DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, Dec. 18, 1991

Note: **G-10**

COUPLED BUNCHES INSTABILITIES IN DAΦNE

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1. Introduction

In DAΦNE the early and primary luminosity goal of a few 10^{32} cm^{-2} sec^{-1} can be achieved by storing in the main ring 30 bunches with a total current of about 1.4 Ampere.

Coupled bunch instabilities, driven by the parasitic HOM of the RF cavity, are one of the main concern in the design of the machine [1]. Fast longitudinal digital feedback systems may provide at best a damping rate of about 100 μsec [2], much faster than the natural radiation damping, not sufficient, however, to damp those relative modes significantly coupled to strong resonances. In fact much faster multibunch instabilities can be excited by some HOM's, unless their shunt impedance is significantly reduced by coupling the cavity to absorbers or by shifting of the HOM frequency.

In this note we compute the rise time of the coupled mode instabilities assuming a single HOM as driving term. Our aim is to get valuable information on the maximum allowed shunt impedance for the most harmful parasitic modes and at the same time investigate on the benefits deriving by a controlled tuning of the HOM's. We refer to the Pellegrini-Wang theory [3], suitable for short bunches, that makes use of rather simple expressions for the instability rise time.

Table I - Machine and sample-HOM parameters.

Machine parameters			**HOM Parameters**	
Length	L	97.690 m		
Energy	E	510 MeV	R_s/Q_r	20
Momentum compaction	α_c	0.017		
Bunch current	I_b	43.76 mA	ω_r	1 - 25 GHz
Bunch length	σ_l	3 cm		
Synchrotron tune	ν_s	0.0128	R_s	1.0, 0.1, 0.01 MΩ
Number of bunches	k_b	30		

2. Unperturbed spectrum of 30 equispaced bunches

The spectrum of the unperturbed motion shows frequencies at

$$\omega_p = (p\, k_b + s + a\, \nu_s)\, \omega_o$$

$$p = 0, \pm 1, \pm 2, ...$$
$$a = 1, 2, 3, ...$$
$$s = 0, 1, ... k_b - 1..$$

where ω_o = 19.295 MHz is the revolution frequency, "a" describes the longitudinal motion in the phase space (dipole mode a = 1, quadrupole mode a = 2 etc), and "s" specifies the longitudinal mode number. As an example we show in Fig.1 the low frequency spectrum for the dipole mode a = 1 in the case of k_b = 30.

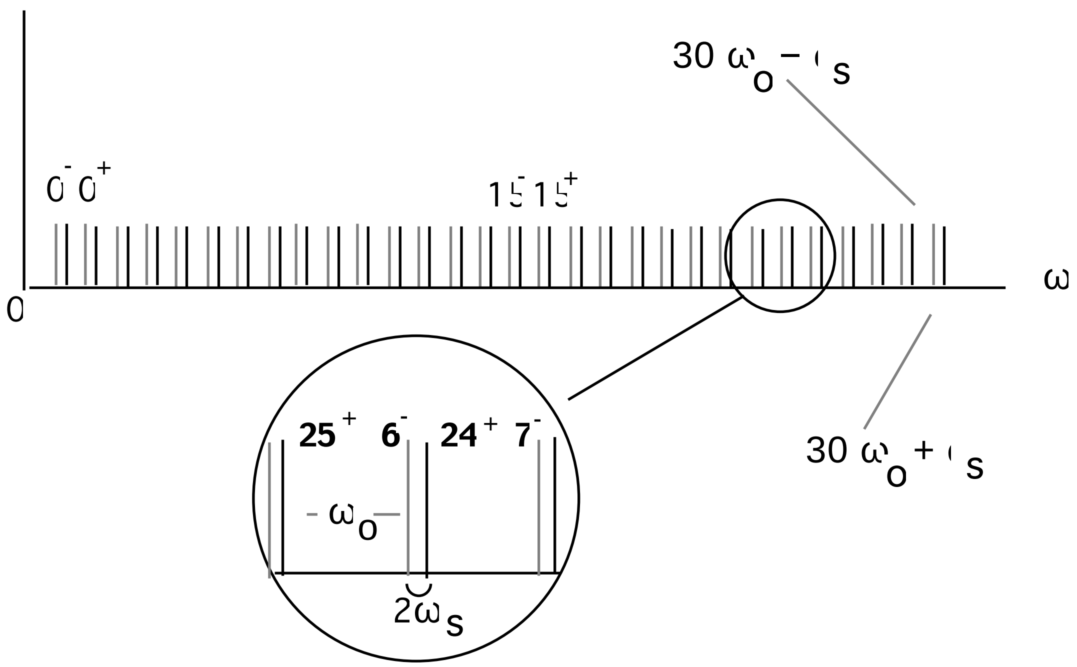

Fig. 1 - Dipole (a=1) stable for p=0 and unstable sidebands for p= -1, k_b=30.

The dashed sidebands are intrinsically stable while the solid ones are unstable. We note that in the displayed range of about 600 MHz for p = 0 and p = 1, two close positive and negative sidebands are spaced by about 2 ω_s (0.494 MHz); their relative modes number differ by 30, (e.g. 24^+ and 6^-). On the other side two consecutive stable or unstable sidebands are spaced of ω_o (19.295 MHz). In case of k_b=120 we have a similar line spectrum with 120 stable and unstable sidebands covering a frequency range of about 2.4 GHz. This scenario repeats over the whole spectrum.

3. Longitudinal instability rise time for the dipole mode a = 1.

When a relative mode of oscillation "s" is excited by the e.m. resonating fields, the motion is intrinsically stable or unstable depending on the sign of the rise time:

$$\tau_{s,1} = \frac{4\pi\,(E/e)\,\nu_s}{k_b I_b \omega_o \alpha_c} \; \frac{1}{\left(Z_{s,1}\right)^{eff}} \qquad (1)$$

Since for a given relative mode "s" the sidebands corresponding to different "p" are quite far away, the effective impedance due to a single HOM can be approximated by the contribution corresponding to the two closest stable "d" (damped) and unstable "u" sidebands:

$$\left(Z_{s,1}\right)^{eff} = \frac{R_s F\left(\omega_p^u\right)}{1 + Q_r^2\left(\frac{\omega_p^u}{\omega_r} - \frac{\omega_r}{\omega_p^u}\right)^2} - \frac{R_s F\left(\omega_p^d\right)}{1 + Q_r^2\left(\frac{\omega_p^d}{\omega_r} - \frac{\omega_r}{\omega_p^d}\right)^2} \qquad (2)$$

Damped and undamped sidebands of the same relative mode are quite apart, the only exception being the mode 0 and $k_b/2$. As an example with 30 bunches, the spectrum is such that the stable and unstable sidebands relative to the modes s = 0 and s = 15 are simultaneously excited by the same HOM.

The form factor $F(\omega_p)$ for the dipole mode, plotted in Fig. 2, for $k_b = 30$, is given by:

$$F\left(\omega_p\right) = \frac{\omega_o}{\omega_p}\,(k_b p + s)^2\, e^{-\left(\frac{(k_b p+s)\,\sigma_l}{R}\right)^2}$$

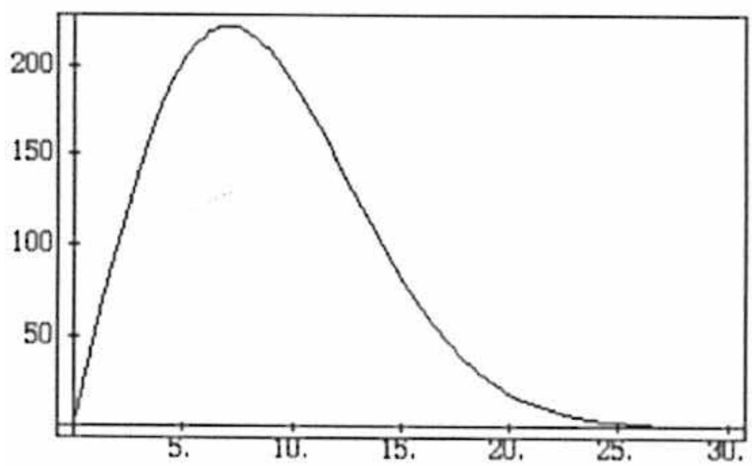

Fig. 2 - Form factor for the dipole mode a = 1 vs. w_r (GHz).

Assuming a HOM at the resonant frequency ω_r, we first compute what is the relative mode "s" excited with the lowest rise time :

$$s^u = Int\left(\frac{\omega_r}{\omega_o} - k_b p - \nu_s\right)$$

Since the sum of two close damped and undamped relative mode number is equal to k_b, we easily find the frequencies of the stable and un-stable sidebands corresponding to the relative mode mostly excited by the HOM:

$$\omega_p^u = \omega_o\left[Int\left(\frac{\omega_r}{\omega_o} - \nu_s\right) + \nu_s\right]$$

For k_b=30 the damped sideband is located at:

$$0 \le s \le 7 \qquad \omega_p^d = \omega_p^u - \left[2s + \nu_s\right]\omega_o$$

$$8 \le s \le 22 \qquad \omega_p^d = \omega_p^u + \left[k_b - 2s - \nu_s\right]\omega_o$$

$$23 \le s \le 29 \qquad \omega_p^d = \omega_p^u + \left[2k_b - 2s - \nu_s\right]\omega_o$$

4. Investigation for k_b = 30, R_s/Q_r = 20, Q_r = 50000, 5000, 500,100.

We analyze here four significative cases assuming a high value of R_s/Q_r = 20 and a total current of about 1.5 A (47 mA in 30 bunches). In the frequency range ω_r = 1 - 25 GHz the resonator bandwidth is of the order of a fraction of MHz for Q_r = 50000 and increases to several tens of MHz for Q_r = 100.

In Fig. 3 we show the rise time due to a HOM with a resonant frequency varying in the region where the maximum of the form factor occurs, i.e. around ω_r = 7.070 GHz. The effect of the interaction with the first 6 relative modes s = 0,1,2,3,4 and 5, consecutively excited is shown. The compensation effect occurring around s = 0 (an identical compensation happens for s = 15) is clearly indicated by the higher maxima at the beginning of the plot. When the resonator frequency is between the stable and unstable sidebands 0^+ and 0^- one observe a Robinson like compensation effect. The risetime of the relative mode s = 0 becomes even negative (damping), thus, at a certain frequency, the lowest rise time is determined by the nearest unstable sidebands s = 1 or s = k_b-1. The minima growth times are at those frequencies where the sidebands are fully coupled. We note that with about 1.5 A stored in the machine, the rise time instability may be as short as fraction of microseconds for Q_r = 50000 (Rs = 1 MΩ) and around few milliseconds for Q_r = 100 (Rs = 2 kΩ). For Q_r<500 (Rs<10 kΩ) it stays above 100 μs.

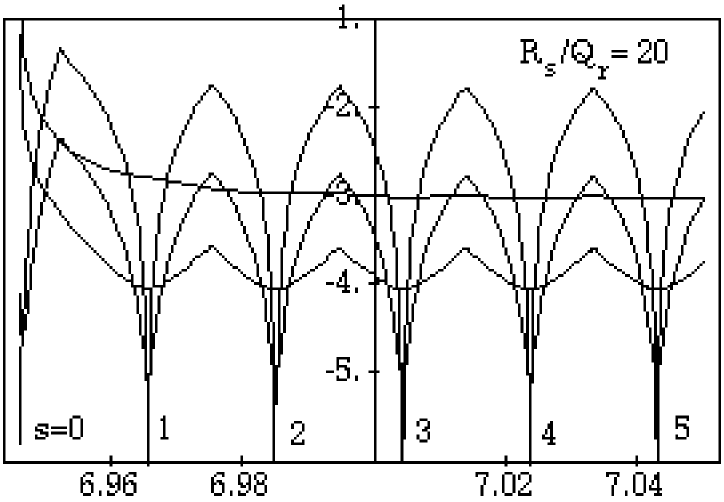

Fig. 3 - Log_{10} [τ(sec)] vs. ω_r (GHz), (Q_r = 50000, 5000, 500, 100), $k_b I_b \approx 1.5$ A.

It is worth noting that reducing the shunt impedance (or decreasing the quality factor Q_r) minima and maxima become closer. This is due to the fact that the minimum value, being caused by a sideband fully coupled, depends only on the shunt impedance R_s, while the maximum, caused by a sideband excited by the tail of the impedance spectrum, depend on both the shunt impedance R_s and the quality factor Q_r. For Q_r = 100 the rise time is practically constant and of the order of few microseconds, with a strong compensation around s = 0 and s = $k_b/2$.

5. Minima and maxima rise times.

The minimum rise time is observed when the HOM is fully coupled to a relative mode sideband. Neglecting the stabilizing effect of the stable sideband we get the following approximate expression for $\omega_r = \omega_p$:

$$\tau_{s,2}^{min}(\omega_r) = \frac{4\pi(E/e)\nu_s}{k_b I_b \alpha_c} \frac{e^{\left(\frac{\omega_r \sigma}{\omega_o R}\right)^2}}{\omega_r R_s}$$

which does not depend on the quality factor Q_r.

In Fig. 4 we plot vs. the frequency what is the shunt impedance exciting the instability with a rise time τ = 100 μs, for a stored current

k_bI_b = 0.1, 0.5, 1.0 and 1.5 Ampere. On the same plot are reported the parasitic shunt impedances of the "low loss" cavity proposed for the "day one" machine operation [5].

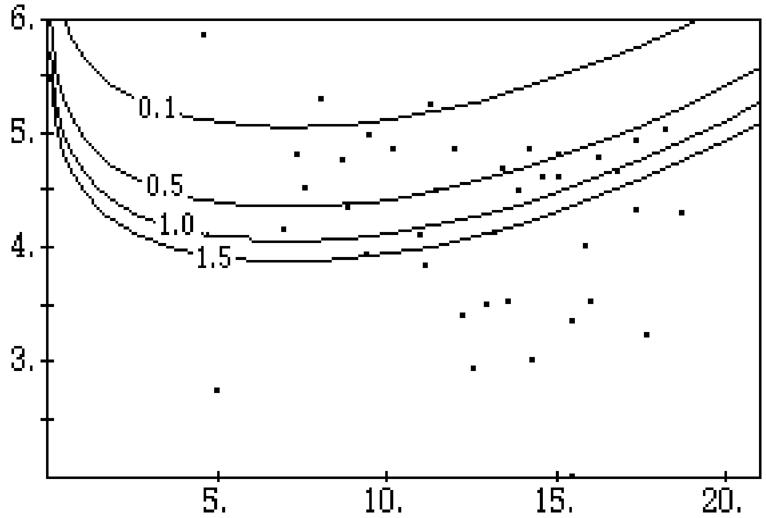

Fig.4. **a=1** $\mathrm{Log}_{10}[R_s(\Omega)]$ vs. ω_r (GHz) for τ =100μs; Io = 0.1, 0.5, 1.0 , 1.5 A.

We see that in spite of the "average" low losses, there are individual modes with a shunt impedance exceeding the allowed limit. Even in the case of 100 mA, there are two modes with a shunt impedance such that a full coupled instability would grow much faster than 100 μs! These two modes can be coupled with a loop antenna or shifted in frequency. It is worth noting that an induced frequency shift of the other modes can be tolerated since their shunt impedance stays anyhow below the threshold. The scenario becomes much worse when we examine the case of 500 mA, where about ten modes more are potentially dangerous. To reach 1.0 and 1.5 Amps it is necessary to damp further 10 modes distributed at higher frequencies.

In the case one is able to develop a reliable system for controlling the HOM frequency, it becomes interesting to know what is the highest rise time, attainable when the HOM frequency is just on the middle of two consecutive unstable sidebands; we get :

$$\tau_{s,1}^{max}(\omega_r) = \left[1+\left(\frac{\omega_o}{2\,\Delta\omega_r}\right)^2\right]\tau_{s,1}^{min}(\omega_r)$$

As expected it is strongly dependent on the revolution frequency and on the resonance bandwidth $2\Delta\omega_r \sim \omega_r/Q_r$. The envelope curves of the maxima rise time values with $R_s/Q_r = 20$ are plotted in Fig. 5.

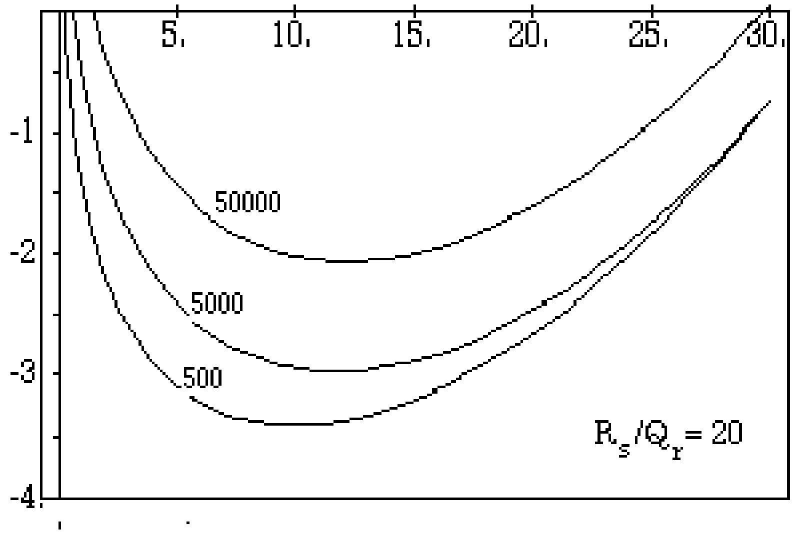

Fig. 5. Envelope of Log_{10} [τ_{max}(sec)] vs.ω_r (GHz) for Q_r = 50000, 5000, 500..

The shift method becomes ineffective when $2\Delta\omega_r \sim \omega_o$. In DAΦNE, in the frequency range of interest, this happens for $Q_r \sim 500$. Below this value the impedance bandwidth is so large that two sidebands at least are simultaneously excited.

It seems hard today to think of controlling the frequencies of so many modes, whereas damping techniques based on the absorbtion of the e.m. energy by means of antennas or waveguide couplers look more promising.

Finally we compute the shunt impedance exciting the quadrupole mode instability with a rise time τ = 100 μs, assuming a full coupling. A single HOM excites instabilities with a rise time given by:

$$\tau_{s,a} = (a-1)!\left(\frac{2}{k_r\sigma}\right)^{2(a-1)}\tau_{s,1}$$

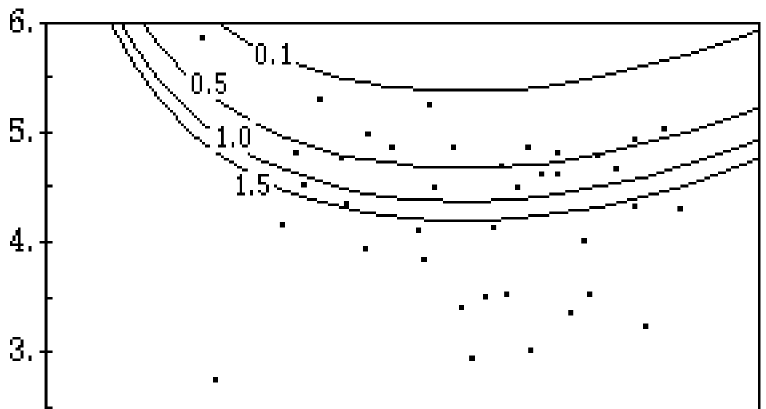

Fig. 6 - **a = 2**: Log_{10} [$R_s(\Omega)$] vs. ω_r (GHz) for τ = 100μs; Io = 0.1, 0.5, 1.0, 1.5 A.

At the moment no feedback system has been envisaged to cure such instabilities, therefore only the Landau damping, estimated of the order of 1 ms, should be taken into account.

7. Transverse Coupled Bunch Instabilities: for the mode "0".

The coherent transverse motion of k_b bunches excites a spectrum analogous to the longitudinal one at frequencies:

$$\omega_{p,a,s} = (k_b p + s + Q_{x,y} + a\nu_s)\,\omega_o$$

For the lowest mode a = 0 we compute the rise time due to a transverse HOM, with shunt impedance R_t (Ω) fully coupled to a sideband:

$$\tau_{s,0}^{min}(\omega_r) = \frac{4\pi (E/e) Q_{x,y}}{k_b I_b \omega_r} \frac{e^{\left(\frac{\omega_r \sigma}{\omega_o R}\right)^2}}{R_t}$$

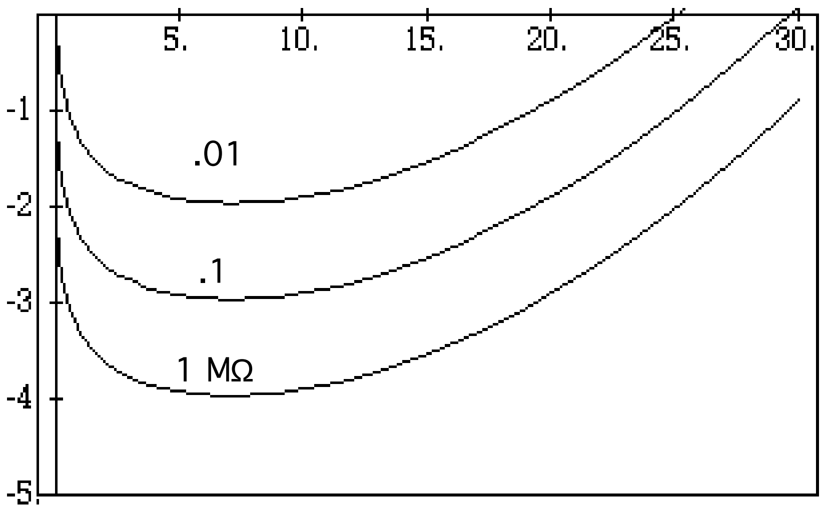

Fig. 7 - Log_{10} [τ_{min}(sec)] vs. ω_r (GHz), (R_t = 1, 0.1, 0.01 MΩ), $k_b I_b \approx 1.5$ A

As one can see the transverse coupled bunch motion is less critical. In fact, by assuming a shunt impedance of 1 MΩ, we have a rise time of 100 μs in the most unfavourable frequency region; this is hundred times better that the rise time obtained for the longitudinal case with the same impedance. In Fig. 8 we show the plot of the shunt impedance calculated for the transverse HOM's of the tapered cavity. There is actually only one point corresponding to 1 MΩ. Few modes have a shunt impedance of 100 kΩ, while all the others are below 10 kΩ. It should be not difficult to damp the transverse instabilities with a feedback system.

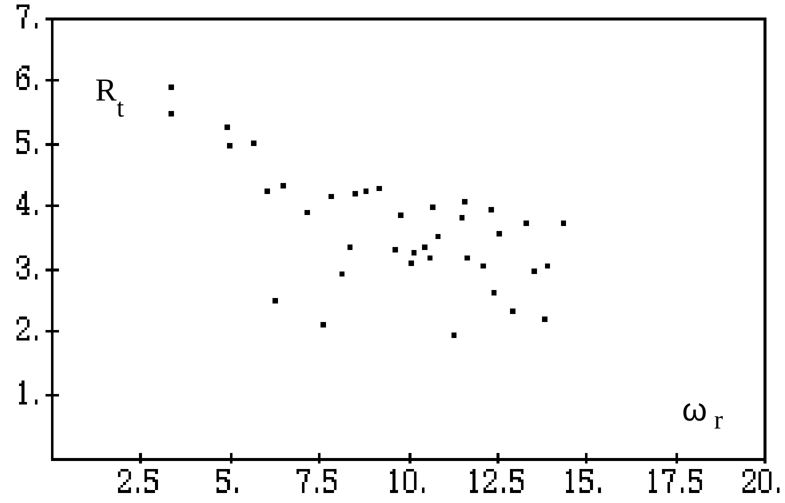

Fig. 8 - Log_{10} [Rt (Ω)] of the tranverse HOM's of the tapered cavity.

8. Conclusions

We present the analysis of the coupled bunch instabilities excited in DAΦNE by equispaced bunches, assuming a single parasitic HOM. We show that the use of a fast feedback system characterized by a damping time of 100 μs can take under control the longitudinal dipole instabilities excited by about 1.5 A, provided the HOM have a resistive impedance lower than 10 kΩ. Less critical appear to be the transverse instabilities, where only few modes seem to be relatively harmful.

For the longitudinal dipole mode, taking as reference a parasitic mode with $R_s/Q_r = 20$, we find that the HOM damping technique ought to reduce the quality factor below 500. On the other side it is also shown that in case a controlled shift of the HOM frequency can be reliably performed under machine operation, the rise time of the fastest instability drops dramatically to unharmful values for a rather small frequency shifts only for the HOM characterized by a high Q.

Analysis of the HOM of the "low loss" cavity proposed for the machine starting operation, shows that despite the small contribution to the overall parasitic loss, many individual modes are characterized by a shunt impedance too high for storing 1.5 Amps. With the present constraints, we do not expect that further optimization's of the cavity shape can significantly reduce the shunt impedances. Furthermore, it seems hard today to think of controlling the frequencies of so many modes, whereas damping techniques based on the absorption of the e.m. energy by means of antennas or waveguides coupler look more promising.

It must be said that the above considerations are derived by a rather pessimistic analysis. In the reality the instability grow rate can be smaller for three main reasons:

- It might be possible to uncouple the bunch and cavity spectra, or at least reduce the coupling, acting on the thermal equilibrium or on the tuner position.

- The instability is initially caused by few bunches only, so that a fast feedback has to work against a weaker instability.

- Experience shows that the actual HOM shunt impedances are smaller than the computed ones. A further improvement can be obtained, for the HOM at high frequencies, by manufacturing the cavity tapers with high resistivity metals. As an example, by using stainless steel one can gain, depending on the intensity of the magnetic fields at the surface, a factor up to 7. Thus, compared to the computed values, a significative reduction of the shunt impedances seems achievable without any damping equipment.

Measurements of HOM on a prototype, and transient coupled bunch instabilities simulations are now necessary in order to verify these expectations.

References

[1] Proposal for Φ-Factory, LNF-90/031 (R).
[2] M. Serio, Preliminary results on Feedback design.
[3] J.M.Wang, BNL - 513002.
[4] R. Boni et al., Preliminary results loaded waveguides HOM damping.
[5] Bartalucci, et al., DAΦNE Technical Note, G-6.