**DAΦNE TECHNICAL NOTE**

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# BUNCH LENGTHENING IN DAΦNE MAIN RING

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## Introduction

The equilibrium distribution of a single bunch in a storage ring is influenced by the RF voltage and by the self induced electromagnetic field.

The self field causes a distortion from the original Gaussian equilibrium distribution, changes the synchrotron frequency and introduces a synchrotron frequency spread. These effects are due to the so called potential well distortion.

In DAΦNE, due to the high circulating current, there is another cause that influences the bunch distribution. Above some threshold current the energy spread of particles starts to increase and the bunch lengthens faster than in the potential well distortion regime. Due to fluctuating microwave signals observed, the effect is called microwave instability or turbulent bunch lengthening.

Preliminary calculations have shown [1] that the nominal bunch current in the DAΦNE main rings would much exceed the threshold value. Unfortunately, there is no reliable method to predict bunch behavior in the turbulent regime. We use here the same method which K. Bane successfully applied to calculate bunch lengthening and bunch shape in the SLC damping rings [2]. In order to find the bunch shape beyond the threshold we:

- scale the energy spread and the bunch length according to the Boussard criterion;

- perform potential well distortion calculations, replacing the "natural" bunch length by that we get after scaling.

In our opinion, a physical justification of such a separation between turbulent and potential well distortion effects might come from the fact that these effects last for considerably different times: the rise time of the microwave instability is much smaller than the synchrotron period, while, to reach equilibrium, the particles have to execute, at least, one synchrotron oscillation in the potential well.

In order to study the bunch lengthening process, the broad-band impedance, as a function of the bunch length σ, has to be estimated.

## DAΦNE main ring impedance

It has been shown that the DAΦNE broad-band impedance is well described by the first four terms of the phenomenological expansion over $\sqrt{\omega}$[3]:

$$\frac{Z}{n} = \left\{ -i\omega_o L(\sigma) + \frac{R(\sigma)}{n} + \frac{(1-i)B(\sigma)}{n^{1/2}} + \frac{(1+i)Z_c(\sigma)}{n^{3/2}} \right\} \Omega \qquad (1)$$

where $\omega_0$ is the angular revolution frequency, $n=\omega/\omega_0$, and the coefficients of the expansion $L,R,B,Z_c$ are extracted by fitting the wake functions calculated by numerical codes TBCI [4] and MAFIA [5] at a given σ.

In general, every broad band impedance model is valid in a finite range of σ as far as the calculated wake functions depend on the actual impedance weighted over the bunch spectrum. A change in the bunch length leads to a change in the spectrum and, in turn, in the broad band model impedance.

For this reason, we calculated the parameters $L,R,B,Z_c$ at different bunch lengths, starting from the natural bunch length up to the nominal σ of 3 cm. As an example, we give the expressions of the broad band DAΦNE impedance for two extreme cases: expression (2) holds for σ=0.5 cm (close to the natural bunch length when the RF peak voltage is 254KV) and (3) for the nominal bunch length of 3 cm:

$$\frac{Z}{n} = \left\{ -i\,0.13 + \frac{126.43}{n} + \frac{(1-i)1.78}{n^{1/2}} + \frac{(1+i)575.89}{n^{3/2}} \right\} \Omega \qquad (2)$$

$$\frac{Z}{n} = \left\{ -i\,0.26 + \frac{24.8}{n} + \frac{(1-i)2.89}{n^{1/2}} + \frac{(1+i)274.42}{n^{3/2}} \right\} \Omega \qquad (3)$$

As we could expect, for shorter bunches the impedance gets more capacitive and losses grow (compare the second and the fourth term in (2) and (3), respectively).

A useful observation has been made using the fitting procedure of the total wake functions of the ring (summed up over all ring elements). The rather complicated expression (1) can be replaced by a simpler one where the only purely inductive and purely resistive terms are kept. For σ = 3 cm the wake function corresponding to the impedance:

$$\frac{Z}{n} = \left\{ -i\,0.36 + \frac{93.83}{n} \right\} \Omega \qquad (4)$$

fits reasonably well with that calculated numerically (see Fig. 1).

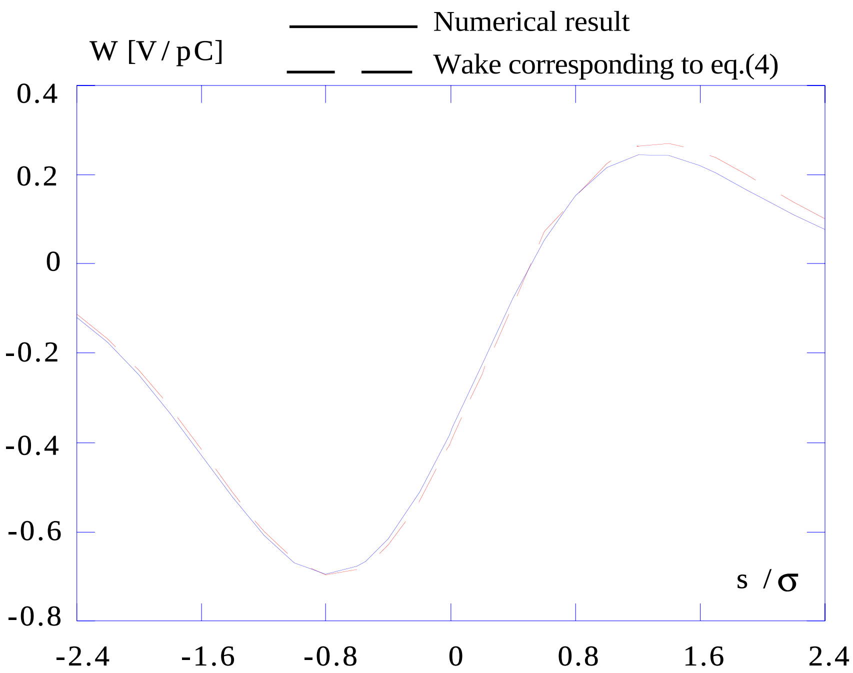


*Fig. 1 - DAΦNE total wake function at σ=3 cm.*

The same is valid for the other extreme case. Figure 2 shows the fit of the wake function for σ = 0.5 cm with the wake function corresponding to the impedance:

$$\frac{Z}{n} = \left\{ -i\,0.17 + \frac{234.4}{n} \right\} \Omega \qquad (5)$$

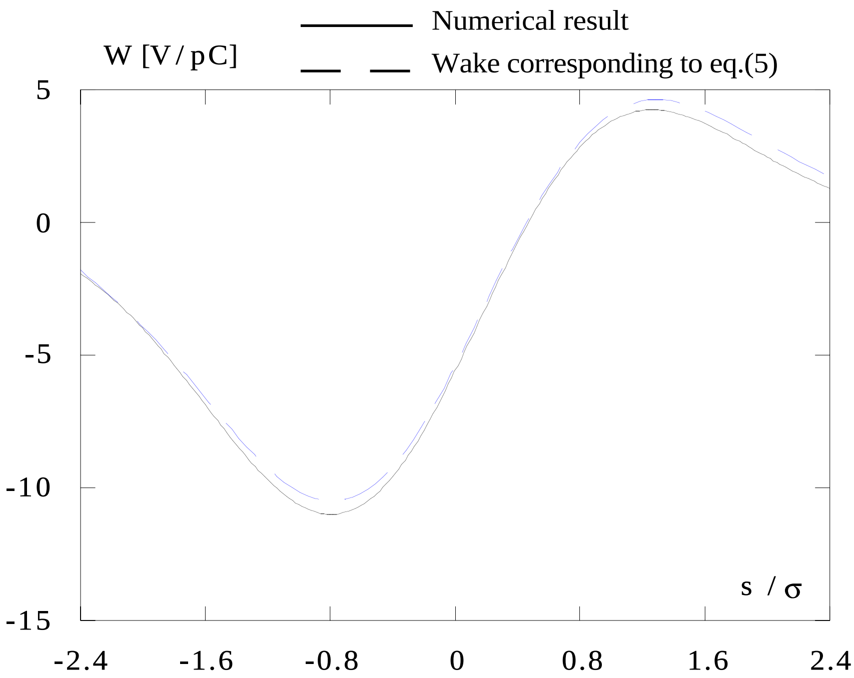


*Fig. 2 - DAΦNE total wake function at σ=0.5 cm.*

This can help in solving Haissinski's equation in the potential well distortion regime, since for the purely inductive and resistive impedances simple analytical expressions for wake functions are known. Moreover, if one of these impedances dominates, the solution of Haissinski's equation can be found analytically.

Fig. 3 and Fig. 4 show the imaginary and resistive parts of DAΦNE normalized impedance versus bunch length. The resistive part is estimated at a typical frequency $\omega_c = c/\sigma$.

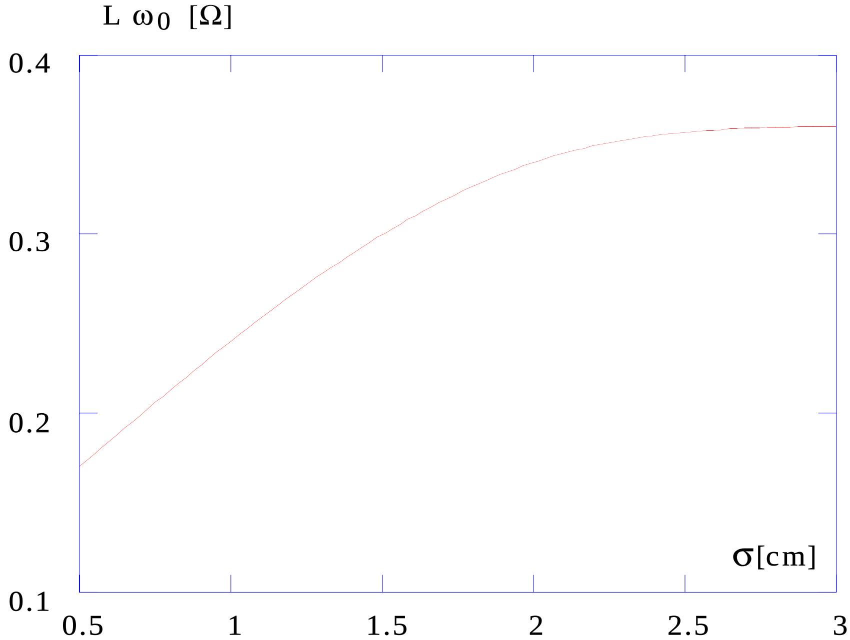


*Fig. 3 - Imaginary part of DAΦNE normalized impedance.*

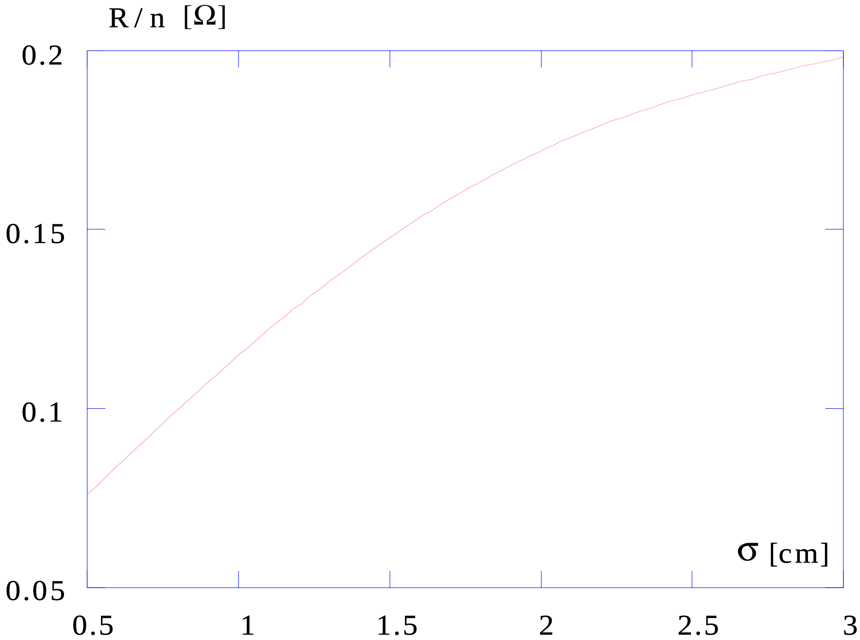


*Fig. 4 - Resistive part of DAΦNE normalized impedance.*

In the turbulent threshold calculations the absolute value of the normalized broad-band impedance |Z/n| is necessary. The dependence of the absolute value of the impedance |Z/n| on the bunch length is shown in Fig. 5.

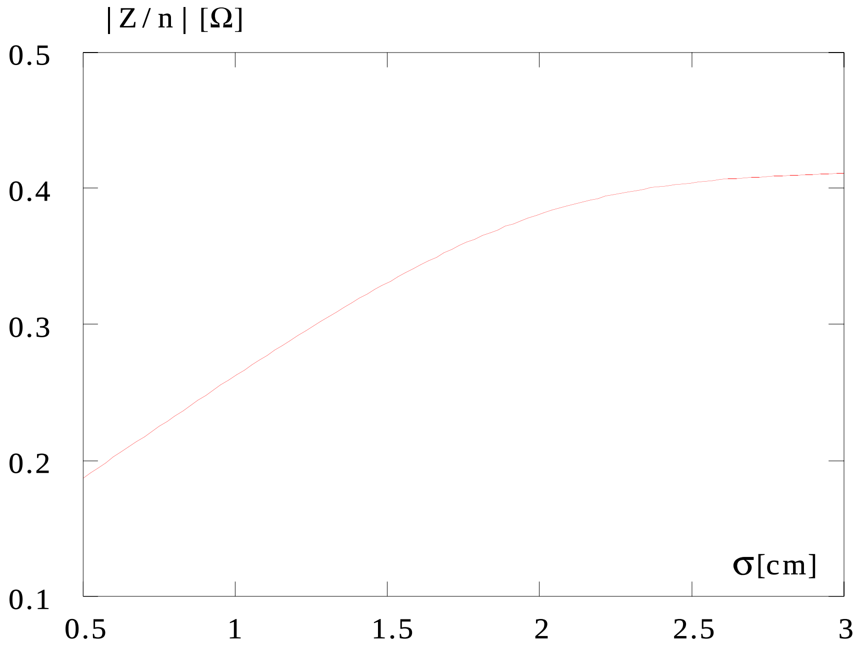


*Fig. 5 - Normalized broad-band impedance versus bunch length.*

## Turbulent lengthening

Above a certain threshold current the energy spread within a bunch starts growing. This effect is known as the microwave instability, because of the high frequency signals that can be observed. The consequent bunch lengthening is very fast and the phenomenon is also called turbulent since the high frequency signals often become fluctuating or turbulent. Since the physical nature of the turbulence is not well understood yet, there is no reliable theoretical method to predict bunch behavior in this regime.

Usually, the empirical Boussard criterion[6] is used to estimate the threshold for the instability. The discussion of the applicability of the criterion can be found, for example, in [7]. Here, as in [2], we use the criterion as a scaling law to find energy spread and bunch length at a given current beyond the threshold:

$$\frac{eI_p |Z(n)/n|}{2\pi\alpha E_o \sigma_E^2} = 1 \qquad (6)$$

with Z the broad-band impedance at $n=\omega_c/\omega_o$, $\alpha \cong 5 \times 10^{-3}$, the momentum compaction, $I_p$ the peak current, $E_o$ the beam energy and $\sigma_E$ the RMS energy spread.

In the criterion expressed by eq.(6), $I_p$ and $|Z/n|$ depend on the bunch length, which is proportional to the energy spread. For a Gaussian bunch the peak current is proportional to $N/\sigma$, where N is a total number of particles per bunch. From the fit shown in Fig. 6 we can see that for the DAΦNE main ring the normalized impedance scales as $\sigma^{1/2}$. In Fig. 6, to obtain the best fit, we used only the range 0.5 ÷ 2 cm of Fig. 5. This range covers completely the turbulent length regime.

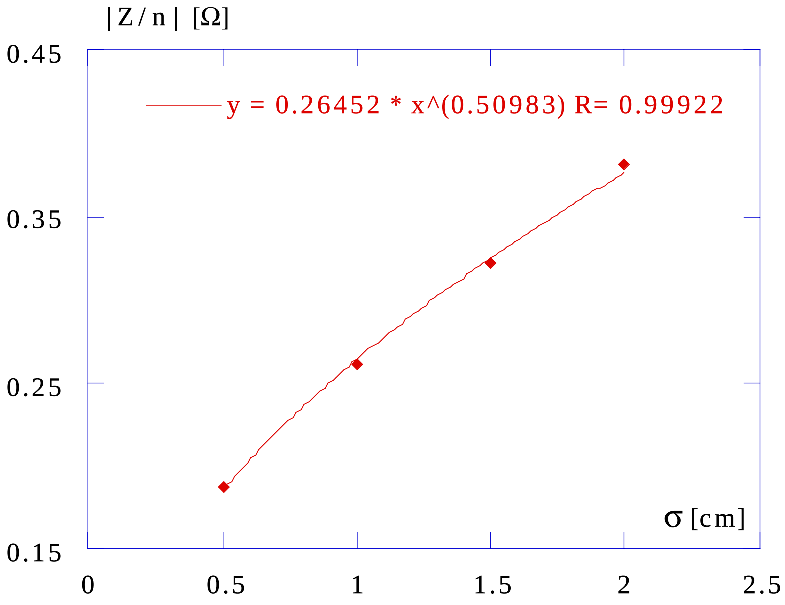


*Fig. 6 - Fit of the normalized impedance versus bunch length.*

All this gives the following scaling for the bunch length in the turbulent regime:

$$\sigma = \sigma_o \left( \frac{N}{N_{th}} \right)^{0.4} \tag{7}$$

where $\sigma_o$ is the natural bunch length and $N_{th}$ is the bunch population at the threshold current.

We also used Boussard's criterion to find $N_{th}$. We should mention that P. Wilson proposed another hypothesis on the threshold criterion [2]. He suggested that the turbulence starts when the slope of the total voltage (RF plus wake-fields) becomes zero at some point within the bunch. It can be shown that for a Gaussian bunch and a purely inductive impedance both criteria are equivalent. Since the DAΦNE main ring impedance is mainly inductive the two criteria are expected to give the close threshold values. Indeed, Boussard's criterion gives $N_{th}=3.96*10^9$ and Wilson's $N_{th} = 4.17*10^9$ at $V_{RF} = 254$ KV.

## Haissinski's equation

Once we get the bunch length above the turbulence threshold, we can start the potential well distortion calculations. An integral transport equation, describing the phase space evolution of electrons in a storage ring is derived taking into account radiation damping and quantum fluctuations[8].

By using the Fokker-Plank method, that is by expanding the integral in time and keeping only the first order terms, from the transport equation it is possible to obtain the diffusion equation

$$\frac{\partial \psi}{\partial t} = -\frac{\partial \psi}{\partial \varepsilon}\frac{\partial H}{\partial \tau} + \frac{\partial \psi}{\partial \tau}\frac{\partial H}{\partial \varepsilon} + \frac{D}{T_o}\left( \psi + \varepsilon \frac{\partial \psi}{\partial \varepsilon} \right) + \frac{1}{2}\frac{\overline{R^2(T_o)}}{T_o}\frac{\partial^2 \psi}{\partial \varepsilon^2} \tag{8}$$

where $\psi(\tau,\varepsilon;t)$ is the longitudinal phase space distribution at a given time t, $\tau$ is the time amplitude of the motion, $\varepsilon$ is the energy deviation, $H(\tau,\varepsilon)$ is the Hamiltonian of a particle (sum of potential and kinetic energy), D is the longitudinal damping constant ($D=2T_o/\tau_\varepsilon$ with $\tau_\varepsilon$ longitudinal damping time) and $\frac{\overline{R^2(T_o)}}{T_o}$ is the variance of the energy radiated by an electron during a revolution period $T_o$.

The physical meaning of eq.(8) is that the change in the phase space distribution in time is due to the phase motion of the unperturbed system (first two terms on RHS), to damping (third term) and to diffusion caused by synchrotron radiation quantum effects (fourth term).

Since we are interested in the stationary distribution, the $\psi(\tau,\varepsilon;t)$ function does not depend on time any more. By using a canonical transformation it is possible to solve the differential equation (8) with respect to the energy, finding the so called Haissinski's equation [9]

$$I(\tau) = C \exp\left( -\frac{E_o}{\alpha H_o} \varphi(\tau) \right) \tag{9}$$

where $I(\tau)$ is the line density distribution, C an integration constant, $E_o$ the nominal energy, $\alpha$ the momentum compaction, $H_o = \frac{1}{2}\overline{R^2(T_o)}\frac{\alpha}{DE_o}$ [10,11], and $\varphi(\tau)$ the potential energy given by

$$\varphi(\tau) = \frac{\alpha}{E_o T_o} \int_0^\tau \left[ eV(x) - U_o \right] dx \tag{10}$$

with V(x) the total voltage, that is the RF plus the self induced voltage, and $U_o$ the energy loss per revolution.

If we define the potential energy given only by the RF voltage as

$$U_{RF} = \frac{\alpha}{E_o T_o} \int_o^\tau \left[ eV_{RF}(x) - U_o \right] dx \approx \frac{\omega_s^2 \tau^2}{2} \tag{11}$$

with $\omega_s$ the synchrotron angular frequency, eq.(9) can be written as

$$I(\tau) = C \exp\left[ -\frac{\tau^2}{2\sigma_{\tau o}^2} + \frac{1}{\dot{V}_{RF}\sigma_{\tau o}^2} \int_o^\tau V_{ind}(x) dx \right] \tag{12}$$

where $\sigma_{\tau o}^2 = \frac{\alpha H_o}{E_o \omega_s^2}$, $\dot{V}_{RF}$ is the slope of the RF voltage at the synchronous phase, and $V_{ind}$ is the induced voltage related to the longitudinal broad band impedance. In the case of purely inductive or resistive impedance, eq.(12) can be easily solved, but for DAΦNE, where the impedance can be approximated by an RL series circuit, it is only possible to solve it numerically.

Defining

$$V_{ind} = L\frac{dI}{d\tau} - RI \qquad (13)$$

and taking the time derivative of both sides of eq.(12), we find

$$\frac{I'(\tau)}{I(\tau)} = -\frac{\tau}{\sigma_{\tau o}^2} - \frac{LI'(\tau)}{\dot{V}_{RF}\sigma_{\tau o}^2} + \frac{RI(\tau)}{\dot{V}_{RF}\sigma_{\tau o}^2} \qquad (14)$$

## Application to DAΦNE

Eq.(14) has to be solved numerically with the boundary condition

$$\int_{-\infty}^{+\infty} I(\tau)d\tau = Q_{tot} \qquad (15)$$

where $Q_{tot}$ is the total charge of a single bunch.

With the estimated impedance given by (4) we have evaluated the new stationary distributions with an RF peak voltage of 254 KV (Fig. 7) and 127 KV (Fig. 8). As an initial value of $\sigma_{\tau o}$ we took σ/c where σ is the bunch length after turbulent lengthening, obtained by eq.(7). We should mention here that, in general, eq.(14) has to be solved in a self-consistent manner, taking into account the dependence of L and R on σ, but since the variation of the inductive and resistive impedance is small in the range of σ above 2 cm, i.e. where Haissinski's equation is solved, L(σ) and R(σ) where replaced in eq.(14) by their average values.

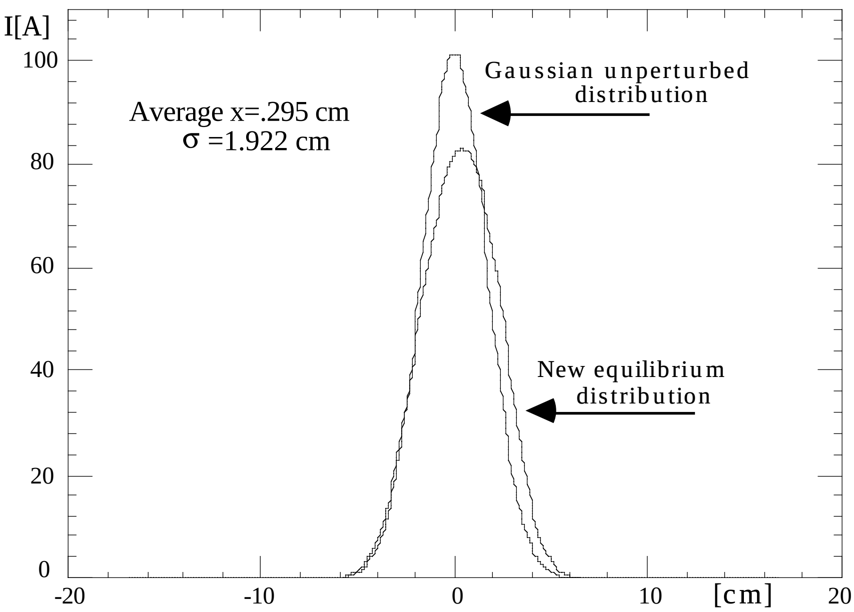


*Fig. 7 - Line density distribution for $V_{RF}$ = 254 KV.*

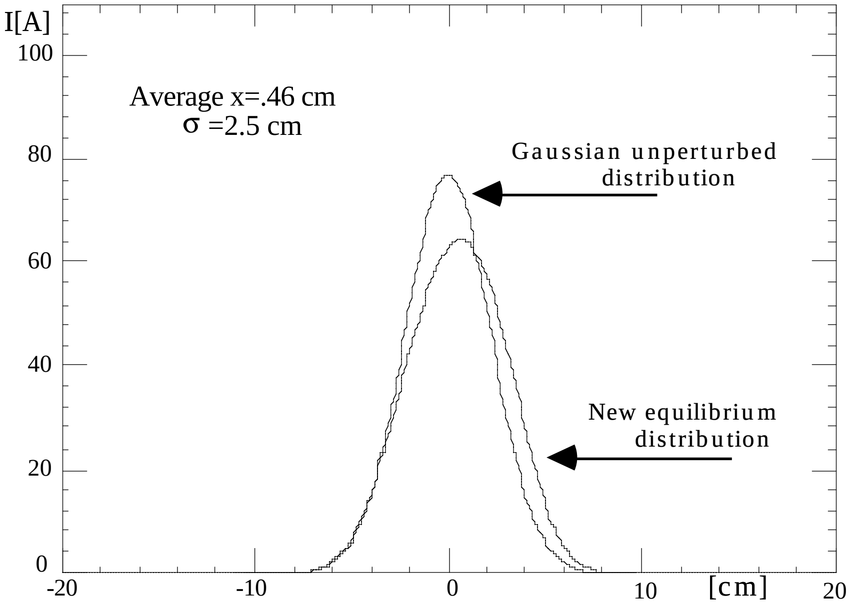


*Fig. 8 - Line density distribution for $V_{RF}$ =127 KV.*

As it can be seen from Fig. 7 and Fig. 8, in order to reach the nominal value of bunch length 3 cm, which has been chosen as a reasonable compromise between different physical demand (i.e. Touschek life time, multibunch instability rise time, RF losses, etc.), the bunch has still to be lengthened. This can be done by reducing the RF voltage, or by introducing additional impedance artificially. Figures 9 and 10 show the bunch distributions for the case when the imaginary part of the impedance is twice the estimated value given by eq.(4).

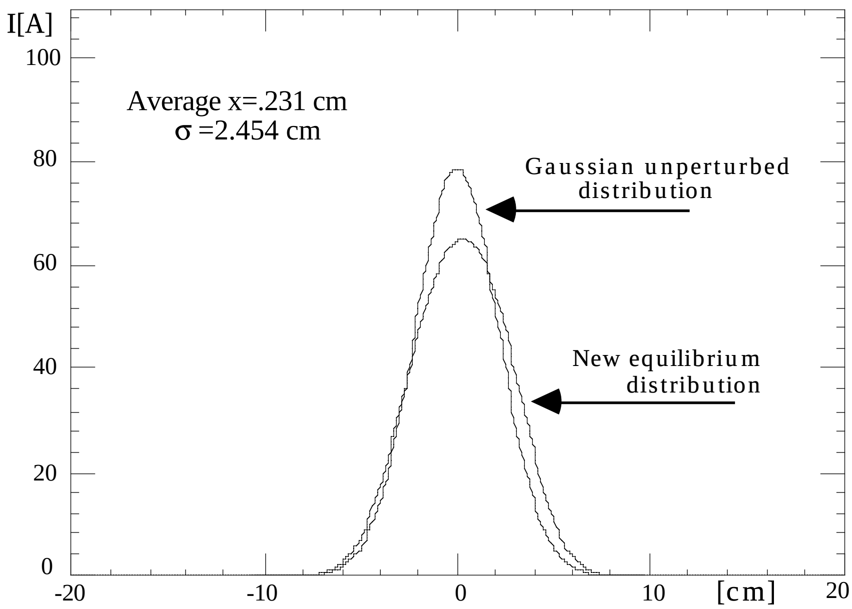


*Fig. 9 - Line density distribution with an impedance double of that estimated and $V_{RF}$ = 254 KV.*

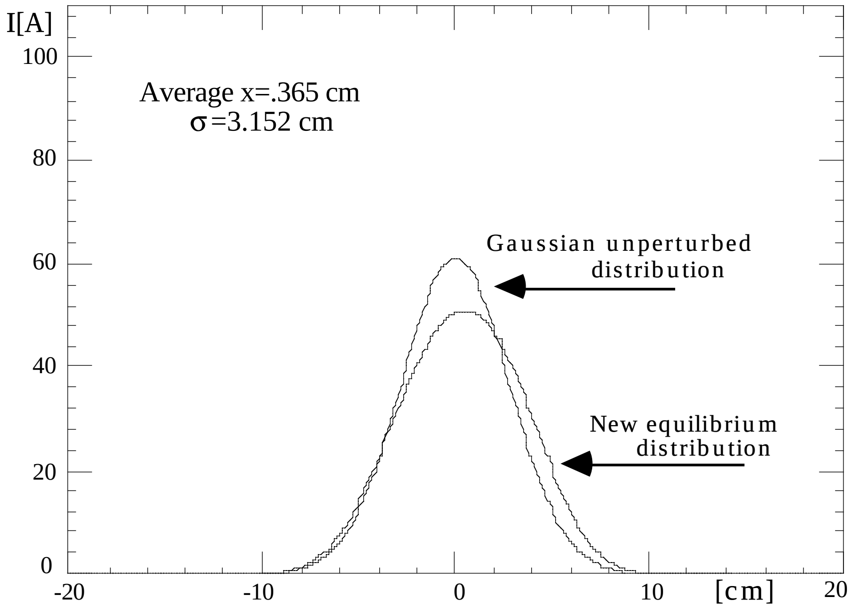


*Fig. 10 - Line density distribution with an impedance double of that estimated and $V_{RF}$ = 127 KV.*

We can see that with an RF peak voltage of 127 KV, the bunch length is close to the nominal value. However, it is worth reminding that a substantial reduction of the RF voltage is not acceptable because it leads to a smaller RF acceptance.

## The incoherent synchrotron tune

The induced voltage changes the potential well seen by a single particle. This distortion causes a spread of the incoherent synchrotron frequency. The potential energy in terms of the line density distribution is[2]

$$\varphi(\tau) = -\sigma_{\tau o}^2 \omega_s^2 \ln\left(\frac{I(\tau)}{I(0)}\right) \tag{16}$$

The equation of motion of a single particle in the well, neglecting the effect of radiation damping, is

$$\frac{d^2\tau}{dt^2} + \dot{\varphi}(\tau) = 0 \tag{17}$$

The period of the synchrotron oscillation of a single particle with initial condition $\tau(0)=\tau_o$ and $\dot{\tau}(0) = 0$ is

$$T_s = \sqrt{2}\int_{\tau_o}^{\tau_1} \frac{d\tau}{\sqrt{\varphi(\tau_o) - \varphi(\tau)}} \tag{18}$$

with $\varphi(\tau_1)=\varphi(\tau_o)$.

By solving eq.(18) numerically with the line density distribution obtained with Haissinski's equation, and by changing $\tau_o$, we express $\omega_s = \frac{2\pi}{T_s}$ as function of $\tau_o$.

In Fig. 11 we show the potential well distortion and in Fig. 12 the synchrotron frequency spread in the case of the distribution obtained (Fig. 7) with an RF peak voltage of 245 KV and the impedance estimated with eq.(4).

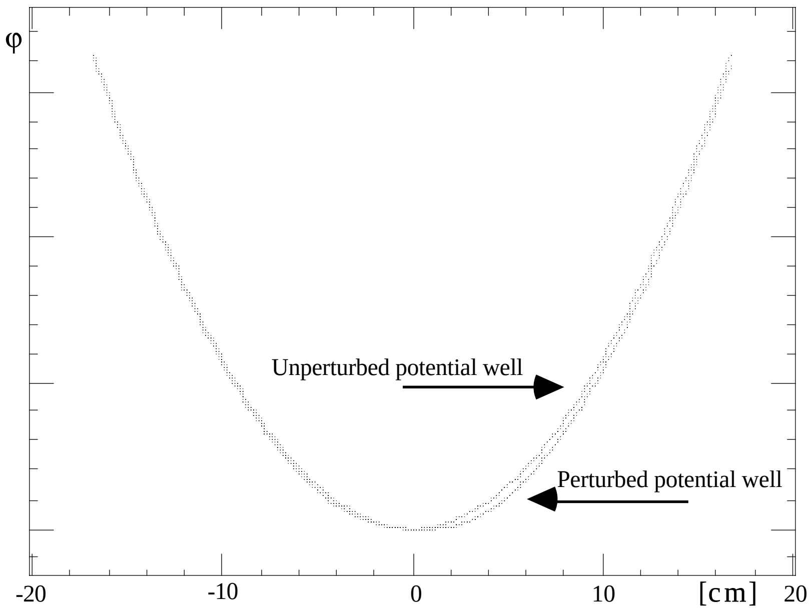


*Fig. 11 - Potential well distortion.*

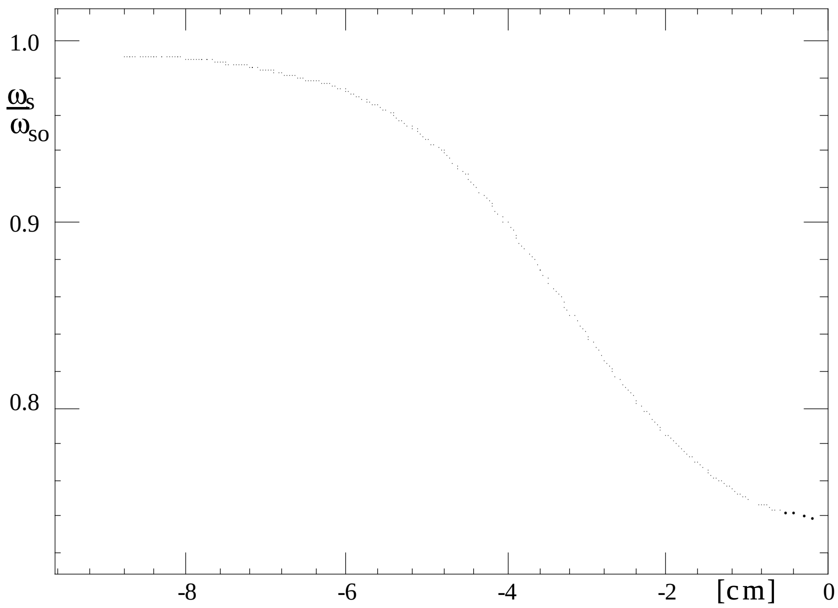


*Fig. 12 - Synchrotron frequency spread.*

## Conclusions

In spite of the fact that there exists a vast literature on bunch lengthening, there is no satisfactory explanation of a turbulent threshold, and no reliable methods are known to predict single bunch behavior at currents above threshold.

The semiempirical Boussard's criterion yields results in good agreement with observations in many machines, e.g. [2].

We have used this criterion to evaluate the bunch length above the turbulence threshold, and, after that, we have applied the potential well distortion theory to obtain the bunch equilibrium distribution. The same procedure has been used successfully to simulate the bunch lengthening process in the SLC damping rings[2].

From the obtained results, we can see that the final bunch length is below the nominal one, and therefore the bunch has to be further lengthened. This can be done by decreasing the RF voltage or by increasing the inductive part of the machine impedance. For example, the nominal value of the bunch length is reached at $V_{RF}$ = 127 KV with an impedance twice the estimated one for the DAΦNE main ring. However, a substantial reduction of the RF voltage is not acceptable as far as it gives a reduction in the RF acceptance. Therefore, additional ways of controlling the bunch length should be studied.

Another reason to develop a system to cope with any eventual differences in the bunch behavior (either shorter or longer) is the lack of a reliable theory to describe the strong turbulent lengthening regime. This is also why the applicability condition of the Boussard's criterion, both for the threshold current and as a scaling law above turbulence, has to be deeply investigated.

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