DAΦNE TECHNICAL NOTE

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SYNCHROBETATRON BEAM-BEAM RESONANCES IN DAΦNE

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Introduction

It is widely believed that the overlap of nonlinear resonances can create a beam-beam limit. The instability occurs at a certain value of the tune shift parameter when the stochastic layers of the resonances touch each other and a particle has a possibility to travel from one resonance to another and eventually to higher amplitudes. For the complete description of the effect see [1].

The synchrobetatron resonances are more closely spaced in phase space than the pure betatron resonances. Clearly, the synchrobetatron resonances tend to overlap before the betatron ones setting a lower limit to the dynamical stability of a beam. Indeed, in the example considered in [2] the beam-beam limit can be reached already at $\xi \sim 0.03 \div 0.04$.

In this paper we make an attempt to choose the DAΦNE working point in order to minimize the harmful influence of beam-beam resonances. In particular, we try to keep a working point far from dangerous sum resonances and to have synchrobetatron resonances well separated.

In the following we consider a single interaction point, head-on collisions, no parasitic crossings, weak-strong interaction. It is supposed that the synchrobetatron sideband resonances are created by the betatron tune modulation due to synchrotron oscillations. Analytical formulae which we use for the resonance parameter calculations can be found in [3].

Synchrobetatron beam-beam resonances

In order to demonstrate the synchrobetatron resonance overlap effect we will use a simple nonlinear mapping analogous to that of [4] but written for the case of a very flat bunch with $R = \sigma_y / \sigma_x << 1$ (we remind that for DAΦNE R = 0.01) :

$$\bar{x}_{n+1} = \bar{x}_n \cos 2\pi\nu_{x0} + \bar{x}'_n \sin 2\pi\nu_{x0}$$

$$\bar{x}'_{n+1} = -\bar{x}_n \sin 2\pi\nu_{x0} + \bar{x}'_n \sin 2\pi\nu_{x0} + \Delta\bar{x}'_{n+1}$$

$$\bar{y}_{n+1} = \bar{y}_n \cos 2\pi\nu_{yn} + \bar{y}'_n \sin 2\pi\nu_{yn} \qquad (1)$$

$$\bar{y}'_{n+1} = -\bar{y}_n \sin 2\pi\nu_{yn} + \bar{y}'_n \sin 2\pi\nu_{yn} + \Delta\bar{y}'_{n+1}$$

$$\nu_{y_{n+1}} = \nu_{yo} + \Delta\nu \sin 2\pi\nu_s n$$

Here the dimensionless variables $\bar{x} = x / \sigma_x ; \bar{y} = y / \sigma_y$ and $\bar{x}' = \beta_x^* x' / \sigma_x ; \bar{y}' = \beta_y^* y' / \sigma_y$ have been introduced with $\beta_{x,y}^*$ being the betatron function at the interaction point. The transverse kicks for the flat bunch are given by [5]:

$$\Delta x' = -\frac{N r_e}{\gamma \sigma_x} \sqrt{\frac{2\pi}{1 - R^2}} \, \mathrm{Im}\{ f_{bb} \}$$

$$\Delta y' = -\frac{N r_e}{\gamma \sigma_x} \sqrt{\frac{2\pi}{1 - R^2}} \, \mathrm{Re}\{ f_{bb} \} \qquad (2)$$

where

$$f_{bb} = W(u + iRv) - \exp\{ -(1 - R\;)(u\;+ v\;) \} W(Ru + iv)$$

$W(z)$ is the complex error function and

$$u = \frac{x}{\sigma_x \sqrt{2(1 - R^2)}} \quad ; \quad v = \frac{y}{\sigma_y \sqrt{2(1 - R^2)}}$$

Figure 1 shows an example of trajectories in the vertical phase space for the case with $\nu_{y0} = 0.7$; $\varsigma_y = 0.01$; $\nu_s = 0.004$; $\Delta\nu = 0.01$. The phase space coordinates of a particle have been plotted once per synchrotron period. For all traced particles $\bar{x} = 0$; $\bar{x}' = 0$ in order to decouple vertical and horizontal motions. The synchrotron sidebands of order 4,5,6 of a 10^{th} order betatron resonance are clearly seen. The other sidebands are not shown just not to overcrowd the picture. The sideband resonances are well separated and narrow stochastic layers are observed around separatrixes and near the points where the resonance islands touch each other.

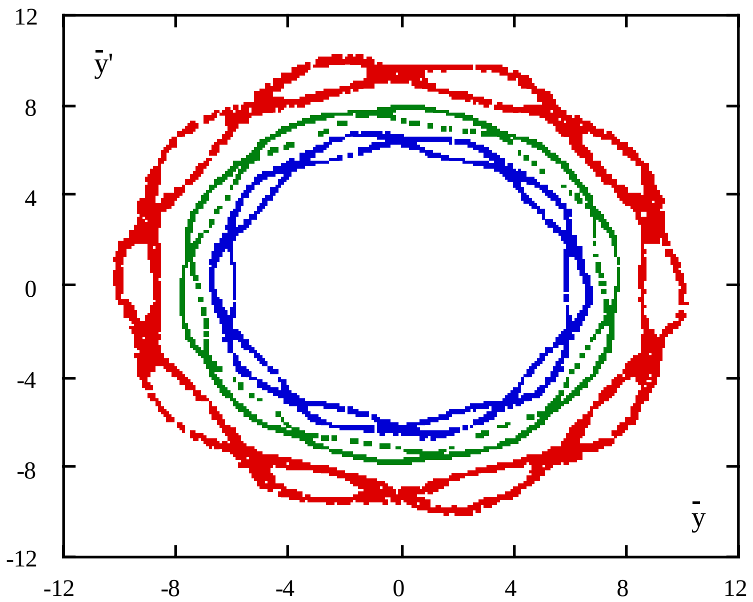

Fig. 1 - The 4,5,6 synchrotron sidebands of the betatron resonance $10\nu_y = \mathrm{int}$.

When the layers overlap the motion in phase space gets stochastic and a particle tends to diffuse to larger amplitudes. This is exactly the case for the working point with $\nu_{x0} = 5.18$; $\nu_{y0} = 6.15$ and $\varsigma_x = \varsigma_y = 0.04$. In Fig. 2 we can see a regular structure of the phase space trajectories near the betatron resonance of the sixth order mentioned in [3] in absence of synchrotron motion.

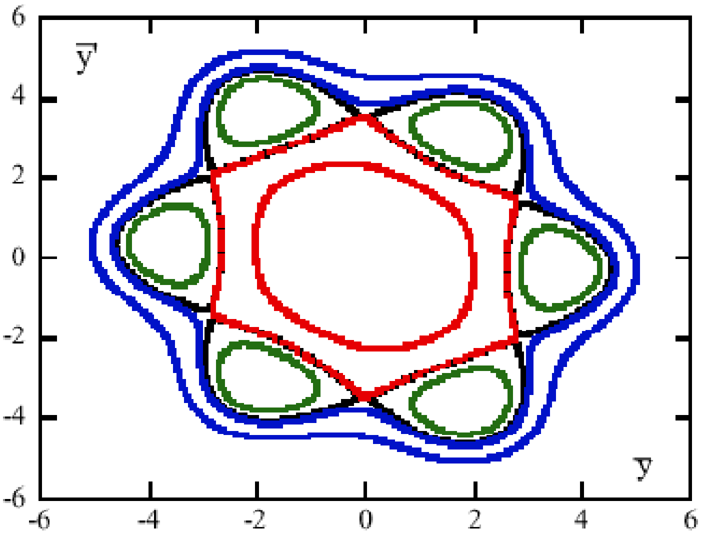

Fig. 2 - Phase space trajectories near the resonance $6\nu_y = 37$.

The synchrotron motion leads to the appearance of synchrotron sidebands and their overlap. Fig. 3 shows the stochastic behavior of the particle executing the synchrotron oscillations with amplitude $c\hat{\tau} = \sigma_z$ and initial transverse coordinates $x = x' = y' = 0$ and $y = 2\sigma_y$.

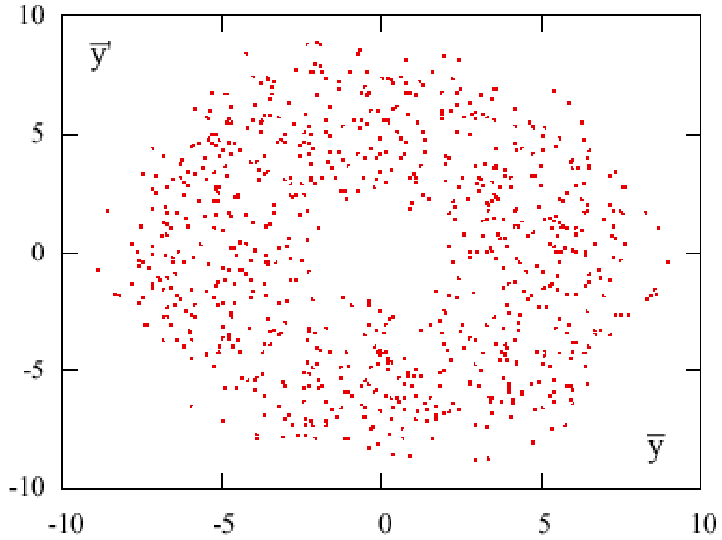

Fig. 3 - Stochastic behaviour of a particle in the phase space.
The particle was traced over 200000 turns.
The transverse coordinates are printed after each synchrotron period.

The nonlinear mapping does not take into account the helpful effect of averaging over the betatron phases for the bunches with a length comparable to the betatron function at the interaction point. In the following we proceed using analytical expressions of [3] to estimate the resonance width and the resonance overlap effect.

In Fig. 4 we show the same sixth order betatron resonance with its synchrotron sidebands.

The resonance widths are shown in the vertical betatron amplitude plane as a function of the amplitude of the synchrotron oscillations. Here again we consider particles with $x = x' = 0$. Due to the symmetry of the beam-beam kicks this allows us to decouple vertical and horizontal oscillations in order to simplify the visualization of the overlap effect.

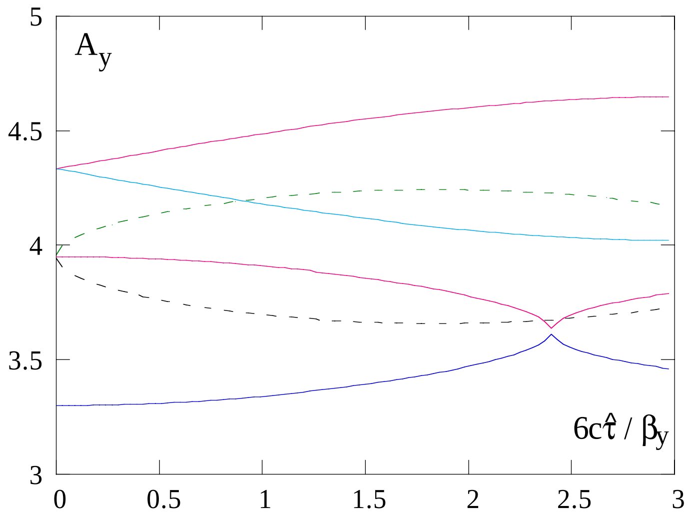

Fig. 4 - Overlap of synchrobetatron sidebands of the resonance $6\nu_y = 37$ *in the amplitude plane. The lower pair of solid lines corresponds to the primary resonance; the upper one - to the second sideband. The dashed lines limit the first sideband.*

The resonance overlap in the amplitude plane not necessarily means overlap in phase space as far as resonance sideband islands are somewhat shifted in phase with respect to each other, as it can be seen, for example, in Fig. 1. But in the above case the deep intersection of the resonance bands in the amplitude plane will certainly lead to stochastic particle motion.

The best way to avoid synchrobetatron sideband overlap is to move the working point into the region of the resonances of higher order. In order to demonstrate this let us move the working point closer to the integer where the bunch footprint is crossed by the betatron resonance of 8^{th} order.

The first four synchrobetatron sidebands for the case are shown Fig. 5.

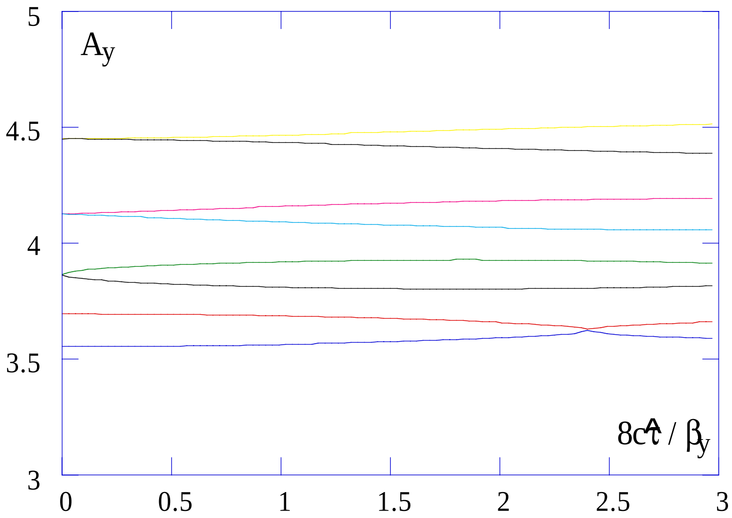

Fig. 5 - Overlap of synchrobetatron sidebands of the resonance $8\nu_y = 49$ *in the amplitude plane.*

The sidebands are well separated in the amplitude space and this guarantees the separation in phase space.

It should be noted that the particle amplitudes at which the resonant conditions are satisfied in Fig. 4 and Fig. 5 are the same. This was done intentionally in order to make a correct comparison between resonances. We remind that the resonance width depends not only on the order of the resonance but also on the particle amplitude at which the resonance condition is satisfied.

On the choice of the working point in DAΦNE

Some cautions must be taken in order to minimize degradation of beam-beam performance due to nonlinear beam-beam resonances:

- possibly avoid strong sum resonances being the source of the beam streaming and phase convection;

- eliminate conditions for resonance overlap. The difference resonances must also be considered in this case.

- strong difference resonances capable to push particles to high amplitudes have to be avoided.

The grids of the sum and difference resonances up to the tenth order are shown in Fig. 6 and Fig. 7, respectively.

The shaded triangular approximates the beam footprint corresponding to the working point with $\nu_x = 5.18$; $\nu_y = 6.15$.

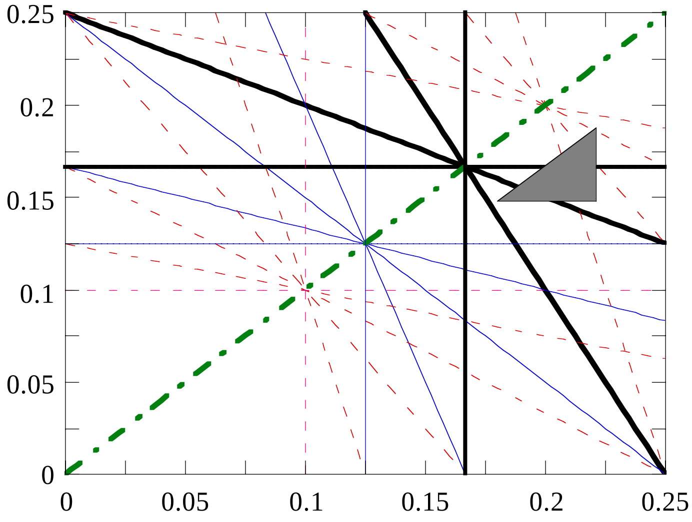

Fig. 6 - Sum resonances: solid thick lines - sixth order resonances;
solid thin lines - eighth order resonances;
dashed line - tenth order resonances.

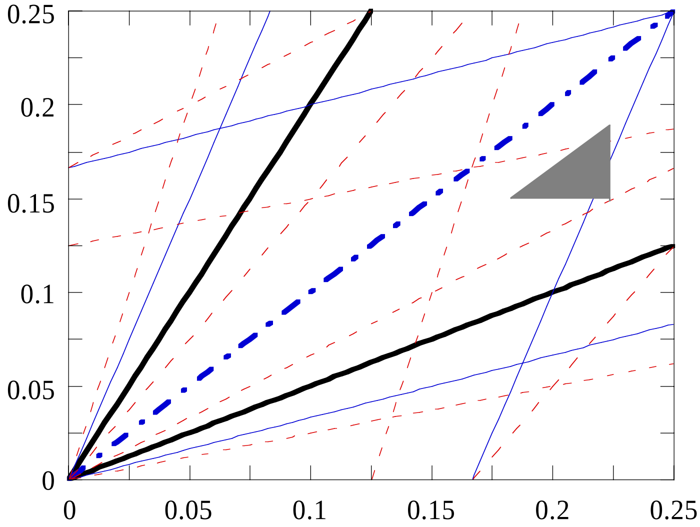

Fig. 7 - Difference resonances

As we have discussed above and in [3], the working point is not the best choice because of the strong sum betatron resonances crossing the beam footprint and the overlap of the synchrobetatron sidebands. So we tried to find a suitable working point in order to reduce the influence of beam-beam resonances on the machine performance. The task is not easy for DAΦNE for the following reasons:

- it is impossible to place the triangle with a side of 0.04 between the strong beam-beam resonances, while keeping an enough distance from integer resonances.
- the working points in the vicinity of the integers have small dynamic apertures in DAΦNE [6];
- the separation between synchrobetatron sidebands is small in DAΦNE due to the small synchrotron tune $\nu_s = 0.0078$.

A working point which could be a possible choice is $\nu_{x0} = 5.15; \quad \nu_{y0} = 6.05$. Let us consider this point in detail. Fig. 8 shows the beam footprint with the intersecting resonances. The numbers in brackets correspond to the resonance numbers: $(l,m,k) \Leftrightarrow l(\nu_x - 5) + m(\nu_y - 6) = k$.

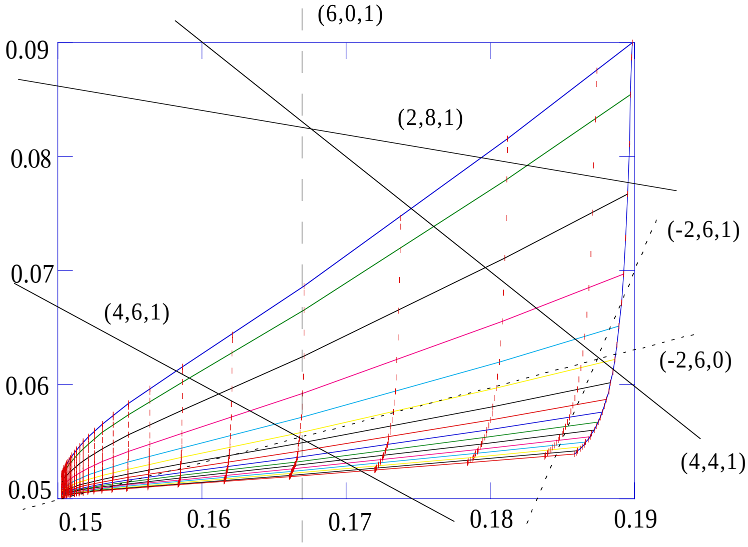

Fig. 8 - Beam footprint for the working point $\nu_x = 5.15; \quad \nu_y = 6.05$

Fig. 9 presents the betatron resonances, except $6\nu_x = 37$, in the transverse amplitude plane. As it can be seen, neither the bunch core nor the bunch tail are affected by the resonances substantially. The resonance widths are relatively small. The difference resonance $6\nu_y = 2\nu_x$ crossing the bunch tail and having the largest width does not contribute to the resonance streaming.

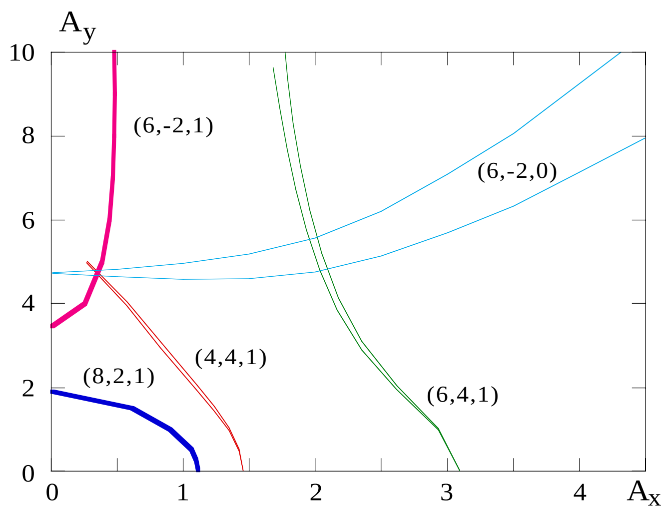

Fig. 9 - Betatron resonances in the amplitude plane
for the working point $\nu_x = 5.15; \quad \nu_y = 6.05$.

For none of these resonances the criterion of synchrobetatron resonance overlap is fulfilled. As an example, we show in Fig. 10 the first sidebands for the resonance $6\nu_y = 2\nu_x$ which is the strongest at the given working point.

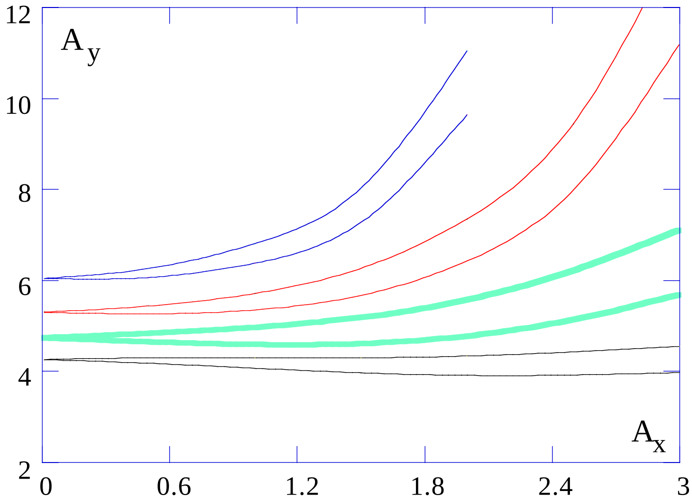

Fig. 10 - Synchrobetatron satellites for the resonance $6\nu_y = 2\nu_x$.
The primary resonance is within thick solid lines.

The sidebands are well separated. In plotting Fig. 10 we suggested that all the sidebands have their maximum width at the same time. This leads to overestimate of the overlap effect. The sidebands have maximum width at different amplitudes of the synchrotron oscillations and therefore are even more separated than in Fig. 10.

Some words must be said about the purely horizontal betatron resonance $6\nu_x = 37$. Its resonance condition and width practically do not depend on the vertical transverse coordinate of the particle. The oscillations in the resonance are executed in the horizontal plane so that the resonance streaming along the vertical direction is hardly possible. The resonance does not have synchrobetatron satellites due to the finite bunch length as far as $\beta_x^* >> \sigma_z$.

But it also can be somewhat dangerous. The resonance region spreads from $A_x = 1.54$ to $A_x = 2.46$ and mutual action of the resonance and radiation damping can distort the particle distribution in that region. While damping reduces the amplitude of oscillations, the resonance pushes it back. If the resonance period is shorter than the damping time the particle distribution in the resonance region can differ from gaussian, i.e. some horizontal blow-up can take place. It is not expected to be serious, although numerical calculations have to be done to check it.

The best way to avoid also the strong horizontal resonance would be to move the working point closer to the horizontal integer. But this is not an acceptable choice because the dynamic aperture is getting rapidly smaller. Numerical simulations showed that it is unacceptably small already for the working point with $\nu_x = 0.09$; $\nu_y = 0.06$ [6].

Another working point we would like to discuss is $\nu_x = 5.13$; $\nu_y = 6.10$. The beam footprint and the betatron resonances affecting it are shown in Fig. 11 and Fig. 12, respectively.

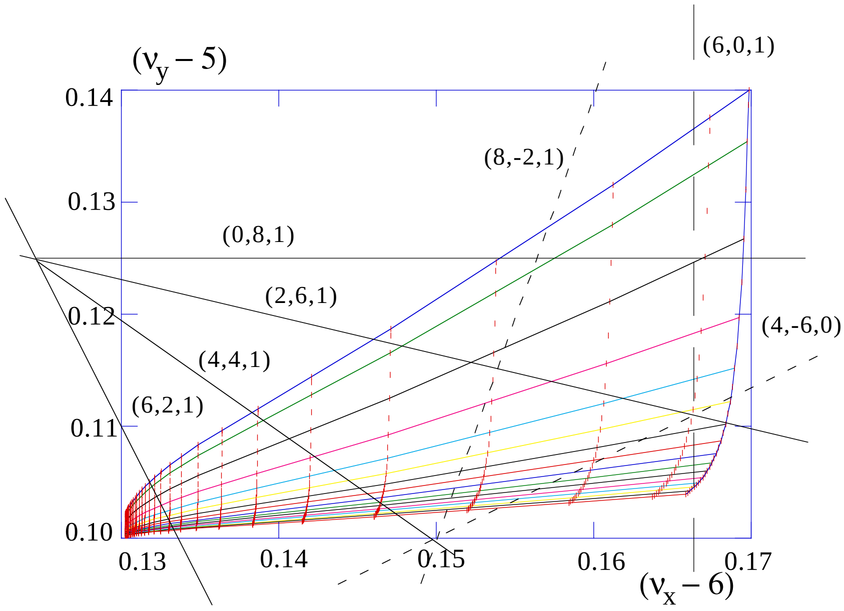

Fig. 11 - Beam footprint for the working point $\nu_x = 5.13; \quad \nu_y = 6.10$

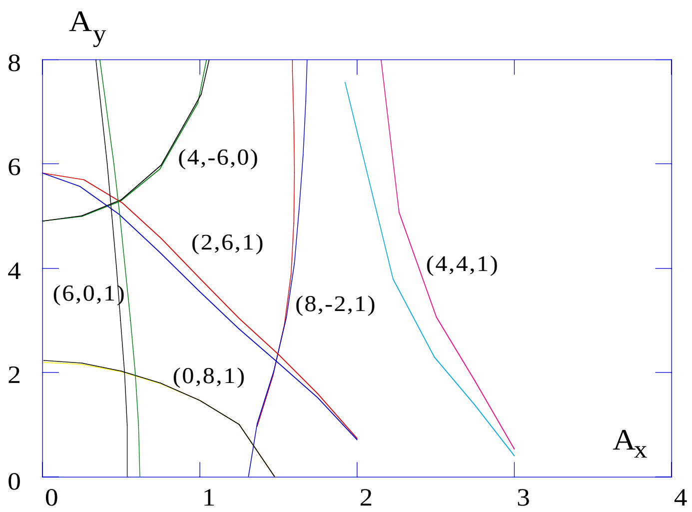

Fig. 12 - Betatron resonances in the amplitude plane for the working point $\nu_x = 5.13; \quad \nu_y = 6.10$.

The resonances crossing the bunch core are weak and not expected to affect the bunch distribution strongly. A small contribution to the background can be given by the sum resonance $4\nu_x + 4\nu_y = 45$ passing through the beam tail. Some problems can arise due to the strong sum resonance $6\nu_x + 2\nu_y = 33$ affecting the extreme tail of the bunch. We show this resonance separately in Fig. 13. For this particular resonance the analytical model can be used only as an approximation because it looses accuracy when the resonance width is comparable to the amplitudes satisfying the resonance condition. Numerical simulations with the mapping (1) have shown that the resonance width is even wider than that shown in Fig. 13.

Certainly, this resonance may transport particles to larger amplitudes and its synchrobetatron sidebands overlap, but it covers a region with low particle density. Its actual influence on machine performance can be evaluated by a detailed numerical simulations.

The resonance could be removed by slightly shifting the horizontal working point towards 6.135 (as an example). However, one should remember that this would increase width of the resonances passing through the bunch core.

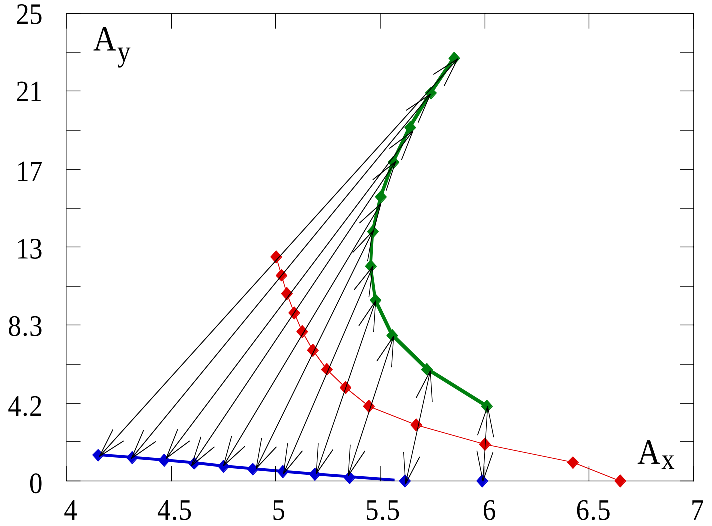

Fig. 13 - Sum resonance $6\nu_x + 2\nu_y = 33$ *in the amplitude plane. Arrows show the direction of oscillations.*

Discussion

1. The main reasons which complicate the optimal choice of the working point for the beam-beam performance are the following:

- influence of the nonlinear beam-beam resonances on the beam dynamics is weaker for working points placed in the vicinity of integers but the dynamic aperture of DAΦNE is small at such points.
- far from the integers the beam footprint with $\varsigma_x = \varsigma_y = 0.04$ is always crossed by one or more strong beam-beam resonances.
- the small synchrotron tune helps the overlap of synchrobetatron resonances.

2. In our opinion, among numerous analyzed working points the most favorable are those with $\nu_x = 5.15$; $\nu_y = 6.05$ and $\nu_x = 6.10$; $\nu_y = 5.13$. But these working points are not free of drawbacks which are discussed in the text above. The numerical simulation taking into account radiation damping, noise and external nonlinearities could give more precise information about these working points.

References

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