DAΦNE TECHNICAL NOTE

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THE CRAB CAVITY IN DAΦNE: DESIGN CRITERIA

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The crab-crossing scheme is one of the optional tools which have been proposed [1] to fine tune the luminosity in DAΦNE.

In both rings, a RF cavity, which is located at 90° betatron phase advance from the interaction point (IP), is used to horizontally tilt the bunch about its centre, thereby making it collide head-on with the opposite bunch at the IP. A symmetrical cavity cancels the tilt after the interaction. This solution should avoid excitation of synchrobetatron resonances, which severely limit the luminosity in all storage ring colliders with finite crossing angle.

We have examined the general features of the deflecting cavity, with the only requirement on the operating frequency to be about 350 MHz. A possible model for such a cavity is presented. More technical problems, which have to be faced in a real design, will be considered subsequently.

1) Optimum choice of the deflecting mode

As it is well-known pure TE (transverse electric) modes do not produce any angular deflection of a charged particle, but only a shift of its transverse position. However, such modes do exist only in pill-box ideal cavities, while in real structures there are always three non-zero components for both $\vec{E}$ and $\vec{H}$ dipolar fields. In a real structure there are always two modes which can be good candidates for the deflecting mode: they are usually identified as TM_{110} and TE_{111} and are close to each other in frequency. Both couple to the beam, magnetically the first, electrically the latter. An optimization criterion is given by the transverse shunt impedance $R_\perp$ [2]:

$$R_{\perp n} = \frac{\left| \frac{1}{k_n} \int \vec{\nabla}_\perp E_{nz}\Big|_0 e^{j\frac{\omega}{v}z} dz \right|^2}{2\, P_n} = \frac{\left|V_{\perp n}\right|^2}{2P_n} \qquad [\Omega]$$

where n refers to a given mode and $\vec{\nabla}_\perp E_{nz}\Big|_0$ is the transverse gradient of the longitudinal electric field evaluated on the cavity axis $(r=0)$.

Note that even if E_z is zero on the axis, in general its gradient will not be. In cylindrical coordinates, $\vec{\nabla}_\perp$ is expressed as $\vec{\nabla}_\perp \equiv \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}$. If we choose the plane $\theta = 0$ to be the deflection plane, the θ-component of the gradient is zero, hence $\vec{\nabla}_\perp E_{nz}\,|_0 = \hat{r}\frac{\partial}{\partial r} E_{nz}\,|_{r=0}$.

If we now look at the definition of (R/Q) as computed by URMEL at the radial position r,

$$(R/Q)_n = \frac{1}{(k_n r)^2} \frac{1}{2\,P_n Q_n} \left| \int_0^L E_{nz}(z,r)\, e^{j\frac{\omega}{v}z} dz \right|^2 \qquad [\Omega]$$

we easily see that, in a region close to $r = 0$, we can linearize $E_{nz}(r,z)$ as $r\frac{\partial}{\partial \theta} E_{nz}\Big|_{r=0}$ and introducing into the above formula we get

$$(R/Q)_n = \frac{1}{(k_n r)^2} \frac{1}{2\,P_n Q_n} r^2 \left| \int_0^L \frac{\partial\, E_{nz}(z)}{\partial r}\,\Big|_0\, e^{j\frac{\omega}{v}z} dz \right|^2 = \frac{R_{\perp n}}{Q_n}$$

so we can compare the deflecting power of the two modes using URMEL results.

2) Calculation of the required deflecting voltage

The change in the transverse momentum produced by the deflecting fields is given by:

$$\Delta p_\perp = \mathrm{Re}\left\{ \int_0^T F_\perp(z,t)\, e^{j(\omega t+\phi)} dt \right\} = \mathrm{Re}\left\{ \int_0^T e\left[E_\perp(z) - j\,\mu\left(\vec{v}\wedge\vec{H}\right)_\perp(z)\right] e^{j(\omega t+\phi)}\, dt \right\}$$

where T is the particle transit time through the cavity gap of length L. Although we don't know the analytical expression of the field profile of a real, axially symmetrical cavity we still can draw some conclusions about its general behaviour. Without loss of generality we write:

$$\Delta p_\perp = \frac{e}{c}\int_0^L \left\{ E_\perp(z)\cos\left(kz+\phi_0+\frac{\omega_{RF}\,s}{c}\right) + \mu\left(\vec{v}\wedge\vec{H}\right)_\perp(z)\sin\left(kz+\phi_0+\frac{\omega_{RF}\,s}{c}\right)\right\}dz$$

assuming the relativistic particle to be at an absolute longitudinal position z=0 at the time t=0; furthermore we neglect the transverse variation of the e.m. fields along the particle trajectory and we schematize the cavity as a pointlike kicker. The phase ϕ of a particle at a longitudinal position s, relative to the bunch centre, is given by $\phi = \phi_0 + \frac{\omega_{RF}\,s}{c}$, where ϕ_0 is the proper phase which ensures no net deflection of the bunch centre (zero-crossing phase). It is easy to see that such phase does exist and $\phi_0 = -\frac{\omega L}{2c}$ in the case of an axially symmetrical cavity with field configuration such that $\overrightarrow{E_\perp}$ is zero both at the centre and at each end of the cavity, as dictated by Maxwell's equations. Thus, after some algebra:

$$\Delta p_\perp = \frac{e}{c}\int_0^L \left[E_\perp(z)\cos\left(kz+\phi_0\right)+\mu\left(\vec{v}\wedge\vec{H}\right)_\perp(z)\sin\left(kz+\phi_0\right)\right]\cos\left(\frac{\omega_{RF}\,s}{c}\right)dz \; -$$

$$-\frac{e}{c}\int_0^L \left[E_\perp(z)\sin\left(kz+\phi_0\right)-\mu\left(\vec{v}\wedge\vec{H}\right)_\perp(z)\cos\left(kz+\phi_0\right)\right]\sin\left(\frac{\omega_{RF}\,s}{c}\right)dz$$

where the first integral is 0 by definition of the zero-crossing phase ϕ_0, while the latter is the maximum effective deflecting voltage, as corrected by the transit time factor and corresponding to a phase $\phi = \frac{\pi}{2} - \phi_0$ Thus we can write: $\Delta p_\perp = \frac{eV_{RF}}{c}\sin\left(\frac{\omega_{RF}s}{c}\right)$.

The required deflection angle is given by $\theta = \frac{s\tan\alpha}{\sqrt{\beta_c\beta^*}}$ where α is half the crossing angle and, since $\Delta p_\perp \approx \theta\, p$ for relativistic particles, we finally obtain the well-known formula of the deflecting voltage for small displacements s: $V_{RF} \approx \frac{cE}{e}\frac{1}{\omega_{RF}}\frac{\tan\alpha}{\sqrt{\beta_c\beta_x}}$.

With the typical DAΦNE parameter values, α = 10 mrad, E = 510 MeV, $\beta_c \cong 10$ m, β_x = 4.5 m, we have $V_{RF} \cong 100$ kV.

For the dipole mode TM_{110} in a pill-box, we have found that

$$\Delta p_\perp \propto \frac{2}{c}\ [\sin(\frac{\omega L}{2c} + \phi) \sin\frac{\omega L}{2c}]\ .$$

If $\phi = \phi_0 = -\frac{\omega L}{2c}$ there's no deflection. The power series development around ϕ_0 reads:

$$\Delta p_\perp \propto \frac{2}{c}\ \sin\frac{\omega L}{2c}\ (\phi - \phi_0)\ \left[1 - \frac{(\phi - \phi_0)^2}{6} + ...\right] = \frac{2}{c}\ \sin\frac{\omega L}{2c}\ \frac{2\pi s}{\lambda}\ \left[1 - \frac{\left(\frac{2\pi s}{\lambda}\right)^2}{6} + ...\right]$$

where λ is the cavity wavelength. With our design parameters σ_s = 3 cm (σ_s is the r.m.s. bunch length), λ = 85 cm, the contribution of the higher order term is less than 1% of that of the first term for a particle at s = 1σ_s. Thus we have a momentum change which is proportional to the particle distance from the bunch centre with good approximation. The amount of non-linearity that may be tolerated has been investigated by computer simulation [3], thereby showing that only particles at large longitudinal amplitudes ($\geq 3\sigma_s$) are affected by synchrobetatron resonances (their transverse amplitude is increased by more than 100%) for DAΦNE.

Finally, we would like to remark that in a pill-box there is no transverse electric field for the mode TM_{110}, whereas in a real structure there is also a non-negligible component of the electric field E_r on the axis, which is orthogonal to the magnetic (deflecting) field H_θ. Note that the longitudinal profile of this component is different from that of H_θ, as a consequence of Maxwell's equations. We checked that this component does not add any non-linear quadratic term to the deflection by the magnetic field.

We made detailed calculations in the case of a pill-box excited in a TM_{11p} mode and found that the contributions of E_r and H_θ add always in the same way, independent on the direction of particle's velocity, and are both linearly dependent on the phase ϕ. Of course, the sign of the global deflection is changed by reversing particle's velocity. We omit these calculations for the sake of brevity. In the case of a real structure, owing to the symmetry properties of the e.m. fields, the same result applies, but the relative contributions of E_r and H_θ cannot be evaluated separately.

3) <u>An example of a crab cavity</u>

In DAΦNE, the requirement on the crab cavity are certainly less stringent than in the other larger $e^+ e^-$ colliders, which are presently under study. The peak voltage needed is only 100kV and the various tolerances are in general quite loose. However, the problem of getting rid of the undesired modes is by no means simpler in our case. In fact, in addition to the TM_{110} mode which is used to tilt the bunch, there exist two modes, which are always trapped in the cavity , and have to be regarded as quite dangerous: the other TM_{110} whose orientation is rotated axially by $\frac{\pi}{2}$ with respect to the deflecting one, and the accelerating TM_{010}, which has a big shunt impedance and needs special attention.

In [4] it has been suggested to consider, as a candidate for the crab cavity, a cell-shaped structure with very large beam holes, like the single-mode cavity proposed by Weiland [5]. He claimed that this structure would keep only a couple of dipolar modes trapped, at the price of a reduced shunt impedance of the fundamental mode. All the other HOMs would propagate through the beam ducts and be easily damped therein.

A similar approach is now widely followed by several laboratories towards a HOM-free resonating structure for the various B-Factory projects. Although this design in certainly best suited for SC cavities, where high voltages have to be developed, we think it is of interest also for us in order to get a 'feeling' of the actual requirements which have to be fulfilled. In fact, there certainly are several different methods of getting rid of the undesired modes (tuning, damping, RF feedback, etc), which all have to be looked at, but we still believe that a careful design of the basic structure is the best starting point .

In the following we present URMEL and TBCI results for a 'Weiland-like' single cell with very long exiting tubes. The geometry has been chosen such that the cavity resonates at f_{RF}=357 MH_z in the TM_{110} mode. URMEL results for the first few dipolar (monopolar) modes are shown in Table I (II).

The cutoff frequency of the beam holes is 371 MH_z. By inspection of Table I and II, we see that only a monopole (the accelerating one) and the two dipole TM_{110} and TE_{111} are left below cutoff. For each dipole mode there are two degenerate, orthogonally polarized modes, which split up in a real cavity, where the cylindrical symmetry is broken. Appropriate measures have to be taken in order to avoid mode rotation, anyway.

TABLE I

SUMMARY OF ALL MODES FOUND (FULL CELL RESULTS)

(VOLTAGE INTEGRATED AT R0= 0.237 METER OFF AXIS)

MODE TYPE	FREQUENCY (MHZ)	(R/Q) (OHM AT RO)	(R/Q)/(K*RO)**2M (OHM)	ACCURACY	CONTAMINATION
1-EE- 1	356.988	26.118	8.306	3.1E-04	0.011428
1-EE- 2	409.267	7.336	1.775	3.2E-04	0.013890
1-EE- 3	477.226	3.641	0.648	4.0E-04	0.016109
1-EE- 4	577.387	1.523	0.185	6.1E-04	0.019803
1-ME- 1	321.535	7.446	2.919	4.7E-04	0.009266
1-ME- 2	416.931	0.197	0.046	4.8E-04	0.012192
1-ME- 3	506.067	0.453	0.072	1.8E-04	0.005700
1-ME- 4	588.084	8.727	1.023	9.0E-04	0.036380

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URMEL ENDED WITHOUT ERROR(S)/WARNING(S)

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** 0 ** ERRORS DETECTED
** 0 ** WARNINGS ISSUED

TABLE II

SUMMARY OF ALL MODES FOUND (FULL CELL RESULTS)

(VOLTAGE INTEGRATED AT R0= 0.000 METER OFF AXIS)

MODE TYPE	FREQUENCY / MHZ	(R/Q)/OHM AT R0	ACCURACY	CONTAMINATION
TM0-EE- 1	267.611	40.481	6.7E-04	0.006229
TM0-EE- 2	490.117	18.250	2.7E-04	0.014562
TM0-EE- 3	525.520	14.703	2.5E-04	0.019976
TM0-EE- 4	575.020	8.236	2.7E-04	0.017589
TM0-ME- 1	487.300	15.899	2.1E-04	0.022399
TM0-ME- 2	511.125	0.084	1.7E-04	0.019110
TM0-ME- 3	566.603	4.727	1.6E-04	0.009178
TM0-ME- 4	649.136	13.213	1.1E-04	0.004614

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URMEL ENDED WITHOUT ERROR(S)/WARNING(S)

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** 0 ** ERRORS DETECTED
** 0 ** WARNINGS ISSUED

For TM_{110} we have: $(R/Q) = 8.3\,\Omega$, $Q = 55000$, while for TE_{111} we have: $(R/Q) = 2.9\,\Omega$, $Q = 64000$, hence the the reason why TM_{11p} modes are preferred, although their Q's are lower than those of TE_{11p} modes. The power dissipation for copper cavities is 11kW and 27 kW, respectively.

The field distribution of mode TM_{110} is shown in Fig. 1, together with the geometry of half the cavity. The field distribution of mode TE_{111} is shown in Fig. 2 for comparison.

To confirm the reduction of HOM effects on particle dynamics, we ran also TBCI for monopolar and dipolar wakefields. We chose $\sigma_s = 3$ cm for the bunch length and $N_b = 9*10^{10}$ electrons for the bunch charge. The resulting energy loss per particle is 932 eV for the monopolar modes and 3 eV for the dipolar modes acting on a particle at 1 cm off-axis. This value may seem quite high, as compared with the radiated energy per turn, but one has to bear in mind that it is proportional to the bunch charge, which is very big in our case. The corresponding wake potentials are shown in Fig. 3. A more detailed study of beam dynamics under the transverse wakefield will be the subject of a subsequent note.

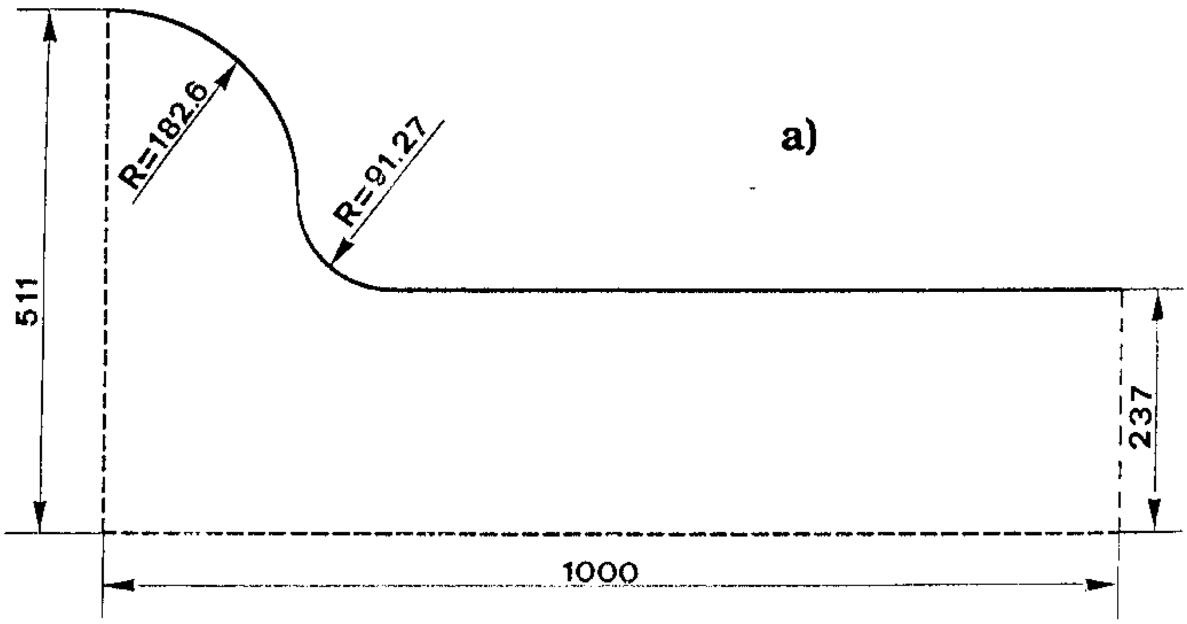
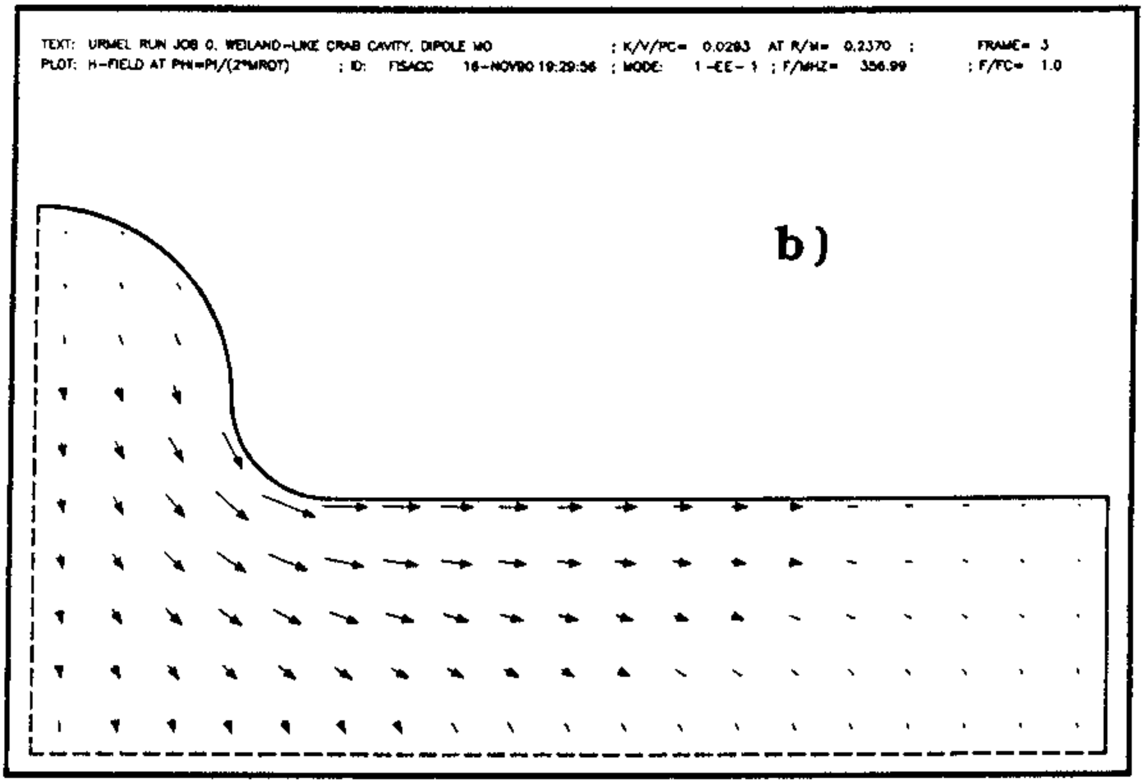
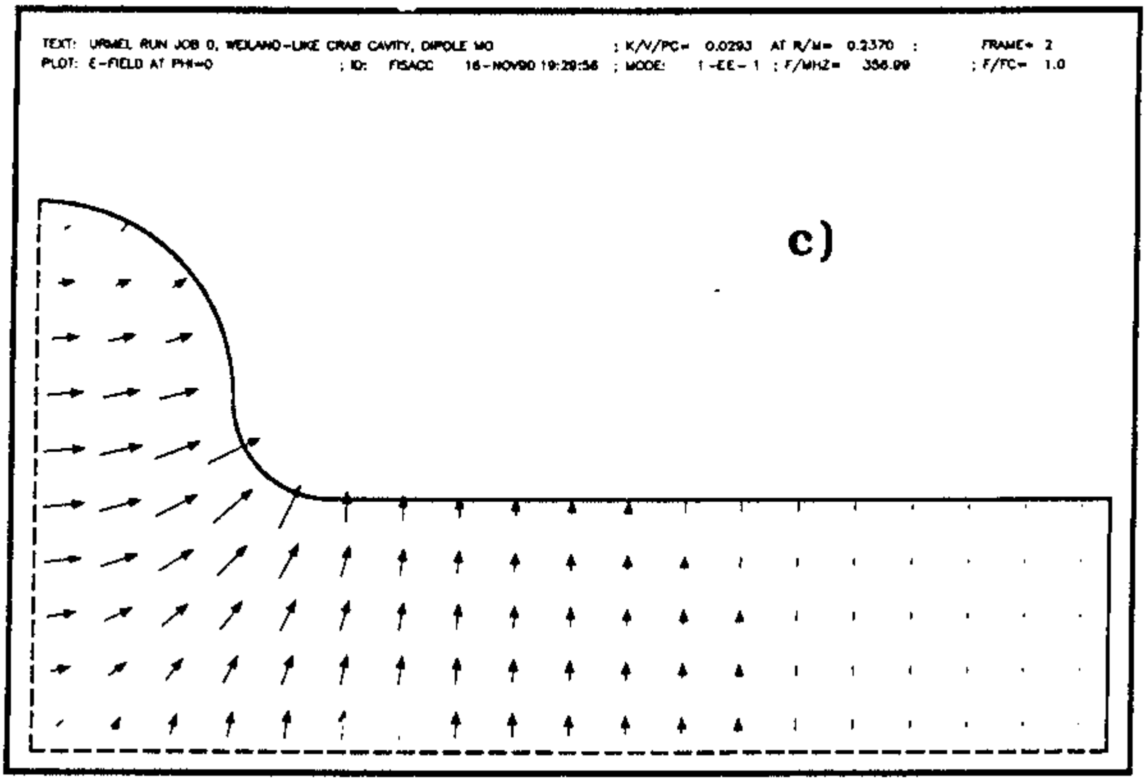
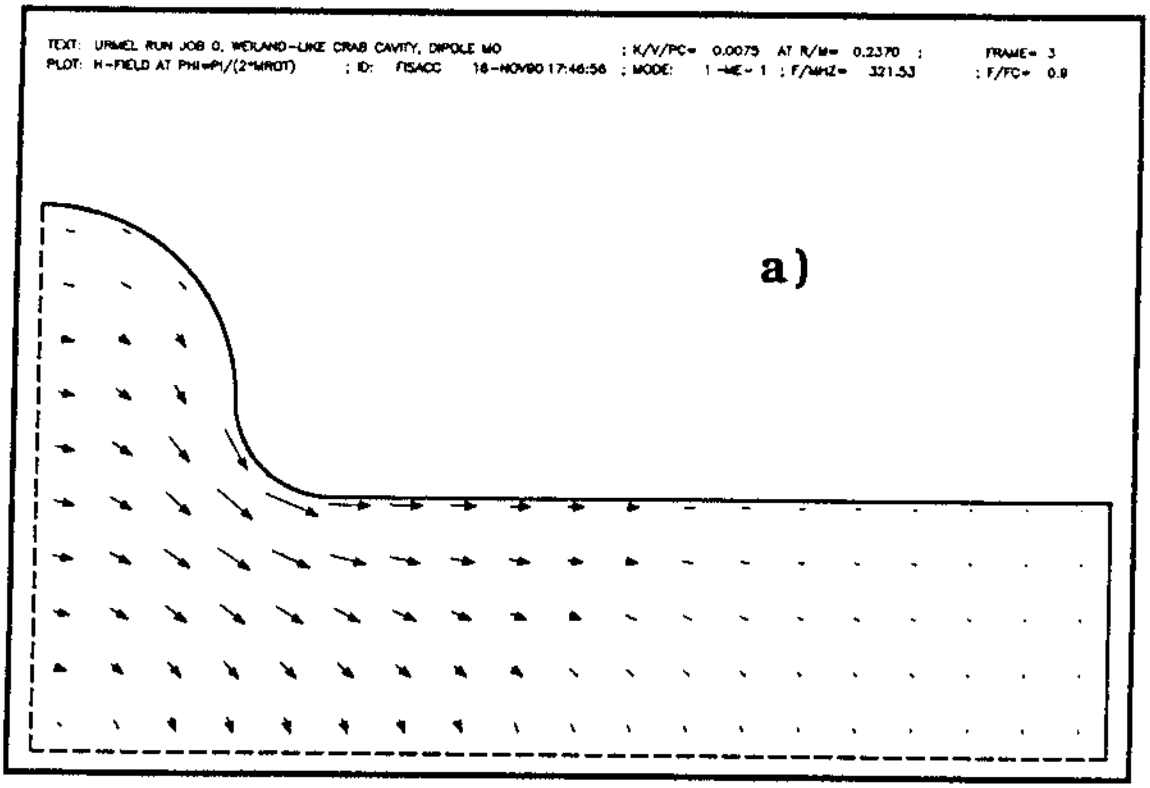


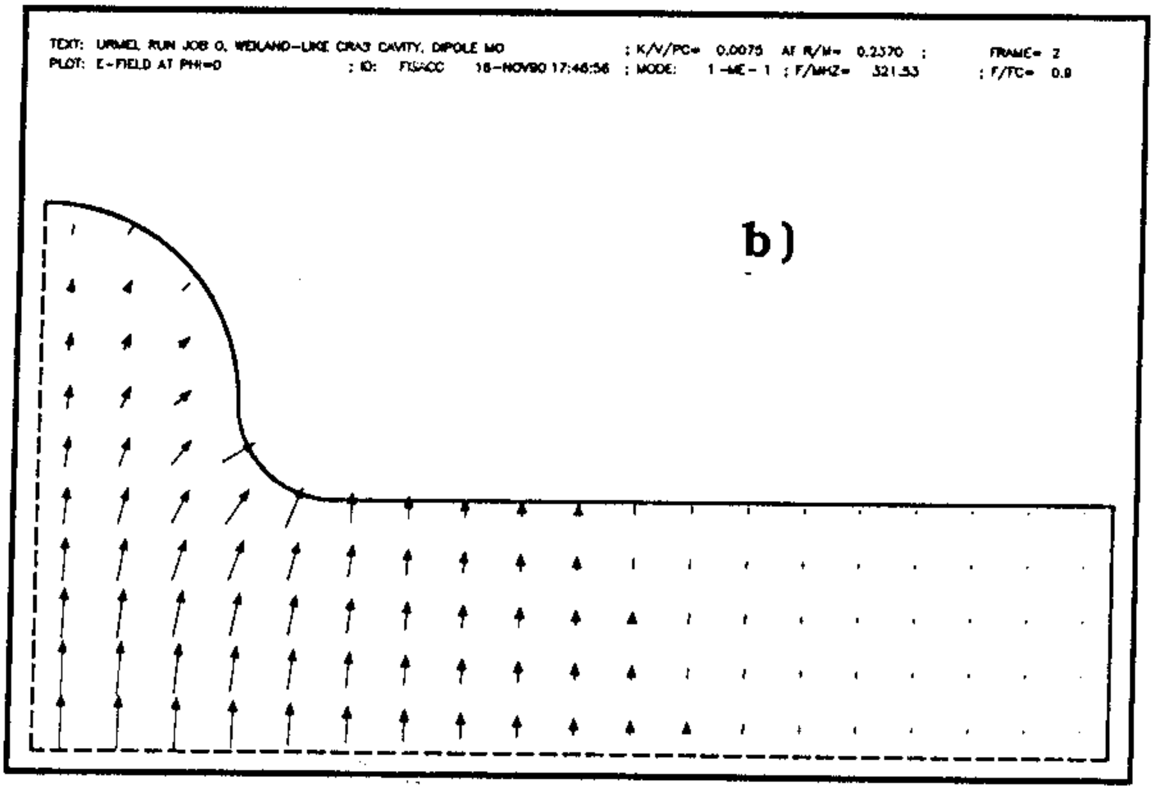
*Fig. **1** a) - Cross section of a half deflecting cavity.*



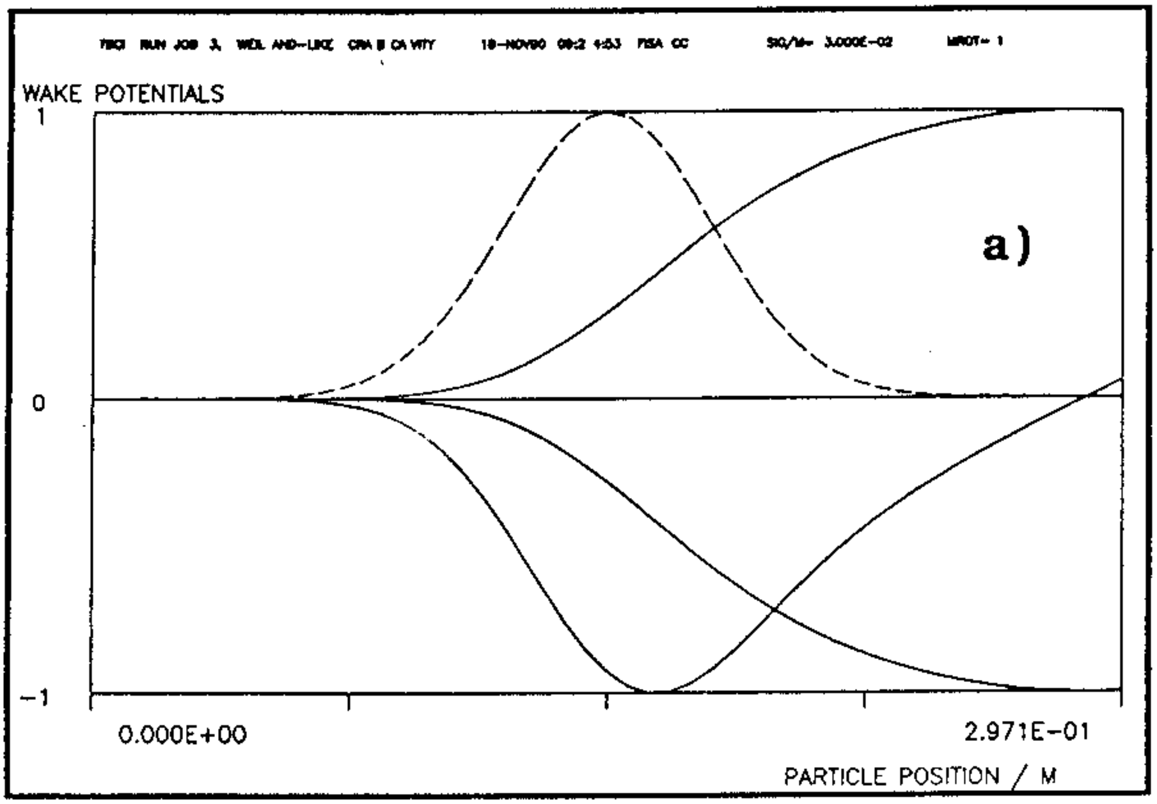


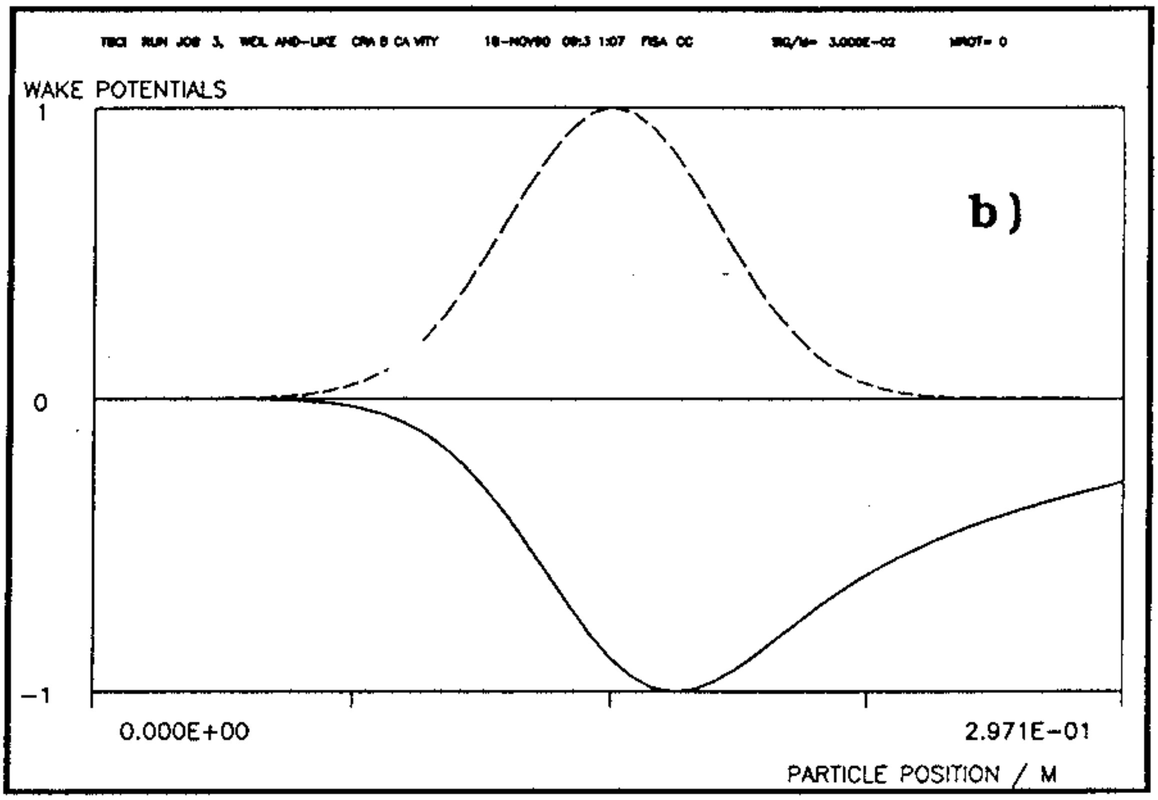
*Figs. **1** b) and 1 c) - Magnetic and electric field distribution of the dipole mode TM_{110}, respectively at $\varphi = 90°$ and $\varphi = 0°$.*





***Figs. 2** a) and 2 b) - Magnetic and electric field distribution of the dipole mode TE_{111}, respectively at $\varphi = 90°$ and $\varphi = 0°$.*





*Figs. **3** a) and 3 b) - Monopolar and dipolar wake potentials.*

4) Conclusion

A very preliminary investigation of a possible crab cavity has been performed. Basic ideas and physical principles have been presented. A computer calculation of the mode structure for a reasonable model of the cavity has been presented as well. Since the power and tolerance requirements in our crab crossing scheme are not very tight, a safe design of the geometry is possible.

We believe that the major concern for this work will be the damping/tuning of the fundamental (accelerating) and the other three trapped modes, while the damping of the other undesired (propagating) HOMs should not bring about too much difficulty.

References

1) Proposal for a Φ-Factory, p. 365, LNF-90/031 (R) (1990).

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3) A. Piwinski, DESY Note HERA 90-04 (1990).

4) S. A. Kheifets, J.M. Paterson and G.A.Voss, SLAC-PUB-5011 (1989).

5) T.Weiland, DESY Report 83-073 (1983).