing length in the bending magnet. Some systematic measurements are still needed: tunes with and without sextupoles at different frequencies for an accurate chromaticity and central orbit frequency measurement.

## REFERENCES

[1] C. Biscari, M. Preger, Numerical constants and initial set points for the first part of the DAФNE injector commissioning, DAФNE Tech. Note C-17, Apr. 95.
[2] A. Battisti et al., Measurement and tuning of the DAФNE Accumulator dipole, DAФNE Tech. Note MM-9, Aug. 95.
[3] B. Bolli et al., Measurements on Tesla Quadrupole prototype for the DAФNE Accumulator and Main Ring, DAФNE Tech. Note MM-4, Dec. 94.
[4] B. Bolli et al., Field quality and alignment of the DAФNE Accumulator quadrupoles, DAФNE Tech. Note MM-8, Aug. 95.
[5] C. Biscari, Quadrupole modelling, DAФNE Tech. Note L-23, Mar. 96.
[6] M. Bassetti, C. Biscari, Analytical Formulae for Magnetic Multipoles, Particle Accelerators, 1996, Vol. 52, pp. 221-250.

## APPENDIX 1 - Lattice Parameters

| QX $-\mathbf{Q Z}$ | 3.147 | 1.161 |
| :--- | :--- | :--- |
| tunes/period | 1.574 | 0.5806 |


| ETA (m) | BX0 (m) | BZ0 (m) | ETAMAX (m) | BXMAX (m) | BZMAX (m) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1340 | 1.526 | 3.372 | 0.9345 | 4.487 | 14.10 |

## SYNCHROTRON RADIATION INTEGRALS (R.H.HELM et al.) :

| I1 (m) | $0.140414177 \mathrm{D}+01$ |  |  |
| :--- | :--- | :--- | :--- |
| I2 (1/m) | $0.571157659 \mathrm{D}+01$ |  |  |
| I3 (1/m²) | $0.519196961 \mathrm{D}+01$ |  |  |
| $\mathbf{I 4}(\mathbf{1} / \mathbf{m})$ | $-0.104880077 \mathrm{D}+01$ |  |  |
| I5 (1/m) | $0.373658343 \mathrm{D}+01$ |  |  |
| D | -.1836 | $0.0000 \mathrm{E}+00$ |  |
| JS,JX,JZ | 1.816 | 1.184 | 1.000 |
| DAMPINGS(ms) | 11.15 | 17.10 | 20.24 |

## TRANSFER MATRIX FOR ONE FULL PERIOD

| $-0.894859457 \mathrm{D}+00$ | $-0.681138094 \mathrm{D}+00$ | $0.253939890 \mathrm{D}+00$ |
| :---: | :---: | ---: |
| $0.292490691 \mathrm{D}+00$ | $-0.894859457 \mathrm{D}+00$ | $-0.391981863 \mathrm{D}-01$ |
|  |  |  |
| $-0.874577878 \mathrm{D}+00$ | $-0.163488108 \mathrm{D}+01$ |  |
| $0.143810787 \mathrm{D}+00$ | $-0.874577878 \mathrm{D}+00$ |  |

BEAM PAR $\& d N / d t$ FOR $T=293 K-P=1 n T o r r-Z(b i a t o m i c)=8:$

| REV. FREQUENCY (MHZ) | $0.920738507 \mathrm{D}+01$ |
| :--- | :---: |
| HARMONIC NUMBER | $0.800000000 \mathrm{D}+01$ |
| RF FREQUENCY (MHZ) | $0.736590806 \mathrm{D}+02$ |
| VRF(KV) | $0.100000000 \mathrm{D}+03$ |
|  |  |
| ENERGY (MEV) | $0.511000000 \mathrm{D}+03$ |
| U0 (KeV) | $0.548285388 \mathrm{D}+01$ |
| MOM. COMPACTION | $0.431247473 \mathrm{D}-01$ |
| F SYNC.(KHz) | $0.301590262 \mathrm{D}+02$ |
| RF ACCEPTANCE | $0.181811692 \mathrm{D}-01$ |
| NAT.BUNCH LENGTH(cm) | $0.299087170 \mathrm{D}+01$ |
| NAT. ENERGY SPREAD | $0.438376940 \mathrm{D}-03$ |



Fig. A1-One period optical functions

## APPENDIX 2 - Accumulator Parameters List

The $\mathrm{K}^{2}$ values are relative to the fit presented in this note for $\mathrm{v}_{\mathrm{x}}=3.147$, $\mathrm{v}_{\mathrm{y}}=1.1605$. The monitor and corrector lengths are set to zero. $\mathrm{A}_{\mathrm{x}}$ and $\mathrm{A}_{\mathrm{y}}$ are the full pipe aperture along the ring.

| TYPE | NAME | L tot (m) | L (m) | K2 ( $\mathrm{m}^{-2}$ ) | $\begin{aligned} & \text { TETA } \\ & (\mathrm{deg}) \end{aligned}$ | RO (m) | $\begin{gathered} \mathbf{A x} \\ (\mathbf{m m}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathbf{A y} \\ (\mathbf{m} \mathbf{m}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0.905375 | 0.905375 |  |  |  | 100 | 30 |
| 7 | SPTA1002 | 1.530375 | 0.625000 |  |  |  | 100 | 30 |
| 99 | FL2A1001 | 1.530375 | 0.000000 |  |  |  | 100 | 30 |
| 1 | 0 | 1.750375 | 0.220000 |  |  |  | 103 | 30 |
| 99 | SIPA1001 | 1.750375 | 0.000000 |  |  |  | 103 | 30 |
| 4 | DHSA1001 | 2.614375 | 0.864000 | -. 373466 | 45.0 | 1.100079 | 103 | 30 |
| 99 | SIPA1002 | 2.614375 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 2.801375 | 0.187000 |  |  |  | 103 | 30 |
| 50 | BPBA1001 | 2.801375 | 0.000000 |  |  |  | 103 | 110 |
| 1 | 0 | 2.982375 | 0.181000 |  |  |  | 103 | 86 |
| 3 | SXPA1001 | 3.082375 | 0.100000 | -7.27 |  |  | 103 | 86 |
| 1 | 0 | 3.240375 | 0.158000 |  |  |  | 103 | 86 |
| 2 | QUAA1001 | 3.540375 | 0.300000 | 4.463165 |  |  | 103 | 86 |
| 1 | O | 3.710375 | 0.170000 |  |  |  | 103 | 86 |
| 3 | SXPA1002 | 3.810375 | 0.100000 | 5.93 |  |  | 103 | 86 |
| 1 | 0 | 3.980375 | 0.170000 |  |  |  | 103 | 86 |
| 2 | QUAA1002 | 4.280375 | 0.300000 | -4.86358 |  |  | 103 | 86 |
| 99 | VUGA1001 | 4.280375 | 0.000000 |  |  |  | 103 | 86 |
| 1 | 0 | 4.736375 | 0.456000 |  |  |  | 103 | 86 |
| 2 | QUAA1003 | 5.036375 | 0.300000 | 4.43619 |  |  | 103 | 86 |
| 1 | 0 | 5.206375 | 0.170000 |  |  |  | 103 | 80 |
| 50 | BPSA1001 | 5.206375 | 0.000000 |  |  |  | 103 | 80 |
| 1 | 0 | 5.226375 | 0.020000 |  |  |  | 103 | 80 |
| 56 | CHVA1001 | 5.226375 | 0.000000 |  |  |  | 103 | 80 |
| 1 | 0 | 5.526375 | 0.300000 |  |  |  | 103 | 30 |
| 99 | SIPA1003 | 5.526375 | 0.000000 |  |  |  | 103 | 30 |
| 4 | DHSA1002 | 6.390375 | 0.864000 | -. 373466 | 45.0 | 1.100079 | 103 | 30 |
| 1 | 0 | 6.577375 | 0.187000 |  |  |  | 103 | 30 |
| 50 | BPBA1002 | 6.577375 | 0.000000 |  |  |  | 103 | 30 |
| 1 | O | 6.697250 | 0.119875 |  |  |  | 103 | 30 |
| 70 | KCKA1001 | 7.542250 | 0.845000 |  |  |  | 186 | 136 |
| 1 | 0 | 7.914750 | 0.372500 |  |  |  | 140 | 140 |
| 99 | SIPA1004 | 7.914750 | 0.000000 |  |  |  | 140 | 140 |
| 56 | CHVA1002 | 7.914750 | 0.000000 |  |  |  | 140 | 140 |


| TYPE | NAME | L tot (m) | $\mathbf{L}(\mathbf{m})$ | $\left.\mathbf{K 2 ~}_{\left(\mathbf{m}^{-2}\right)}\right)$ | TETA <br> $(\mathbf{d e g})$ | RO (m) | Ax <br> $(\mathbf{m m})$ | Ay <br> $(\mathbf{m m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{O}$ | 8.140750 | 0.226000 |  |  |  | 140 | 140 |
| $\mathbf{1}$ | $\mathbf{O}$ | 8.366750 | 0.226000 |  |  |  | 140 | 140 |
| $\mathbf{1}$ | $\mathbf{O}$ | 8.739250 | 0.372500 |  |  |  | 140 | 140 |
| $\mathbf{7 0}$ | KCKA2001 | 9.584250 | 0.845000 |  |  |  | 186 | 136 |
| $\mathbf{1}$ | $\mathbf{0}$ | 9.704125 | 0.119875 |  |  |  | 103 | 30 |
| $\mathbf{5 0}$ | BPBA2001 | 9.704125 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{1}$ | $\mathbf{O}$ | 9.891125 | 0.187000 |  |  |  | 103 | 30 |
| $\mathbf{4}$ | DHSA2001 | 10.755125 | 0.864000 | -.373466 | 45.0 | 1.100079 | 103 | 30 |
| $\mathbf{9 9}$ | SIPA2001 | 10.755125 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{1}$ | $\mathbf{O}$ | 11.055125 | 0.300000 |  |  |  | 103 | 75 |
| $\mathbf{5 6}$ | CHVA2001 | 11.055125 | $\mathbf{0 . 0 0 0 0 0 0}$ |  |  |  | 103 | 75 |
| $\mathbf{1}$ | $\mathbf{O}$ | 11.075125 | 0.020000 |  |  |  | 103 | 75 |
| $\mathbf{5 0}$ | BPSA2001 | 11.075125 | 0.000000 |  |  |  | 103 | 75 |
| $\mathbf{1}$ | $\mathbf{O}$ | 11.245125 | 0.170000 |  |  |  | 103 | 86 |
| $\mathbf{2}$ | QUAA2001 | 11.545125 | 0.300000 | 4.43619 |  |  | 103 | 86 |
| $\mathbf{9 9}$ | VUGA2001 | 11.545125 | 0.000000 |  |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 12.001125 | 0.456000 |  |  |  | 103 | 86 |
| $\mathbf{2}$ | QUAA2002 | 12.301125 | 0.300000 | -4.86358 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 12.471125 | 0.170000 |  |  |  | 103 | 86 |
| $\mathbf{3}$ | SXPA2001 | 12.571125 | 0.100000 | 5.93 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 12.741125 | 0.170000 |  |  |  | 103 | 86 |
| $\mathbf{2}$ | QUAA2003 | 13.041125 | 0.300000 | 4.463165 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 13.199125 | 0.158000 |  |  |  | 103 | 86 |
| $\mathbf{3}$ | SXPA2002 | 13.299125 | 0.100000 | -7.27 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 13.480125 | 0.181000 |  |  |  | 103 | 86 |
| $\mathbf{5 0}$ | BPBA2002 | 13.480125 | 0.000000 |  |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 13.667125 | 0.187000 |  |  |  | 103 | 30 |
| $\mathbf{9 9}$ | SIPA2002 | 13.667125 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{4}$ | DHSA2002 | 14.531125 | 0.864000 | -.373466 | 45.0 | 1.100079 | 103 | 30 |
| $\mathbf{9 9}$ | SIPA2003 | 14.531125 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{1}$ | $\mathbf{O}$ | 14.751125 | 0.220000 |  |  |  | 100 | 30 |
| $\mathbf{9 9}$ | FL2A2001 | 14.751125 | 0.000000 |  |  |  | 100 | 30 |
| $\mathbf{7}$ | SPTA2001 | 15.376125 | 0.625000 |  |  |  | 100 | 30 |
| $\mathbf{1}$ | $\mathbf{O}$ | 16.281500 | 0.905375 |  |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{0}$ | 17.501875 | 1.220375 |  |  |  | 86 | 75 |
| $\mathbf{9 9}$ | DCMA3001 | 17.501875 | 0.000000 |  |  |  | 86 | 75 |
| $\mathbf{5 6}$ | CHVA3001 | 17.501875 | $\mathbf{0 . 0 0 0 0 0 0}$ |  |  |  | 86 | 75 |
| $\mathbf{1}$ | $\mathbf{O}$ | 17.721875 | 0.220000 |  |  |  | 86 | 75 |
| $\mathbf{5 0}$ | BPSA3001 | 17.721875 | $\mathbf{0 . 0 0 0 0 0 0}$ |  |  |  | 86 | 77 |
| $\mathbf{1}$ | $\mathbf{O}$ | 18.031875 | 0.310000 |  |  |  | 103 | 30 |
| $\mathbf{9 9}$ | SIPA3001 | 18.031875 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{4}$ | DHSA3001 | 18.895875 | 0.864000 | -.373466 | 45.0 | 1.100079 | 103 | 30 |


| TYPE | NAME | L tot (m) | $\mathbf{L}$ (m) | $\mathrm{K} 2\left(\mathrm{~m}^{-2}\right)$ | $\begin{aligned} & \text { TETA } \\ & \text { (deg) } \end{aligned}$ | RO (m) | $\begin{gathered} \mathbf{A x} \\ (\mathbf{m m}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathbf{A y} \\ (\mathbf{m} \mathbf{m}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 99 | SIPA3002 | 18.895875 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 19.082875 | 0.187000 |  |  |  | 103 | 30 |
| 50 | BPBA3001 | 19.082875 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 19.263875 | 0.181000 |  |  |  | 103 | 30 |
| 3 | SXPA3001 | 19.363875 | 0.100000 | -7.27 |  |  | 103 | 30 |
| 1 | 0 | 19.521875 | 0.158000 |  |  |  | 103 | 30 |
| 2 | QUAA3001 | 19.821875 | 0.300000 | 4.463165 |  |  | 103 | 86 |
| 1 | 0 | 19.991875 | 0.170000 |  |  |  | 103 | 86 |
| 3 | SXPA3002 | 20.091875 | 0.100000 | 5.93 |  |  | 103 | 86 |
| 1 | 0 | 20.261875 | 0.170000 |  |  |  | 103 | 86 |
| 2 | QUAA3002 | 20.561875 | 0.300000 | -4.86358 |  |  | 103 | 86 |
| 1 | 0 | 21.017875 | 0.456000 |  |  |  | 103 | 86 |
| 2 | QUAA3003 | 21.317875 | 0.300000 | 4.43619 |  |  | 103 | 86 |
| 1 | 0 | 21.487875 | 0.170000 |  |  |  | 103 | 86 |
| 1 | 0 | 21.537875 | 0.050000 |  |  |  | 103 | 86 |
| 56 | CHVA3002 | 21.537875 | 0.000000 |  |  |  | 103 | 86 |
| 1 | 0 | 21.807875 | 0.270000 |  |  |  | 103 | 30 |
| 99 | SIPA3003 | 21.807875 | 0.000000 |  |  |  | 103 | 30 |
| 4 | DHSA3002 | 22.671875 | 0.864000 | -. 373466 | 45.0 | 1.100079 | 103 | 30 |
| 99 | SIPA3004 | 22.671875 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 22.858875 | 0.187000 |  |  |  | 103 | 30 |
| 50 | BPBA3002 | 22.858875 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 23.047250 | 0.188375 |  |  |  | 103 | 30 |
| 70 | KCKA3001 | 23.597250 | 0.550000 |  |  |  | 186 | 136 |
| 1 | 0 | 24.422250 | 0.825000 |  |  |  | 103 | 30 |
| 99 | SIPA3005 | 24.422250 | 0.000000 |  |  |  | 103 | 30 |
| 99 | VUGA3001 | 24.422250 | 0.000000 |  |  |  | 103 | 30 |
| 100 | RFCA3001 | 24.422250 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 25.087250 | 0.665000 |  |  |  | 103 | 30 |
| 56 | CHVA4001 | 25.087250 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 25.247250 | 0.160000 |  |  |  | 103 | 30 |
| 70 | KCKA4001 | 25.797250 | 0.550000 |  |  |  | 186 | 136 |
| 1 | 0 | 25.985625 | 0.188375 |  |  |  | 103 | 30 |
| 50 | BPBA4001 | 25.985625 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 26.172625 | 0.187000 |  |  |  | 103 | 30 |
| 99 | SIPA4001 | 26.172625 | 0.000000 |  |  |  | 103 | 30 |
| 4 | DHSA4001 | 27.036625 | 0.864000 | -. 373466 | 45.0 | 1.100079 | 103 | 30 |
| 99 | SIPA4002 | 27.036625 | 0.000000 |  |  |  | 103 | 30 |
| 1 | 0 | 27.306625 | 0.270000 |  |  |  | 103 | 86 |
| 56 | CHVA4002 | 27.306625 | 0.000000 |  |  |  | 103 | 86 |
| 1 | O | 27.356625 | 0.050000 |  |  |  | 103 | 86 |
| 1 | 0 | 27.526625 | 0.170000 |  |  |  | 103 | 86 |


| TYPE | NAME | L tot (m) | $\mathbf{L}(\mathbf{m})$ | $\mathbf{K 2 ~}_{\left(\mathbf{m}^{-2}\right)}\left(\begin{array}{c}\text { TETA } \\ (\mathbf{d e g})\end{array}\right.$ | RO (m) | Ax <br> $(\mathbf{m m})$ | Ay <br> $(\mathbf{m m})$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2}$ | QUAA4001 | 27.826625 | 0.300000 | 4.43032 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 28.282625 | 0.456000 |  |  |  | 103 | 86 |
| $\mathbf{2}$ | QUAA4002 | 28.582625 | 0.300000 | -4.86358 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 28.752625 | 0.170000 |  |  |  | 103 | 86 |
| $\mathbf{3}$ | SXPA4001 | 28.852625 | 0.100000 | 5.93 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 29.022625 | 0.170000 |  |  |  | 103 | 86 |
| $\mathbf{2}$ | QUAA4003 | 29.322625 | 0.300000 | 4.463165 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 29.480625 | 0.158000 |  |  |  | 103 | 86 |
| $\mathbf{3}$ | SXPA4002 | 29.580625 | 0.100000 | -7.27 |  |  | 103 | 86 |
| $\mathbf{1}$ | $\mathbf{O}$ | 29.761625 | 0.181000 |  |  |  | 103 | 30 |
| $\mathbf{5 0}$ | BPBA4002 | 29.761625 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{1}$ | $\mathbf{O}$ | 29.948625 | 0.187000 |  |  |  | 103 | 30 |
| $\mathbf{9 9}$ | SIPA4003 | 29.948625 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{4}$ | DHSA4002 | 30.812625 | 0.864000 | -.373466 | 45.0 | 1.100079 | 103 | 30 |
| $\mathbf{9 9}$ | SIPA4004 | 30.812625 | 0.000000 |  |  |  | 103 | 30 |
| $\mathbf{1}$ | $\mathbf{O}$ | 31.122625 | 0.310000 |  |  |  | 103 | 77 |
| $\mathbf{5 0}$ | BPSA4001 | 31.122625 | $\mathbf{0 . 0 0 0 0 0 0}$ |  |  |  | 86 | 75 |
| $\mathbf{1}$ | $\mathbf{O}$ | 31.342625 | 0.220000 |  |  |  | 86 | 75 |
| $\mathbf{5 6}$ | CHVA4003 | 31.342625 | $\mathbf{0 . 0 0 0 0 0 0}$ |  |  |  | 86 | 75 |
| $\mathbf{9 9}$ | KHTA4001 | 31.342625 | 0.000000 |  |  |  | 86 | 75 |
| $\mathbf{9 9}$ | KVTA4001 | 31.342625 | 0.000000 |  |  |  | 86 | 75 |
| $\mathbf{9 9}$ | WCMA4001 | 31.342625 | 0.000000 |  |  |  | 86 | 75 |
| $\mathbf{9 9}$ | PHTA4001 | 31.342625 | 0.000000 |  |  |  | 86 | 75 |
| $\mathbf{9 9}$ | PVTA4001 | 31.342625 | 0.000000 |  |  |  | 86 | 75 |
| $\mathbf{1}$ | $\mathbf{O}$ | 31.822625 | 0.480000 |  |  |  | 75 | 75 |
| $\mathbf{1}$ | $\mathbf{O}$ | 32.563000 | 0.740375 |  |  |  | 103 | 86 |

