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SYNCHROTRON TUNE SHIFT AND TUNE SPREAD DUE TO BEAM-BEAM COLLISIONS WITH A CROSSING ANGLE

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Introduction

Experimental observations and measurements at DAΦNE have shown that beam-beam collisions can damp the longitudinal coupled bunch instability [1]. Bringing into collisions a high current electron beam with an unstable positron one was stabilizing the synchrotron oscillations of the e+ beam, even with the longitudinal feedback system switched off. Besides, a negative frequency shift of positron beam synchrotron sidebands has been observed when colliding the beams.

We attribute these two effects to a nonlinear longitudinal kick arising due to beam-beam interaction under a finite crossing angle. It is worthwhile to note here that we have observed this effect clearly only after implementation of the crab waist scheme of beam-beam collisions at DAΦNE having twice larger horizontal crossing angle with respect to the previous operations with the standard collision scheme [2].

In this Note we obtain an analytical expression for the synchrotron tune shift, that is also a measure of the synchrotron tune spread, and compare the formula with numerical simulations.

Tune shift

In collisions with a crossing angle the longitudinal kick of a test particle is created due to a projection of the transverse electromagnetic fields of the opposite beam onto the longitudinal axis of the particle. The kicks that the test particle receives while passing the strong beam with rms sizes σ_x, σ_y, σ_z under a horizontal crossing angle θ are [3]:

$$x' = \frac{2r_e N}{\gamma}\left(x - ztg(\theta/2)\right)\int_0^\infty dw \frac{\exp\left\{-\frac{\left(x - ztg(\theta/2)\right)^2}{\left(2\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right) + w\right)} - \frac{y^2}{\left(2\sigma_y^2 + w\right)}\right\}}{\left(2\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right) + w\right)^{3/2}\left(2\sigma_y^2 + w\right)^{1/2}}$$

$$y' = \frac{2r_e N}{\gamma} y \int_0^\infty dw \frac{\exp\left\{-\frac{\left(x - ztg(\theta/2)\right)^2}{\left(2\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right) + w\right)} - \frac{y^2}{\left(2\sigma_y^2 + w\right)}\right\}}{\left(2\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right) + w\right)^{1/2}\left(2\sigma_y^2 + w\right)^{3/2}} \qquad (1)$$

$$z' = x' tg(\theta/2)$$

where x, y, z are the horizontal, vertical and longitudinal deviations from the synchronous particle travelling on-axis, respectively. N is the number of particles in the strong bunch, γ is the relativistic factor of the weak beam. Then, for the on-axis test particle (x = y = 0) the longitudinal kick is given by:

$$z' = -\frac{2r_e N}{\gamma} z tg^2(\theta/2) \int_0^\infty dw \frac{\exp\left\{-\frac{(z tg(\theta/2))^2}{\left(2(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)) + w\right)}\right\}}{\left(2(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)) + w\right)^{3/2} \left(2\sigma_y^2 + w\right)^{1/2}} \tag{2}$$

For small synchrotron oscillations $z << \sigma_z$ the exponential factor in the integral can be approximated by 1 and taking into account that

$$\int_0^\infty dt \frac{1}{(a+t)^{3/2}(b+t)^{1/2}} = \frac{2}{a + \sqrt{a}\sqrt{b}} \tag{3}$$

we obtain an expression for the linearized longitudinal kick:

$$z' = -\frac{2r_e N}{\gamma} z tg^2(\theta/2) \frac{1}{\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right) + \sqrt{\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right)\sigma_y^2}} \tag{4}$$

Then, analogously to the transverse cases, we can write the expression for the synchrotron tune shift:

$$\xi_z = -\frac{r_e N}{2\pi\gamma} \beta_z \frac{tg^2(\theta/2)}{\left(\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right) + \sigma_y \sqrt{\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right)}\right)} \tag{5}$$

Remembering that the longitudinal beta function can be written as:

$$\beta_z = \frac{c|\eta|}{\nu_{z0}\omega_0} = \frac{\sigma_{z0}}{(\sigma_E/E)_0} \tag{6}$$

with c being the velocity of light; η the slippage factor, ν_{z0} the unperturbed synchrotron frequency and ω_0 the angular revolution frequency, we obtain the final expression for the linear tune shift:

$$\xi_z = -\frac{r_e N^{strong}}{2\pi\gamma^{weak}} \frac{\left(\frac{\sigma_{z0}}{(\sigma_E/E)}\right)^{weak} tg^2(\theta/2)}{\left(\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right) + \sigma_y^2 \sqrt{\left(\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)\right)}\right)^{strong}} \tag{7}$$

Here we have added notations "weak" and "strong" just not to forget which beam parameters we should use in tune shift calculations.

For the case of flat beams with $\left(\sigma_y << \sqrt{\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)}\right)$ the tune shift expression can be further simplified to

$$\xi_z = -\frac{r_e N^{strong}}{2\pi\gamma^{weak}} \frac{\left(\frac{\sigma_{z0}}{(\sigma_E/E)}\right)^{weak}}{\left(\left(\frac{\sigma_x}{tg(\theta/2)}\right)^2 + \sigma_z^2\right)^{strong}} \tag{8}$$

As we see from (8), for the flat bunches the synchrotron tune shift practically does not depend on the vertical beam parameters. So, one should not expect any big variations due to crabbing and/or hour-glass effect.

Since particles with very large synchrotron amplitudes practically do not "see" the opposite beam (except for a small fraction of synchrotron period) their synchrotron frequencies remain very close to the unperturbed value ν_{z0}. For this reason, like in the transverse cases, the linear tune shift can be used as a measure of the nonlinear tune spread.

Numerical Simulations

In order to check validity of (7) we performed numerical simulations with the beam-beam code LIFETRAC. The synchrotron and betatron tunes in the presence of beam-beam effects are calculated by tracking in the following way. First of all a test particle is tracked for one turn with the initial conditions:

$$X_i = \delta(i,j)\sigma_i q, \quad i = 1,2,..,6, \quad q << 1, \quad \delta(i,j) = \{\begin{matrix} 0, & if & i \neq j \\ 1, & if & i = j \end{matrix} \tag{9}$$

where X_i are the coordinates in the 6D phase space and σ_i are the respective rms sizes.

Doing this 6 times for j = 1,2,..,6 we obtain the 6×6 revolution matrix. Then the matrix eigenvalues are calculated, those give us the tunes. For these simulations we use a simple model of a collider with linear transformations from IP to IP. In order to reproduce correctly the Gaussian longitudinal distribution we divide a strong bunch in much more longitudinal slices than in ordinary beam-beam simulations. In these conditions the following equation is valid:

$$\cos(2\pi\nu_z) = \cos(2\pi\nu_{z0}) - 2\pi\xi_z \sin(2\pi\nu_{z0}) \tag{10}$$

where ν_{z0} is the initial synchrotron tune without beam-beam interaction, and ν_z is the tune calculated by tracking. Thus we can find the synchrotron tune shift ξ_z.

For the sake of comparison we use typical parameters of SuperB [4] and DAΦNE listed in Table 1. The last three rows show the ξ_z calculated analytically from (7), the nominal synchrotron tune and the tune in beam-beam collisions obtained from (10), respectively.

Table 1. DAΦNE and SuperB parameters and synchrotron tune shifts

Parameters	SuperB	DAΦNE
N, strong beam	5.74×10^{10}	3.3×10^{10}
γ, weak beam	8180	998
σ_z, mm, weak beam	5	12.8
σ_z, mm, strong beam	5	19
σ_x, μm, strong beam	5.65	255
$\sigma_{E/E}$, weak beam	6.57×10^{-4}	5.0×10^{-4}
θ, mrad	60	50
ϕ, weak beam	16.58	1.255
ξ_z, analytical	-0.00102	-0.000811
ν_{z0}	0.0100	0.01150
ν_z	0.0893	0.01066

First, our numerical simulations have confirmed that, in accordance with (8), the synchrotron tune shift does not depend on parameters of the vertical motion, such as β_y and ν_y. Second, an agreement between the analytical and numerical estimates is quite reasonable for the horizontal tunes far from integers, see Fig. 1. Quite naturally, in a scheme with a horizontal crossing angle synchrotron oscillations are coupled with the horizontal betatron oscillations. One of the coupling's side effects is the ν_z dependence on ν_x, which becomes stronger in vicinity of the main coupling resonances $\nu_x \pm \nu_z = k$. Obviously this effect is not accounted in (7) and (8), so in order to make comparisons with the analytical formula we need to choose the horizontal betatron tune ν_x closer to half-integer, where its influence on ν_z is weaker. The coupling vanishes for very large Piwinski angles, that is why the ν_z dependence on ν_x is stronger for DAΦNE with respect to that of SuperB.

Since ν_x for DAΦNE is rather close to the coupling resonance, we will use numerical simulations in order to compare the calculated synchrotron tune shift with the measured one described in [1]. In particular, when colliding the weak positron beam with ≈ 500 mA electron beam, the measured synchrotron frequency shift was about -630 Hz (peak-to-peak). In our simulations we use the DAΦNE beam parameters listed in Table 1 with respectively lower bunch current ($N = 0.9 \times 10^{10}$) and shorter bunch length ($\sigma_z = 1.6$ cm). This results in the synchrotron tune shift of – 0.000232 corresponding to the frequency shift of -720 Hz. In our opinion the agreement is good considering experimental measurement errors and the finite width of the synchrotron sidebands (see Fig. 12 and Fig. 13 in [1]).

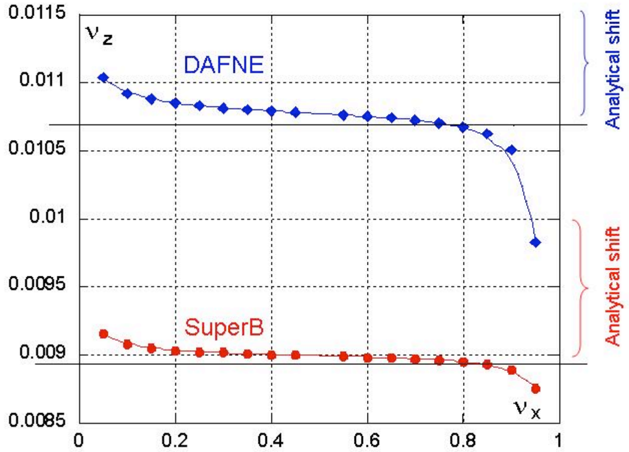

Figure 1. Synchrotron tune dependence on the horizontal tune. The solid straight lines correspond to the analytically predicted synchrotron tunes (last row in Table 1).

As the next step we have calculated numerically the synchrotron tune dependence on synchrotron oscillation amplitudes since namely this amplitude dependent spread of synchrotron frequencies can give Landau damping of the longitudinal coupled bunch mode instability. For this purpose we track on-axis particles with different initial longitudinal coordinates over 2048 turns and perform the Fourier transform in order to extract respective synchrotron frequencies.

In Fig. 2 the blue curve shows the calculated synchrotron tune dependence on the normalized synchrotron amplitude for the DAΦNE "weak" positron beam interacting with the "strong" electron beam with the current of 1.7 A. For comparison, the green curve shows the tune dependence on amplitude arising due to nonlinearity of the RF voltage. As we can see, the synchrotron tune spread due to the beam-beam interaction is notably larger than that due to the RF voltage alone, at least within 5 σ_z. In the past it was shown that the RF voltage nonlinearity is strong enough to damp quadrupole longitudinal couple bunch mode instability [5]. So, we can expect a strong Landau damping of longitudinal coupled bunch oscillations by the beam-beam collision. This conclusion is in accordance with performed measurements reported in [1].

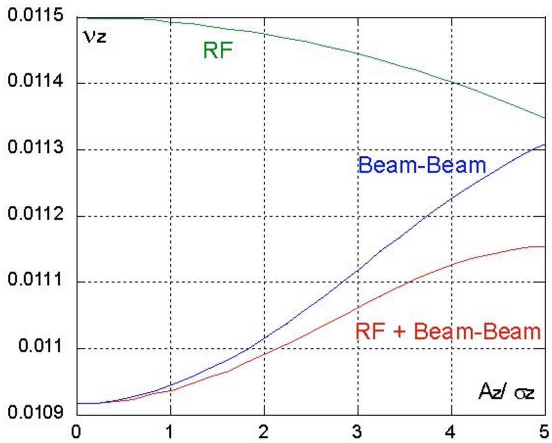

Figure 2. Synchrotron tune dependence on normalized amplitude of synchrotron oscillations (blue curve – tune dependence created by beam-beam collisions alone, green – RF nonlinearity alone, red – both contributions).

Conclusions

1. We have obtained a simple analytical formula for evaluation of the synchrotron tune shift and tune spread due to beam-beam collisions with a crossing angle.
2. The formula agrees well with the simulations when the horizontal tune is far from the synchro-betatron resonances $\nu_x \pm \nu_z = k$. The agreement is better for larger Piwinski angles.
3. Measured and obtained by simulations synchrotron frequency shifts are in a good agreement.
4. Calculations have shown that at high beam currents the synchrotron tune spread induced by the beam-beam interaction at DAΦNE can be larger than the tune spread due to the nonlinearity of the RF voltage. This may result in additional Landau damping of the longitudinal coupled bunch oscillations.

References

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