DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, Oct. 2, 1992

Note: **G-15**

TRANSVERSE RESISTIVE WALL INSTABILITY IN DAΦNE

L. Palumbo, M. Zobov

Introduction

Performances of most storage rings are limited by coherent instabilities. Transverse instabilities are destructive for a stored beam. One of the instabilities, the resistive wall instability, is driven by both short range and long range wake-fields. It can be dangerous in single bunch operation as well as in multibunch one, where coupled bunch modes must be considered too.

In this note we compute the rise times of the transverse resistive wall instability for the DAΦNE accumulator and the Main Rings. In our calculations we refer to Sacherer-Zotter theory [1]. Parameters of the accumulator and of the Main Rings are taken from [2] and [3].

Possible ways to increase the instability rise time and to dump the instability are discussed.

Resistive wall instability

An unperturbed beam motion is characterized by two mode numbers. For k_b bunches in a storage ring there are k_b coupled-bunch modes with mode number s = 0, 1,...., k_b-1, specifying the phase shift $\Delta\varphi = (2\pi s / k_b)$ between bunches. Another mode number m = 0,1,2, ..,∞ is needed to describe an individual bunch motion in the synchrotron phase space within s^{th} coupled-bunch mode.

The unperturbed modes are at frequencies:

$$\omega_p^T = (pk_b + s + \nu_{x,y} + m\nu_s)\,\omega_0$$

$$p = 0, \pm 1, \pm 2, \ldots, \pm\infty \qquad (1)$$

where $\nu_{x,y}$ is the transverse betatron tune, ν_s is the synchrotron tune; ω_0 is the angular revolution frequency.

The perturbed frequency of the coherent oscillation mode is modified from the unperturbed value by a complex coherent frequency shift:

$$\Delta\omega^{T}_{s,m} = j \frac{I_b c^2}{2 \nu_{x,y} \omega_0 (E/e) L} \frac{1}{(m+1)} [Z_T]^{s,m}_{eff} \tag{2}$$

where I_b is the average bunch current; E is the total energy of a particle; $L = 4\sigma_l$ is the total bunch length and:

$$[Z_T]^{s,m}_{eff} = \frac{\sum_{p=-\infty}^{+\infty} Z_T(\omega^T_p)\, h_m(\omega^T_p - \omega_\xi)}{\sum_{p=-\infty}^{+\infty} h_m(\omega^T_p - \omega_\xi)} \tag{3}$$

with:

$$\omega_\xi = \frac{\xi}{\alpha_c} \omega_0 \nu_{x,y} \tag{4}$$

and:

$$\xi = \frac{(d\nu/\nu)_{x,y}}{dp/p} \tag{5}$$

Here $h_m(\omega)$ is a bunch mode spectrum which for a gaussian bunch is given by:

$$h_m(\omega) = \left(\frac{\omega \sigma_l}{\beta c}\right)^{2m} \exp\left\{-\left(\frac{\omega \sigma_l}{\beta c}\right)^2\right\} \tag{6}$$

An instability occurs when the imaginary part of the frequency shift of the coherent oscillation mode is negative:

$$\mathrm{Im}(\Delta\omega^{T}_{s,m}) < 0 \tag{7}$$

So the instability can be driven by the real part of the resistive wall impedance:

$$Z^{rw}_T(\omega) = (1+j) \frac{R Z_0}{b^3} \delta_0 \sqrt{\frac{\omega_0}{\omega}} \tag{8}$$

where:

$$\delta_0 = \sqrt{\frac{2\,\rho\,R}{Z_0}} \tag{9}$$

ρ is the resistivity of the vacuum chamber material; R is the mean storage ring radius;
b is the radius of the ring vacuum pipe;
Z_0 free space impedance (=377 Ω).

If one of the lines in the bunch spectrum is very close to the origin in the negative frequency region (see Fig. 1) the resistive wall instability takes place since this line is associated with a very large negative resistance:

$$\mathrm{Re}\left\{Z_T^{rw}(\omega_p)\right\} = -\frac{R\,Z_0\,\delta_0}{b^3}\sqrt{\frac{1}{|\mathrm{Fraq}\,\nu_{x,y} + m\nu_s|}} \tag{10}$$

where $|\mathrm{Fraq.}\nu_{x,y}| = |\mathrm{integer} - \nu_{x,y}| < 1$.

Neighboring lines are associated with the impedance by a factor:

$$\sqrt{|(\mathrm{Fraq}\,\nu_{x,y} + m\nu_s)/(\mathrm{Fraq}\,\nu_{x,y} + m\nu_s \pm k_b)|} \tag{11}$$

Therefore, to a certain extent, we can consider that coherent motion is driven only by the line closest to the origin and use the formula given in [4] to estimate the coherent frequency shift:

$$\Delta\omega^{T}_{s,m} = -\frac{e\,I}{4\,\pi\,(|m|+1)\,m_0 c\,\gamma}\; j\,\frac{R\,Z_0\,\delta_o}{b^3}\sqrt{\frac{1}{|\mathrm{Fraq}\,\nu_{x,y}+m\nu_s|}}\; F_m\{(\omega_\xi - \mathrm{Fraq}\,\nu_{x,y} - m\nu_s)\frac{\tau_L}{2}\} \tag{12}$$

where the form factor F_m for a gaussian distribution is given by:

$$F_m(\Delta\varphi) = \frac{4\,(|m|+1)}{\Delta\varphi^2}\int_0^\infty J_m^2(\varphi)\,\varphi\,\exp\left(-2\left(\frac{\varphi}{\Delta\varphi}\right)^2\right)d\varphi \tag{13}$$

J_m is the Bessel function of order m; τ_L represents the full time bunch length ($\tau_L = L/c$ for gaussian distribution); $I = k_b * I_b$ is the average beam current. The form factor F_m for modes m=0 and m=1 is shown in Fig. 2.

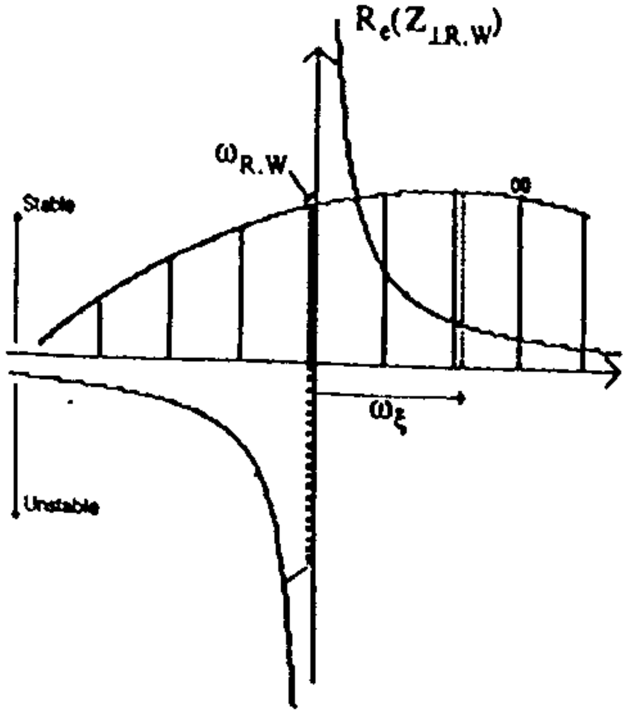

Fig. 1 - Real part of the transverse resistive wall impedance with superimposed bunch spectrum - the case of transverse instability.

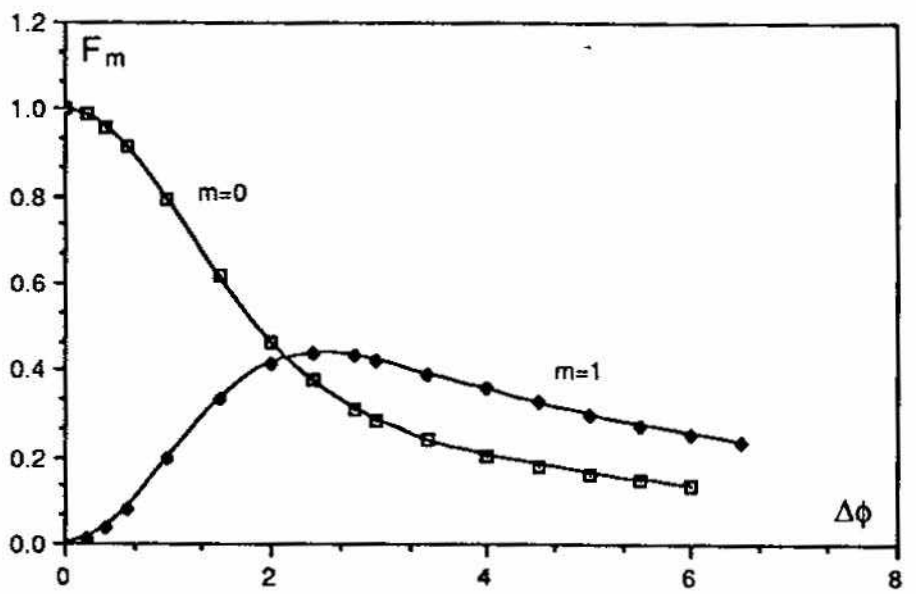

Fig. 2 - Form factor F_m for mode m=0 and m=1.

The single line approximation works better for multibunch regime with a number of bunches k_b>>1. But we should expect that this approximation is not reliable at ξ>0. We explain it taking the mode m=0 as an example. By increasing ξ the bunch spectrum is removed in higher frequency region and relative contribution of the line closest to the origin gets small (the form factor F_0 drops) meantime the contribution of other lines within the bunch spectrum grows. In this case we must take into account all these lines.

Here-under we give the results of a single line approximation at ξ=0 and calculate dependencies of the instability rise time on ξ for DAΦNE accumulator and the main ring using (1)-(9).

Resistive wall instability in accumulator

For DAΦNE accumulator with k_b=1 the most dangerous dipole mode (m=0) shows frequencies at:

$$\omega_p^T = (p + \nu_{x,y})\omega_0 \tag{14}$$

The choice of a transverse tune below an integer will create the situation when one of the lines in the unstable region is closer to the origin than the line nearest to the origin in the stable region (see Fig. 1). In this case the coherent motion can be unstable.

In the first accumulator design the β-tune was ν_x = 2.89, ν_y = 0.91. Both transverse tunes are slightly below an integer and, so, the resistive wall instability takes place. We calculated the instability rise time:

$$\tau_m = \frac{1}{\mathrm{Im}(\Delta\omega_{s,m}^T)} \tag{15}$$

for two most dangerous mode m=0 and m=1. Fig. 3 shows dependencies of the rise times of these modes on the machine chromaticity considering stainless steel as a material for the accumulator vacuum chamber. We can see that the rise time for mode m=0 at ξ=0 is equal to 0.5 ms (single line approximation gives 0.42 ms) that is much less than the radiation damping time which is 21.42 ms for the accumulator. So special measures should be undertaken to dump the instability.

The situation can be improved by increasing the chromaticity. We can do it up to the point until the mode m=1 gets dominating (at ξ=0.65). At this point $\tau_0=\tau_1$=2.5 ms.

Since the resistive wall impedance is proportional to $\rho^{1/2}$ we can gain a factor of 6 using Aluminum instead of stainless steel for the vacuum chamber.

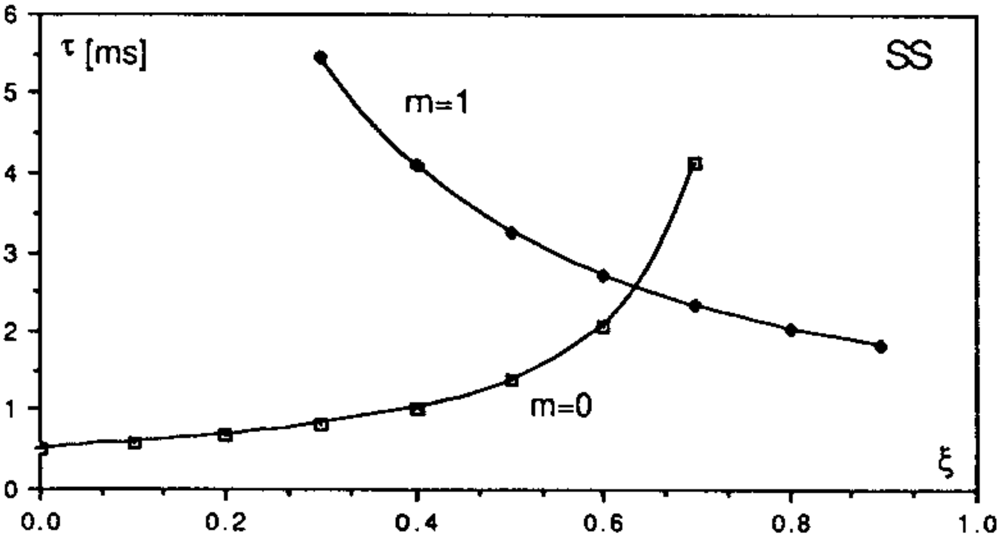

Fig. 3 - Rise times of the resistive wall instability for modes m=0 and m=1 in the DAΦNE accumulator for stainless steel vacuum chamber (ν_x = 2.89, ν_y= 0.91) versus chromaticity.

To eliminate the resistive wall instability completely it is necessary to change the transverse tune making it slightly above an integer. In this case mode m=0 is stable and the rise time for the mode m=1 is much higher than the radiation damping time.

In the accumulator where only a single bunch rotates the most dangerous mode m=0 can be dumped changing the β-tune. In the multi-bunch regime by changing the tune we can not find a condition when all dipole coupled-bunch modes s (m=0) are stable. Always a half of the dipole coupled-bunch modes are stable and a half of them are unstable.

Resistive wall instability in the Main Rings

Let us now consider 30 bunches in the DAΦNE main ring. Both β-tunes are below the integer ν_x = 4.85 and ν_y = 4.87. Among the dipole modes the mode s=25 is most unstable because the spectrum line with p=-1, s=25 is the closest to the origin in the unstable region ω_{-1}(p=-1, s=25, m=0, $\nu\psi$ = 4.87) = 0.13ω_0. Fig. 4 shows the rise time of the modes m=0 and m=1 with the coupled-bunch mode number s=25 for different values of the chromaticity. At ξ=0 the rise time of the mode m=0 is rather small: τ_0 = 140 μs for stainless steel and τ_0 = 840 μs for Al, much shorter than the radiation dumping time in the main ring τ = 36 ms. The same results gives the single line approximation.

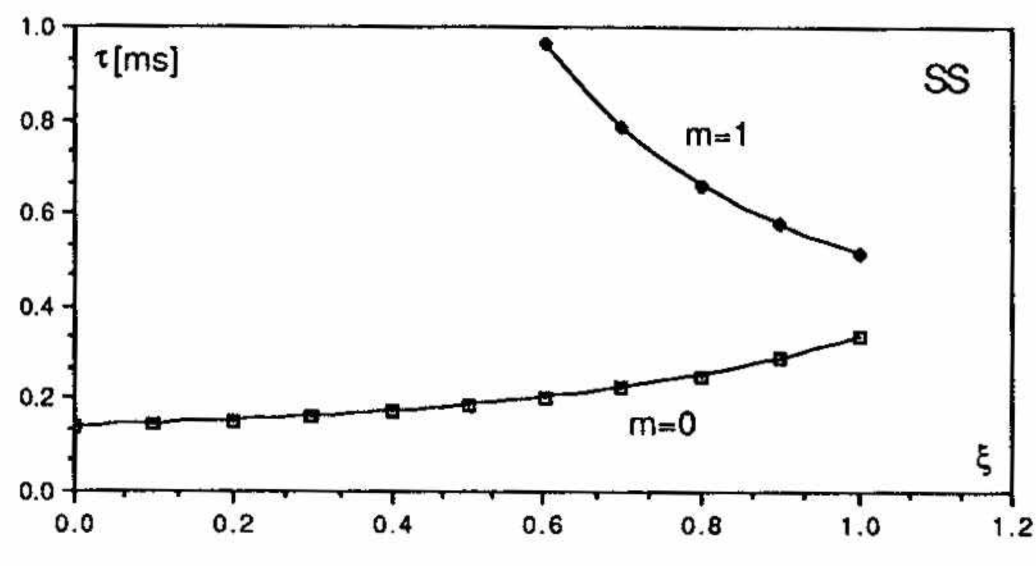

Fig. 4a) - Rise times for modes m=0 and m=1 with the coupled=bunch mode number s=25 in the DAΦNE main ring (for stainless steel vacuum chambr) versus chromaticity.

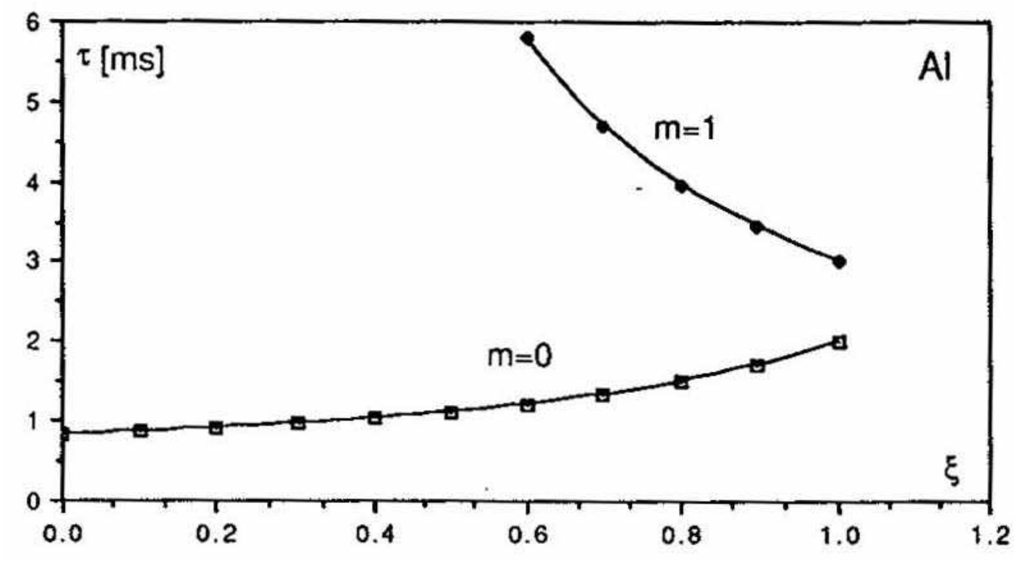

Fig. 4b) - Rise times for modes m=0 and m=1 with the coupled=bunch mode number s=25 in the DAΦNE main ring versus chromaticity - the case of aluminium vacuum chamber.

The results of instability rise time calculations with BBI code [5] presented by B. Zotter for DAΦNE main ring are in Fig. 5. The results correspond to these shown in Fig. 4 except the region with $\xi \sim 1$. It is explained by the fact that in BBI calculations the transverse broad-band resonator impedance with resonant frequency f_r = 1.85 GHz, quality factor Q=1 and shunt impedance R=44 Ohm/m was taken into account and there are additional contribution of lines situated far from the origin, but close to f_r where the resistive part of the impedance has a maximum.

As far as the resistive wall impedance is proportional to $\omega^{1/2}$ we can increase the rise time of the instability choosing the transverse tune above an integer. It means that we move dangerous line in the bunch spectrum far from the origin. For example, if we choose ν_y = 5.09 instead of 4.87 in the horizontal plane the rise time will be increased approximately by a factor 2.7. So a fast feedback system can be effectively used to dump this instability. Another way to increase the rise time is to increase the machine chromaticity ξ. We can see in Fig. 4 that the rise time at ξ = 1 is by a factor 3 higher than that at ξ = 0.

One of the possible cure for the instability is an octupole field, providing amplitude dependence of frequency. A sufficient spread in the bunch frequencies prevents the instability. A criteria of a stable motion is that the rms spread in betatron frequencies should exceed the growth rate (Landau damping).

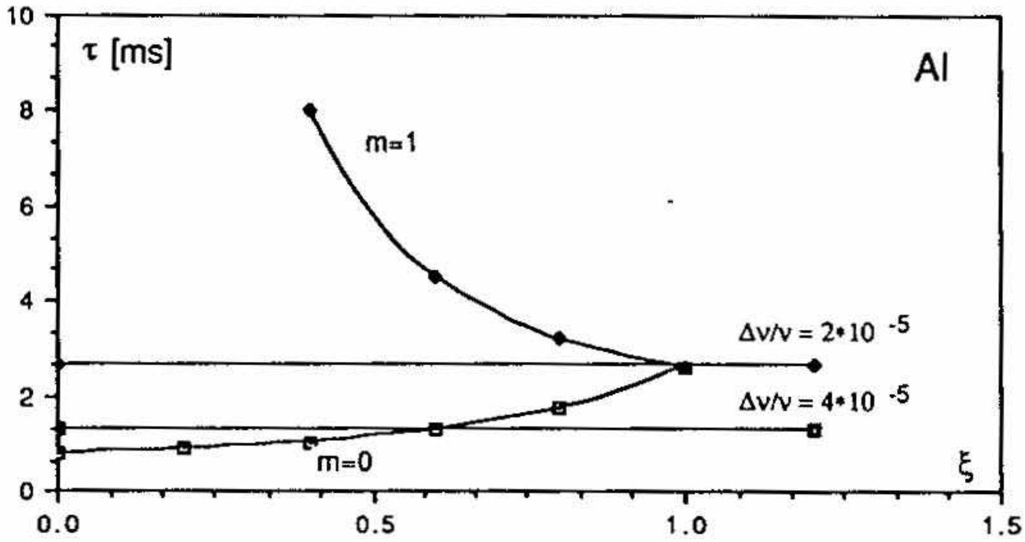

Fig. 5- Results of instability rise time calculation with BBI code for modes m=0 and m=1 for the DAΦNE main ring (for Al). Straight lines correspond to Landau damping time due to octupole - introduced tune spread Δν/ν.

The straight lines in Fig. 5 correspond to Landau damping time because of octupole-introduced tune spread $\Delta\nu/\nu$ (BBI code results).

For example, if we introduce the tune spread $\Delta\nu/\nu = 4*10^{-5}$, than the motion will be stable at $\xi>0.6$.

Momentum spread at non-zero chromaticity also introduces the betatron tune spread but it does not contribute to Landau damping because of the averaging of particle momenta by synchrotron motion.

Conclusions

1) The resistive wall instability in the accumulator ring can be completely eliminated by choosing the transverse tune just above an integer.

2) In the main ring the instability rise time is 140 μs for stainless steel and 840 μs for Al at $\xi=0$ considering 30 bunches in the ring.

   - The instability rise time will increase approximately by a factor 3 by changing the transverse tunes (4.85,4.87) and making them slightly above an integer.
   - Increasing of the machine chromaticity has a positive effect: the instability rise time increases (up to 2 ms for Al at $\xi=+1$). But this gives rise to the problem of a dynamic aperture.
   - The instability can be damped introducing octupoles in the lattice.
   - A feedback system seems to be the most reliable cure of the instability.

Final Remarks

1) A new working point above the integer in both plans ($\nu_x = 3.12$, $\nu_y = 1.14$) has been chosen for the DAΦNE accumulator to avoid the resistive wall instability [6].

2) For the Main Rings a lattice with $\nu_x = 5.12$ and $\nu_y = 5.16$ is under consideration [7] at the present moment. Such a choice allows to increase the rise time of the resistive wall instability up to 380 μs for stainless steel and 2.275 ms for Al at $\xi = 0$ having 30 bunches in the ring. The rise time scales inversely with the number of bunches.

Acknowledgment

The authors would like to thank B. Zotter for clarifying discussion.

References

[1] B. Zotter and F. Sacherer, "Transverse instabilities of relativistic particle beams", Proceedings of the First Course of International School of Particle Accelerators, Erice 1976, CERN 77-13, p175.

[2] S. Guiducci et. al., "DAΦNE accumulator update-2", DAΦNE Technical Note I-4, 1 Oct. 1991.

[3] M.E. Biagini et. al., "DAΦNE lattice update", DAΦNE Technical Note L-4, 2 Dec. 1991.

[4] J.L. Laclare, "Bunched beam coherent instabilities", CERN Accelerator School, September 1991.

[5] M. Gygi-Hanney et al., "Programme BBI- Bunched Beam Instabilities in High Energy Physics", CERN/LEP-TH/83-2, January 1983.

[6] M.R. Masullo, C. Milardi, M.A. Preger, "DAΦNE Accumulator Update-3", DAΦNE Technical Note I-9, May 13, 1992.

[7] M.E. Biagini, private communication.