**DAΦNE TECHNICAL NOTE**

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# OFF-ENERGY STABILITY OF BETATRON AND SYNCHROTRON OSCILLATIONS

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The damping partition numbers can be written as:

$$J_x = 1 - D$$

$$J_y = 1$$

$$J_s = 2 + D$$

where the parameter D, determined by the magnetic structure, is defined as[1]:

$$D = \frac{\int \frac{\psi}{\rho}\left(\left(\frac{1}{\rho^2}\right) + \frac{2}{\rho B}\frac{\partial B}{\partial x}\right) ds}{I_2}$$

where $\psi$ is the dispersion function, and $I_2$ is the second synchrotron integral:

$$I_2 = \int \frac{ds}{\rho(s)^2}$$

For the synchronous particle the radius of curvature does not vanish only in the dipoles, and D can be written as:

$$D = \frac{\rho}{2\pi}\int \frac{\psi}{\rho^3}[1 - 2n]\, ds$$

where:

$$n = -\frac{\rho}{B}\frac{\partial B}{\partial x}$$

is the field index, which is zero in the main rings and 0.5 in the accumulator.

For particles, having a different energy or passing off axis in the quadrupoles, the radius of curvature is not zero, also inside the quadrupoles, and must be taken into account in the estimate of D. It can be approximated by[2]:

$$D = \frac{\Delta E}{E} \frac{\int 2\, K^4(s)\, \psi^2(s)\, ds}{I_2}$$

with:

$$K^2 = \frac{1}{B\rho(s)} \frac{\partial B}{\partial x}$$

Since the oscillations are stable when the partition numbers are positive, the betatron oscillation in the horizontal plane becomes unstable for D = 1 and the synchrotron one for D = -2.

From the expression of D and from:

$$\frac{\Delta E}{E} = - \frac{1}{\alpha_c} \frac{\Delta f_{rf}}{f_{rf}}$$

we obtain that the allowed RF frequency range for the horizontal betatron oscillation is given by:

$$\frac{\Delta f_{rf}}{f_{rf}} = - \frac{\alpha_c I_2}{\int 2\, K^4(s)\, \psi^2(s)\, ds}$$

while for the longitudinal plane we have twice this value on the opposite side.

For the main rings the contributions to the integrals of the short and the long arc, corresponding to the lattice D19[3] are summarized in Table I ($I_2$ is computed considering both dipoles and wiggler contributions; the IR contribution is neglected because the dispersion function is very small).

Table I

| | $I_2$ ($m^{-1}$) | $\int 2\, K^4(s)\, \psi^2(s)\, ds$ ($m^{-1}$) |
|---|---|---|
| Short | 4.6606 | 12.3472 |
| Long | 5.1092 | 16.7438 |
| Total | 9.7698 | 29.0910 |

The RF frequency stability range and the corresponding energy deviation are listed in Table II; they are well above the momentum acceptance of the machine. This result holds also for different optical configurations of the main rings within few percent.

Table II

| | $\frac{\Delta f_{rf}}{f_{rf}}$ | $\frac{\Delta E}{E}$ |
|---|---|---|
| D = 1 | - 0.002 | 0.34 |
| D = -2 | 0.004 | -0.68 |

For the accumulator D = 1 for:

$$\frac{\Delta f_{rf}}{f_{rf}} = 0.008$$

corresponding to:

$$\frac{\Delta E}{E} = 24\%$$


[1] M. Sands: 'The Physics of Electron Storage Rings - An Introduction' , SLAC 121 (1970).

[2] M. Bassetti: 'Struttura di Adone con 6 Qf shuntati. Variazione dei damping con la $f_{RF}$', ADONE note T-65 (1974).

[3] C. Biscari: 'Optimization of the Day-One Interaction Region', DAΦNE Technical Note L-17 (June 1994).