DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

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Note: **IR-7**

CORRECTING COILS IN LOW β INSERTION QUADRUPOLES

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Introduction

The design of DAΦNE Interaction Regions (IRs) for the two experiments KLOE and FI.NU.DA. adopt the Rotating Frame Method (RFM)[1], in order to correct the coupling introduced by the detector solenoids. Let' s recall here the main characteristics of the RFM.

A solenoid rotates the transverse planes by an angle proportional to the integral of the longitudinal component of the magnetic field on the axis, B_s. To avoid betatron coupling outside the IR, the condition

$$\int_{IR} B_s \, ds = 0$$

must be fulfilled. This is achieved by means of two compensating solenoids with the field opposite to the main solenoid and placed outside the detector. Thanks to the IR symmetry, the transverse rotation at the Interaction Point (IP) is also cancelled.

The low-β quadrupoles are placed inside the detector (KLOE and FI.NU.DA.) or between the detector and the compensator (FI.NU.DA.), i.e. in the region where the normal modes are not horizontal and vertical, but tilted following the rotation of the transverse plane; it is then clear that in order to act as normal quadrupoles they must be tilted by an angle

$$\theta_{rot}^{i} = \frac{1}{2 B_s \rho} \int_0^{s_i} B_s \, ds$$

where s_i is the longitudinal distance of the ith quadrupole center from the IP.

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1 M. Bassetti, M.E. Biagini, C. Biscari, M.A. Preger, G. Raffone, G. Vignola - "DAΦNE Interaction Region Design" - Proceedings of PAC 93, Washington.

There is still a residual coupling due to the change in θ_{rot} along the quadrupole length. As it is well known the coupling coming from any source can be cancelled by four independent coupling elements such as skew quads or solenoids. The four elements that cancel our residual coupling are three small adjustments of the rotation angles of the quads (which are equivalent to skew terms) plus a correction of the field in the compensator solenoids. This method provides decoupling at the IP for a perfectly symmetric IR around the IP and in absence of alignment errors.

Even if sources of coupling come from the whole ring, it is the IR errors which probably would provide the strongest contribution to the ring residual coupling, so it is very much convenient to have inside the IR the four knobs per side necessary to cancel the coupling at the IP, independently from the rest of the ring. This is true especially for FI.NU.DA., which foresees to work at two different values of the detector field, and therefore needs the possibility of modifying the RFM parameters. One knob is the magnetic field of the compensator; the other three can be chosen equivalently between a rotation of the quadrupoles supports or correcting skew coils superposed on the quadrupoles. This note provides a quantification of the required coil strengths.

Furthermore it is recommended to have gradient correcting coils on the quadrupoles of KLOE, which are all permanent magnets. The required strength of these coils is given simply as the fraction of the nominal gradient to be corrected.

Skew coils

A quadrupole tilted by an angle α with respect to the orientation of the normal modes couples the betatron motions in two transverse planes. To eliminate this coupling acting directly on the quadrupole, there are two possibilities: the first one is of course to eliminate the tilt mechanically, which is not always possible, the second is the use of superimposed skew coils. Let's see what is the relationship between the quadrupole strength, the tilting angle and the skew strength.

Let's consider the quadrupole matrix and the skew quadrupole matrix as thin lenses. In the thin lens approximation the matrix of a normal quadrupole (i.e. with its axis normal to the normal modes, what means mounted rotated by an angle θ_{rot} when inserted in the IR) of magnetic length L and strength $k = \frac{1}{B\rho}\frac{{}_y}{\partial x}$ is:

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ kL & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -kL & 1 \end{pmatrix} \qquad (1)$$

The matrix of a quadrupole tilted with respect to the normal modes by any angle α is:

$$\mathbf{Q}_{\alpha} = \mathbf{R}(\alpha)\mathbf{Q}\,\mathbf{R}(-\alpha) \tag{2}$$

where

$$\mathbf{R}(\alpha) = \begin{pmatrix} \mathbf{I}\cos\alpha & \mathbf{I}\sin\alpha \\ -\mathbf{I}\sin\alpha & \mathbf{I}\cos\alpha \end{pmatrix} \tag{3}$$

is the rotation matrix and $\mathbf{I}$ the (2x2) identity matrix. Multiplying the three matrices :

$$\mathbf{Q}_{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ kL\cos 2\alpha & 1 & -kL\sin 2\alpha & 0 \\ 0 & 0 & 1 & 0 \\ -kL\sin 2\alpha & 0 & -kL\cos 2\alpha & 1 \end{pmatrix} \tag{4}$$

Applying (2) to a skew quadrupole $(\alpha = \pi/4)$ with the same length L and strength $k_{sk} = \frac{\quad}{B\rho}\frac{x}{\partial x}$, the matrix of the quadrupole results:

$$\mathbf{Q}_{sk} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k_{sk}L & 0 \\ 0 & 0 & 1 & 0 \\ -k_{sk}L & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

In the case of skew coils superimposed to a quadrupole tilted by an angle α we must apply (4) substituting $\alpha \rightarrow \frac{\pi}{4} + \alpha$ and obtain:

$$\mathbf{Q}_{sk(\alpha)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -k_{sk}L\sin 2\alpha & 1 & -k_{sk}L\cos 2\alpha & 0 \\ 0 & 0 & 1 & 0 \\ -k_{sk}L\cos 2\alpha & 0 & k_{sk}L\sin 2\alpha & 1 \end{pmatrix} \tag{6}$$

Let's now distinguish two cases:

a) A quadrupole, tilted by an angle α, with skew coils whose orientation is independent of the quadrupole axes; the matrix is represented by the product of the two matrices (4) and (5):

$$\mathbf{Q}_{sk}\,\mathbf{Q}_{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ kL\cos 2\alpha & 1 & -k_{sk}L - kL\sin 2\alpha & 0 \\ 0 & 0 & 1 & 0 \\ k_{sk}L - kL\sin 2\alpha & 0 & -kL\cos 2\alpha & 1 \end{pmatrix} \tag{7}$$

The matrix results uncoupled if:

$$k_{sk} = -k \sin 2\alpha \tag{8}$$

and corresponds to a normal quadrupole with a strength k_n given by:

$$k_n = k + \Delta k = k \cos 2\alpha \tag{9}$$

b) A quadrupole, tilted by an angle α, plus skew coils whose orientation is defined by the quadrupole axes; the matrix is represented by the product of the two matrices (4) and (6):

$$\mathbf{Q}_{sk(\alpha)}\,\mathbf{Q}_{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -k_{sk} L \sin 2\alpha + kL \cos 2\alpha & 1 & -k_{sk} L \cos 2\alpha - kL \sin 2\alpha & 0 \\ 0 & 0 & 1 & 0 \\ -k_{sk} L \cos 2\alpha - kL \sin 2\alpha & 0 & k_{sk} L \sin 2\alpha + kL \cos 2\alpha & 1 \end{pmatrix} \tag{10}$$

The matrix results uncoupled if:

$$k_{sk} = -k \tan 2\alpha \tag{11}$$

and corresponds to a normal quadrupole with a strength k_n given by:

$$k_n = k + \Delta k = \frac{k}{\cos 2\alpha} \tag{12}$$

For small α the two cases are of course equivalent.

The required strength of a skew coil used to correct a quadrupole tilt angle is proportional to the quadrupole gradient: coils with the same length of the corresponding quadrupole must yield ~ 3.5% of the nominal gradient to correct an angle of 1° (see Fig. 1).

The strength of the quadrupole remains unchanged.

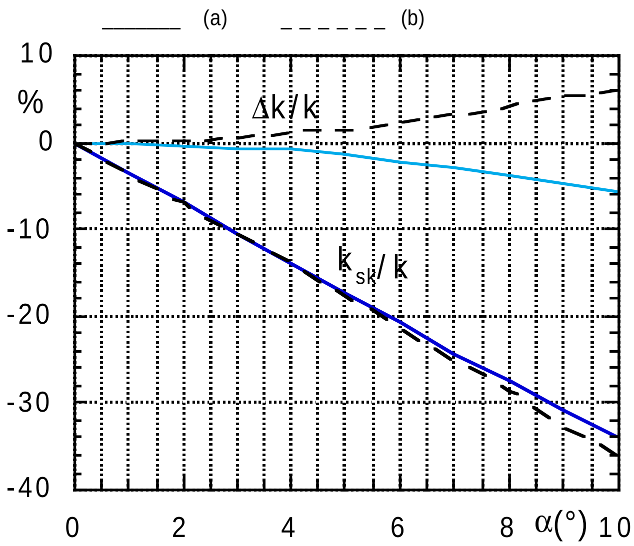

Figure 1 - Strength of the skew coils and change of the quadrupole strength with α.

KLOE

The three permanent magnet quadrupoles will be mounted on the same support, with their nominal tilt angle. This support will have the possibility of being rotated by a small angle (within ±1°).

The first quadrupole QF1 will not have correcting coils, because of lack of space. It is then necessary to have the skew correcting coils on the other two, QD and QF2.

The nominal lengths and gradients of the KLOE quadrupoles are listed in Table I, together with the gradients which coils should yield to correct the angle by 1°.

Gradient correcting coils should be mounted also on QD and QF2, to give ~0.5% of the nominal gradients.

Table I - KLOE quads

Name	Length (m)	K^2 (m^{-2})	G (T/m)	θ (°)	K skew (m^{-2})	G skew (T/m)
QF1	0.20	3.5	6.0	5.9	0	0
QD	0.35	5.7	9.7	10.5	0.19	0.32
QF2	0.27	2.8	4.7	15.5	0.10	0.16

FI.NU.DA.

There are only two permanent magnet quadrupoles inside the detector (per side) while two normal conducting quadrupoles are placed outside and mounted on a support which can rotate up to 22.5°. It is clear that the external quads do not need any correcting coil. Furthermore the gradient correcting coils are not necessary on the two permanent magnet quadrupoles because small optical mismatches can be corrected with the two outer quads.

In the definition of the correcting coils it must be taken into account that FI.NU.DA. could work at different magnetic field values, which means that the nominal tilt angles of the quadrupoles must change proportionally.

One solution is that skew coils are mounted on the two first quads. Table II shows as in KLOE case the value of the skew used to correct the angle by 1°.

Table II - FINUDA quads

Name	Length (m)	K^2 (m^{-2})	G (T/m)	θ (°)	K skew (m^{-2})	G skew (T/m)
Q1	0.1575	5.8	9.8	8.6	0.20	0.33
Q2	0.3000	6.9	11.7	14.8	0.24	0.41

If the detector magnetic field (B_o) changes, considering the relationship between the skew strength, the rotation angle and the quadrupole strength, the required gradient from the skew coils is represented in Fig. 2.

Table III shows the rotation angles and the quadrupole strengths for three different values of B/B_o.

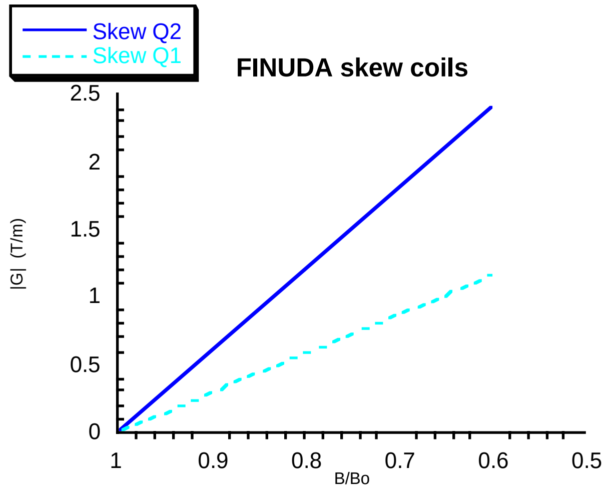

Figure 2 - Gradient of skew coils on Q1, Q2 of FI.NU.DA. IR

Table III - FINUDA configuration with different B/B_o

B/B_o = 1.

Name	Length (m)	K^2 (m^{-2})	G (T/m)	θ (°)	K skew (m^{-2})	G skew (T/m)
Q1	0.1575	5.8	9.4	8.6	0.	0.
Q2	0.3000	6.9	10.8	14.8	0.	0.
Q3	0.4000	2.0	3.4	22.3	0.	0.
Q4	0.4000	1.3	2.2	22.3	0.	0.

B/B_o = .8

Name	Length (m)	K^2 (m^{-2})	G (T/m)	θ (°)	K skew (m^{-2})	G skew (T/m)
Q1	0.1575	5.8	9.4	8.6	0.3	0.6
Q2	0.3000	6.9	10.8	14.8	0.7	1.2
Q3	0.4000	2.4	4.1	13.3	0.	0.
Q4	0.4000	1.9	3.2	13.3	0.	0.

B/B_o = .6

Name	Length (m)	K^2 (m^{-2})	G (T/m)	θ (°)	K skew (m^{-2})	G skew (T/m)
Q1	0.1575	5.8	9.4	8.6	0.7	1.2
Q2	0.3000	6.9	10.8	14.8	1.4	2.4
Q3	0.4000	2.6	4.3	17.8	0.	0.
Q4	0.4000	2.1	3.6	17.8	0.	0.

Another solution consists in using the two permanent magnet quadrupoles support rotation as one of the knobs, and skew coils only on one of the quads. In this case, considering that the coil strength is proportional to the quadrupole gradient, it is convenient to mount the coils on the first quadrupole, which, furthermore, is also the shorter one.

Table IV shows the necessary skew strengths for the three values of B/B_o as above, if the support position is nominal for the nominal value of B. To allow larger flexibility in the change of B, for the same maximum strength of the skew coils, the nominal support position could correspond to the mean value of B/B_o, as for example is shown in Table V.

Table IV - Q1 Skew strength and support rotation as function of B/B_o in the hypothesis of nominal support position at B/B_o = 1.

B/B_o	1.	0.8	0.6
Support rotation (°)	0.	-2.96	-5.92
θ_{Q1} (°)	8.6	5.64	2.68
θ_{Q2} (°)	14.8	11.84	8.88
$\Delta\theta_{Q1}$ (°) due to skew	0.	1.24	2.48
K_{skew} Q1 (m^{-2})	0.	0.25	0.50
G_{skew} Q1 (T/m)	0.	0.43	0.86

Table V - Q1 Skew strength and support rotation as function of B/B_o in the hypothesis of nominal support position at B/B_o = 0.8

B/B_o	1.	0.8	0.6
Support rotation (°)	2.96	0	-2.96
θ_{Q1} (°)	8.6	6.88	3.92
θ_{Q2} (°)	14.8	11.84	8.88
$\Delta\theta_{Q1}$ (°) due to skew	-1.24	0.	1.24
K_{skew} Q1 (m^{-2})	-0.25	0.	0.25
G_{skew} Q1 (T/m)	-0.43	0.	0.43