DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, Sept. 2, 1991

Note: **G-7**

ENERGY LOSS DUE TO THE BROAD-BAND IMPEDANCE IN DAΦNE

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1. INTRODUCTION

The Broad-Band (BB) Resonator model is often used to fit the actual longitudinal coupling impedance of a storage ring, in order to simplify the evaluation of single-bunch instability thresholds and parasitic losses [1]. Lacking detailed data on the ring impedance, a rough estimate of the parasitic losses can nevertheless be worked out, in spite of the oversimplification implied by the model. In this note we exploit the BB model to make an estimate of the RF power which must be supplied in order to make up the parasitic losses.

2. PARASITIC LOSSES

The ring impedance is summarized by that of a poor-quality resonator with a resonant angular frequency ω_r, shunt impedance R_s and quality factor Q~1:

$$\hat{Z}_{BB}(j\omega) = \frac{R_s}{1 - j\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \quad ; \tag{1}$$

ω_r is usually chosen (somehow arbitrarily) equal to the lowest cut-off frequency of the TM waveguide modes propagating in the vacuum pipe.

The normalized impedance (Z_{BB}/n) is often used in instability calculations. $n = \omega/\omega_0$ is the mode number, i.e. the ratio between the (angular) frequency ω and the revolution frequency ω_0. The magnitude of the normalized impedance $|(Z_{BB}/n)|$ is almost constant in the region between DC and ω_r. This leads to the customary definition of a "ring BB impedance" $(Z_{BB}/n)_0$, descriptive of the impedance per unit length of a ring, independent of the machine size and useful for comparison and scaling purposes.

$(Z_{BB}/n)_0$ and R_s are related by

$$\left(\frac{Z_{BB}}{n}\right)_0 = R_s\left(\frac{\omega_0}{\omega_r}\right) \quad . \tag{2}$$

In order to evaluate the parasitic losses, we assume a gaussian distribution of the instantaneous bunch current $i(t)$:

$$i(t) = I_p \exp\left(-t^2/2\sigma_t^2\right) \quad ; \quad I_p = \frac{<I>T_0}{\sqrt{2\pi}\,\sigma_t} \tag{3}$$

where <I> is the average bunch current, $T_0 = \omega_0/2\pi$ the revolution period and σ_t the rms bunch duration. The spectral density of the bunch current is

$$I(\omega) = <I>T_0 \exp\left(-\omega^2\sigma_t^2/2\right) \quad . \tag{4}$$

The total energy loss E_{BB}, due to the interaction of the bunch with the ring BB impedance, is

$$E_{BB} = \frac{1}{\pi}\int_0^\infty |I(\omega)|^2 \,\mathrm{Re}\left\{\hat{Z}_{BB}(\omega)\right\} d\omega = \left(<I>T_0\right)^2 k_{BB}(\sigma_t) \quad , \tag{5}$$

where Re stands for “real part of ” and $k_{BB}(\sigma_t)$ is the loss parameter, depending solely on geometrical factors and on the bunch length,

$$k_{BB}(\sigma_t) = \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left\{\hat{Z}_{BB}(\omega)\right\} \exp(-\omega^2\sigma_t^2)\, d\omega \qquad [V/C] \tag{6}$$

i.e. the resistive part of the BB impedance weighted by the beam power spectrum.

According to the above terminology, we have:

E_{BB} = energy lost by one bunch (bunch current = <I>)

$$E_{BB} = \left(<I>T_0\right)^2 k_{BB}(\sigma_t) \quad ; \tag{7}$$

U_{BB} = energy lost by one particle (N_e = Number of particles/bunch, e = electron charge)

$$U_{BB} = \frac{E_{BB}}{N_e} = e < I > \frac{2\pi}{\omega_0} * k_{BB}(\sigma_t) \quad ; \tag{8}$$

W_B = power lost by one bunch

$$W_B = < I >^2 \frac{2\pi}{\omega_0} * k_{BB}(\sigma_t) \quad ; \tag{9}$$

W_t = total power lost by n_b bunches (total current = n_b<I>)

$$W_T = n_b < I >^2 \frac{2\pi}{\omega_0} * k_{BB}(\sigma_t) \quad . \tag{10}$$

3. EVALUATION OF THE LOSS PARAMETER [1]

According to (1) and (6), k_{BB} is

$$k_{BB}(\sigma_t) = \frac{R_s}{\pi} \int_0^\infty \frac{\exp\left(-\omega^2 \sigma_t^{\ 2}\right)}{1 + \left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)^2} d\omega \quad ; \tag{11}$$

using (2) and letting $x = \omega\sigma_t$ and $x_r = \omega_r \sigma_t$, we can write

$$k_{BB} = \left(\frac{Z}{n}\right)_0 \left(\frac{\omega_r}{\omega_0}\right) \frac{1}{\pi\sigma_t} \int_0^\infty \frac{x^2 x_r^2 \exp\left(-x^2\right)}{x^4 - x^2 x_r^2 + x_r^4} dx = \left(\frac{Z}{n}\right)_0 \left(\frac{\omega_r}{\omega_0}\right) \frac{1}{\pi\sigma_t} * A \quad . \tag{12}$$

We have evaluated the loss parameter for several values of the resonant frequency ω_r , with an rms bunch length σ_t =100 ps and at a nominal bunch current of 46 <mA>, which are the DAΦNE design parameters. The results obtained are summarized in Fig. 1 and in Table I.

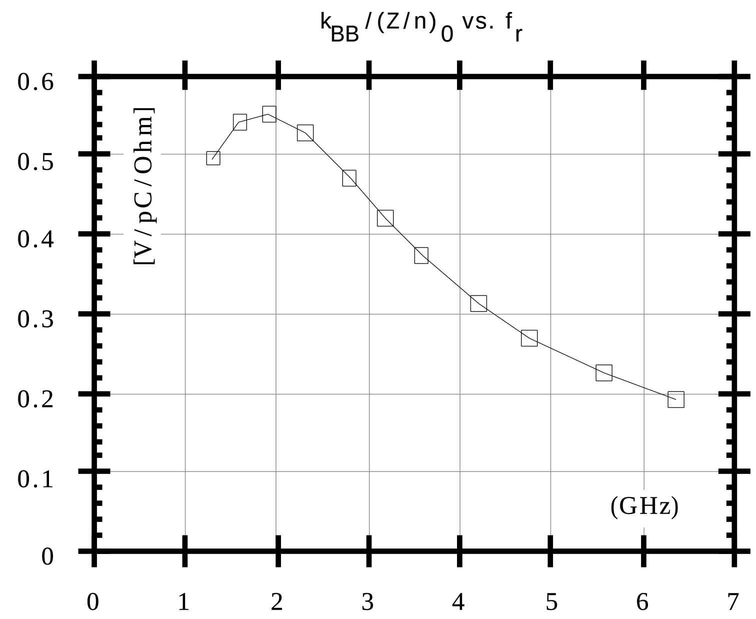

Fig.1 - Loss parameter per unit impedance as a function of the resonant frequency f_r of the model BB resonator.

TABLE I

<I> = 46 mA n_b = 120

$\omega_r/2\pi$ [GHz]	x_r (σ_t=100 ps)	A	$k_{BB}/(Z/n)_0$ [V/pC/Ω]	$U_{BB}/(Z/n)_0$ [KeV/Ω]	$W_B/(Z/n)_0$ [KW/Ω]	$W_t/(Z/n)_0$ [KW/Ω]
1.3	0.817	0.3803	0.496	7.192	0.331	39.7
1.59	1	0.3397	0.542	7.859	0.361	43.4
1.91	**1.2**	**0.2884**	**0.553**	**8.018**	**0.369**	**44.3**
2.31	1.45	0.2276	0.528	7.656	0.352	42.3
2.79	1.75	0.1687	0.472	6.844	0.315	37.8
3.18	2	0.1318	0.421	6.104	0.281	33.7
3.58	2.25	0.1041	0.374	5.423	0.249	29.9
4.2	**2.64**	**0.0743**	**0.313**	**4.538**	**0.209**	**25.1**
4.77	3	0.0563	0.269	3.900	0.179	21.5
5.57	3.5	0.0403	0.225	3.262	0.150	18.0
6.37	4	0.0302	0.193	2.798	0.129	15.4

4. CONCLUSIONS

Assuming a rectangular shape of 40x80 mm^2 for most length of the vacuum chamber, the cut-off frequency of the lowest TM mode is ~ 4.2 GHz. The resonant frequency of the model BB resonator should be at or near this value. However, the estimated loss parameter (at σ_t = 100 ps) varies largely according to the resonant frequency of the model resonator, with a maximum at ~ 2GHz.

Therefore, if we choose this last value for the model resonant frequency, we obtain a conservative estimate of the upper limit of power losses at the nominal bunch current and duration: we suggest to be conservative and use for all further evaluations the results obtained with a resonant frequency around ~ 2GHz, where the loss parameter is a maximum.

REFERENCES

[1] A. Hofmann, J. R. Maidment - LEP Note 168 (28/6/79).