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OPTICAL ANALYSIS OF THE SYNCHROTRON RADIATION MONITORS IN DAΦNE

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Introduction

The radiation range that will be used in DAΦNE synchrotron light monitors has been suggested to be 400 ÷ 600 nm [1]. In order to minimize the chromatic aberration two achromats will be used to image the electron and positron beams onto the photocathodes of detectors. By using achromats other kinds of aberrations will also be much smaller. Unfortunately, every achromat has some residual chromatic aberration that must be analysed in order to get a diffraction limited measurement of the beam size. Given the commercially available achromats such analysis will be done in this paper and the result shows that it is possible to set up an optical system with achromats to keep the aberration error smaller than the errors of beam itself, the diffraction limit error and the depth of field error[1]. Other related issues will also be discussed in this paper.

This note includes: 1 - Chromatic aberration and its connection with the beam curvature error and depth of field error; 2 - Geometric aberrations; 3 - Quality of mirrors; 4 - Size of the tilted mirror and of the second lens; 5 - Discussion of a possible phenomena during the detector position tuning.

1. Chromatic Aberration

1.1. The aberration of the two-achromat system

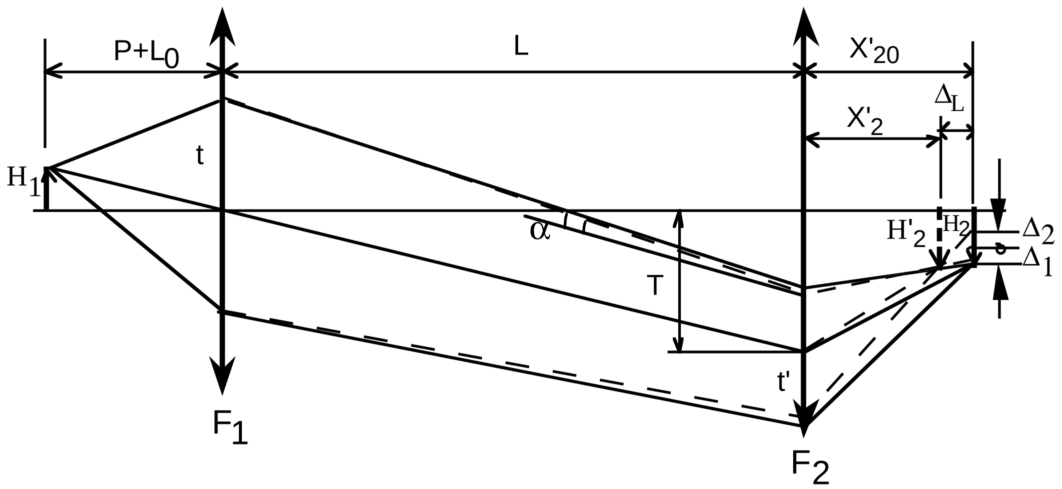

Figure 1 - Schematic drawing of the synchrotron monitor optical system.

A typical two-achromat system is shown in Fig. 1, where F_1 and F_2 represent two achromats. The physical meanings of all the symbols in Fig. 1 are:

H_1	Transverse beam dimension in storage ring.
H_2	Image of H_1 without chromatic aberration.
H'_2	Image of H_1 at a specific wavelength with chromatic focal length shifts on both lenses.
Δ_L	Image longitudinal deviation due to of chromatic aberration.
t	Illuminated area on the first achromat by a point source of synchrotron light. In vertical plane, $t = (p+L_0)\psi_{typ}$, where ψ_{typ} is the divergence of the synchrotron light; In horizontal plane, t = a, the half width of the entrance slit.
L_1	The ideal focal length, or average focal length of the first achromat.
L_2	The ideal focal length, or average focal length of the second achromat .
$f_1 = L_1+\delta_1$	The focus of F_1 at a given wavelength, δ_1 is the chromatic focus shift.
$f_2 = L_2+\delta_2$	The focus of F_2 at a given wavelength, δ_2 is the chromatic focus shift.
L	Distance between the two lenses.
P	Longitudinal distance from beam to the ideal focal position of F_1. P may be caused by longitudinal misalignment or be set deliberately in order to change the image magnification.
β_0	Transverse magnification.

Because of the separation between H_2 and H'_2, a point beam source will have an image spot on the vertical plane at H_2 position, with the vertical aberration parameters:

Δ_2	Half width of the spot.
Δ_1	Spot center deviation to the ideal image point, or the average image point.

The formulas of Δ_1 and Δ_2 for such system are given in Appendix A. With a set of commercially available data of F_1 and F_2 [2]: L_1 = 2254.18 mm, L_2 = 1250.70 mm, δ_1 = 1.17 mm, and δ_2 = 0.61 mm for wavelength from 488 nm to 633 nm, we get the aberration parameters in Table 1. Other parameters used for the calculation of Table 1 are: L = (25000 - P) mm, a = 2 mm, ψ_{typ} = 0.00468 rad, beam vertical size H_{1v} = 0.28 mm, horizontal size H_{1h} = 2.529 mm. In Table 1, the spot center relative shift is valid for both the vertical and horizontal images, i.e., $\Delta_{1h}/H_{2h} = \Delta_{1v}/H_{2v} = \Delta_1/H_2$. From reference [1], the relative errors without image aberration are: vertical ΔY = 0.02901, horizontal ΔX = 0.007955. The total relative errors in Table 1 are got with: $\Delta Y_t = [(1+\Delta Y)^2 + \Sigma(\Delta_v/H_v)^2]^{0.5} -1$ and $\Delta X_t = [(1+\Delta X)^2 + \Sigma(\Delta_h/H_h)^2]^{0.5} -1$.

(i) Comparing the last two rows in the table with the original ΔY and ΔX, it can be seen that the image aberration has little influence on the total image relative errors. The total errors are still mainly determined by the parameters of the beam itself, the geometric errors and diffraction errors

(ii) In horizontal direction Δ_1 is the dominant chromatic error and in vertical plane Δ_2 is the dominant term.

(iii) The biggest relative error caused by chromatic aberration is the vertical relative spot size Δ_{2v}/H_{2v}.

Here we suppose that the detector is put on one end of the longitudinal aberration spot. If we tune the detector to the middle of the spot, the relative aberration error in Table 1 will be half.

Table 1. Typical parameters for the two-lens system (unit: mm)

Parameters	P = -100 mm	P = -50 mm	P = -20 mm	P = -10 mm	P = 0 mm
Longitudinal deviation Δ_L	0.7932	0.8569	0.9163	0.9415	0.9702
Spot center shift Δ_1/H_2	0.01485	0.01223	0.01073	0.01024	0.009749
Horizontal spot size Δ_{2h}/H_{2h}	0.001920	0.001466	0.001239	0.001170	0.001105
Vertical spot size Δ_{2v}/H_{2v}	0.08756	0.06834	0.05853	0.05554	0.05268
Magnification H_2/H_1 (β_0)	0.3894	0.4578	0.5115	0.5323	0.5548
Image position $X'_2(\Delta_L=0)$	1272.30	1263.40	1256.38	1253.65	1250.70
Total relative error-horizontal	0.00807	0.00803	0.00801	0.00801	0.00800
Total relative error-vertical	0.03283	0.03135	0.03073	0.03056	0.03040

p = 0 mm	p = 10 mm	p = 20 mm	p = 50 mm	p = 100 mm	Parameters
0.9702	1.0030	1.0409	1.1957	1.7313	Longitudinal deviation Δ_L
0.009749	0.009268	0.008792	0.007393	0.005154	Spot center shift Δ_1/H_2
0.001105	0.001043	0.000985	0.000830	0.000633	Horizontal spot size Δ_{2h}/H_{2h}
0.05268	0.04995	0.04736	0.04041	0.03151	Vertical spot size Δ_{2v}/H_{2v}
0.5548	0.5793	0.6061	0.7032	0.9584	Magnification H_2/H_1 (β_0)
1250.70	1247.49	1243.97	1231.19	1197.53	Image position $X'_2(\Delta_L=0)$
0.00800	0.00800	0.00799	0.00798	0.00797	Total relative error-horizontal
0.03040	0.03026	0.03014	0.02983	0.02951	Total relative error-vertical

1.2. The connection with curvature and depth of field errors

Such connection between the chromatic errors and the geometric errors are analysed in Appendix B. The result is:

in horizontal plane:

$$\Delta_1 / H_{2h} = \frac{a}{H_{1h}+a}\,\frac{\Delta_L(\delta_1=0)}{Z'}\,\frac{2L\,L_1}{L_1^2-P(L-L_1)}\left(\Delta X_C / H_{1h}\right)$$

$$\approx \frac{\Delta_L(\delta_1=0)}{Z'}\,\frac{L\,L_1}{L_1^2-P(L-L_1)}\left(\Delta X_C / H_{1h}\right), \quad \text{when } a \approx H_{1h} \tag{1.1}$$

in vertical plane:

$$\frac{\Delta_{2V}}{H_{2V}} = \frac{\Delta_L}{Z'}\,\frac{\Delta X_{DF}}{H_{1V}} \tag{1.2}$$

where the beam image length $Z' = Z\beta_0^2$ and in our system Z = 9.299 mm [1]; ΔX_c is beam curvature error and ΔX_{DF} is depth of field error.

We know form Table 1 that the biggest relative chromatic error is Δ_{2V}/H_{2V}, and from eq. (1.2) we know that if chromatic longitudinal spot size $\Delta_L < Z'$ this chromatic error should be smaller than the depth of field error. The smaller the Δ_L, the smaller the chromatic transverse errors. This also enables us to make a judgment if chromatic aberration will be serious or not just from the size of Δ_L.

Table 2. Comparison between the spot longitudinal size and the image length

Parameters	P = -100 mm	P = -50 mm	P = -20 mm	P = -10 mm	P = 0 mm
$\Delta_L(\delta_1=0)$	0.6313	0.6225	0.6155	0.6129	0.6100
Δ_L	0.7932	0.8569	0.9163	0.9415	0.9702
Beam image length Z'	1.40975	1.94876	2.43310	2.63489	2.86263
Δ_L/Z'	0.5627	0.4397	0.3766	0.3573	0.3389
$\frac{\Delta_L(\delta_1=0)L L_1}{Z'\left[L_1^2-P(L-L_1)\right]}$	3.4258	2.8934	2.5750	2.4691	2.3633

P = 0 mm	P = 10 mm	P = 20 mm	P = 50 mm	P = 100 mm	parameters
0.6100	0.6069	0.6035	0.5911	0.5592	$\Delta_L(\delta_1=0)$
0.9702	1.0030	1.0409	1.1957	1.7313	Δ_L
2.86263	3.12098	3.41565	4.59867	8.54061	beam image length Z'
0.3389	0.3214	0.3047	0.2600	0.2027	Δ_L/Z'
2.3633	2.2575	2.1519	1.8355	1.3100	$\frac{\Delta_L(\delta_1=0)L L_1}{Z'\left[L_1^2-P(L-L_1)\right]}$

$\Delta_L(\delta=0)$, Δ_L, Δ_L/Z' and $[\Delta_L(\delta_1=0)/Z']\{LL_1/[L_1^2-P(L-L_1)]\}$ are listed in Table 2. It can be seen that in our system, in vertical direction, the chromatic error should be several times smaller than the depth of field error because of the small value of Δ_L/Z'; and in horizontal direction, the chromatic error will be ~ 1.5 to 3.5 times as much as the curvature error whose contribution to the total error however is very small [1]. That explains again that the chromatic error of such system should be very small.

1.3. Magnification flexible

From Table 1 we know that we may increase or decrease the image magnification by increasing or decreasing P.

The relative aberrations decease as the magnification increases (see Table 1) because of the decrease of Δ_L/Z' and $\Delta_L(\delta=0)/Z'$. However the photon flux per unit image area (or per photocathode pix) will also decrease as $|\beta_0|$ increases: flux/pix $\propto \beta^{-2}(L+P)^{-1}$.

Once the positions of the two lenses is fixed, the magnification of the system can be experimentally measured. The magnification can also be calculated if two of the three parameters are precisely measured: P, L and X'_{20}.

Using equation (A.11) and the two other β_0 expressions (A.11a, A.11b), the possible calculation errors could respectively be:

$$\frac{\Delta\beta_0}{\beta_0} = \left(1+\beta_0\frac{2L_1+P}{L_2}\right)\frac{\Delta L_1}{L_1} + \left(1+\beta_0\frac{P}{L_1}\right)\frac{\Delta L_2}{L_2} - \frac{\beta_0}{L_1L_2}\left[(L-L_1-L_2)\Delta P + P\Delta L\right] \tag{1.3}$$

$$\frac{\Delta\beta_0}{\beta_0} = \left\{\left[L\left(L_2 - X'_{20}\right) + X'_{20}L_2\right]\frac{\Delta L_1}{L_1} - X'_{20}(L-L_1)\frac{\Delta L_2}{L_2} - \left(L_2 - X'_{20}\right)\Delta L + (L-L_1-L_2)\Delta X'_{20}\right\}\times \frac{1}{L_1L_2\beta_0} \tag{1.4}$$

and

$$\frac{\Delta\beta_0}{\beta_0} = \frac{\Delta L_1}{L_1} + \frac{X'_{20}}{L_2 - X'_{20}}\frac{\Delta L_2}{L_2} - \frac{\Delta X'_{20}}{L_2 - X'_{20}} - \frac{\Delta P}{P} \tag{1.5}$$

The manufacturing tolerance of focal length could be 1% [2]. If we suppose the distance measurement errors be: ΔL = 10 mm, ΔP = 5 mm and $\Delta X'_{20}$ = 1 mm, the r.m.s from all the terms in each equation will be:

β_o	-0.3894 (P=-100 mm)	-0.5548 (P=0 mm)	-0.9584 (P=100 mm)	
$\lvert\Delta\beta/\beta_o\rvert$	0.0184	0.0254	0.0455	with eq.(1.3)
	0.2645	0.1827	0.1017	eq.(1.4)
	0.5930	infinite	0.2317	eq.(1.5)

It seems possible for us to calculate the magnification with a precision ~ 2-5%, depending on P, L or X'_{20} measurement precision, with eq.(A.11).

2. Geometric Aberrations

Geometric aberrations have been classified with different names and their vertical aberration sizes have different relations with optic ray parameters: spherical aberration, $\propto r^3$; coma, $\propto r^2h$; astigmatism and field curvature, $\propto rh^2$; and distortion, $\propto h^3$, where r is the aperture size and h is the image vertical size, the off axis distance[3]. Normally, the optical manufacturing companies design the achromats in such a way to let the Abbe sine conditions be almost satisfied. Therefore both the spherical aberration and coma are very small when the wavelength is close to the design one (page A24 of [2]; page 5-2 of [4]; page B-76 of [5]).

The Modulation Transfer Function (MTF) is normally used experimentally to describe the image quality of an optical system [4]. MTF curves describe the ability of a lens or system, as a function of spatial frequency, to transfer object contrast to the image. For a spatially modulated source with intensities T_{max} and T_{min}, if the image has the intensities I_{max} and I_{min}, at a given spatial frequency, the MTF will be:

$$MTF = \frac{(I_{MAX} - I_{MIN})/(I_{MAX} + I_{MIN})}{(T_{MAX} - T_{MIN})/(T_{MAX} + T_{MIN})}$$

The MTF can describe the total effects of all kinds of aberrations. In our system, if we want all other aberrations smaller than the chromatic errors, for example in the vertical direction, $\Delta_{2v} \approx h_{1v}\beta(\Delta_{2v}/h_{2v}) \approx 0.28*0.5*0.05 \approx 0.007$ mm, we need a satisfied MTF for spatial frequency up to 140 cycle/mm. The MTF data also depends on the lens focal ratio. For the second lens of our system, the vertical illuminated area will be $d = 2(T+t') \approx 45$ mm, the effective focal ratio will be $f/d = 1250/45 \approx 28$.

On page 5-4 of reference[4], a typical on-axis MTF curve of an achromat with f/d = 3.77, which is ~ 1/7 of the vertical effective focal ratio of our system, has been given. That figure shows that at the spatial frequency of 140 the MTF will be 0.27. Considering the spherical aberration being proportional to r^3, or d^3, we can believe that the on-axis spherical aberration of our system with such kind of achromats will be very small and negligible.

However, for the off-axis image, which may be caused by the size of the source or misalignment, the astigmatism aberration will be introduced. In the figure of page 5-6 of reference [4] it has been shown that with a field angle of 2.5°, when spatial frequency is 14 cycle/mm, the MTF will decrease to 0.2 (The Rayleigh range of a perfect system corresponding to MTF = 0.22). Because the astigmatism is proportional to rh^2, the spatial resolution of the second lens in our system might be ~ 7 times better than 14 cycle/mm, that is only 100 cycle/mm at the 2.5° field angle. On the other hand, because the astigmatism is proportional to the square of field angle, we can get a better spatial resolution by keeping a smaller field angle. So, roughly, less than 2° is basically required for our system.

In conclusion, if the achromats have similar quality to those described in reference [4], the spherical aberration and coma could be very small and negligible and the astigmatism could also be small enough when we keep the field angle, roughly saying, smaller than 2°. The field angle caused by beam transverse size is very small and negligible; We just need to pay attention to alignment in order to keep a small field angle.

3. The Quality of Mirrors

The following is only a rough discussion about this issue.

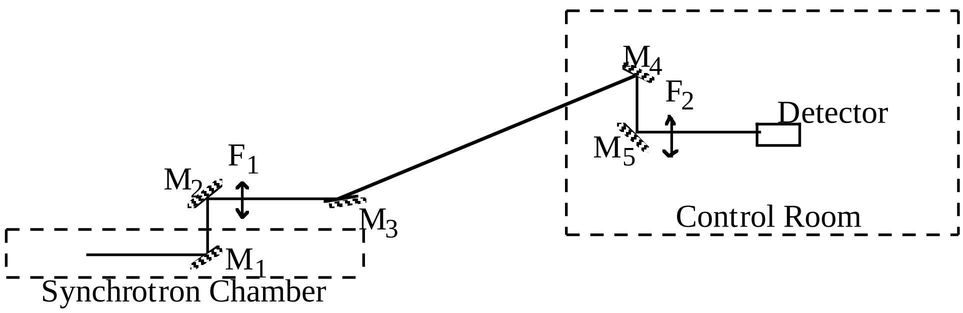

Figure 2 - Position of the five mirrors

Five mirrors will possibly be used in the optical system: two before the first lens and three others between the two lenses, as shown in Fig. 2. Some company describes the mirror surface error with "spherical error" and "irregular error", which represent the spherical deviation and irregular deviation from an ideal surface respectively [5].

For simplicity, we treat defect mirror surface as an average spherical shallow (or convex) one, as shown in Fig. 3(b), and we assume that such spherical surface influence on image will not be so perfect that can be compensated just by finely tuning the position of detector, therefore it will be an effect of image distortion.

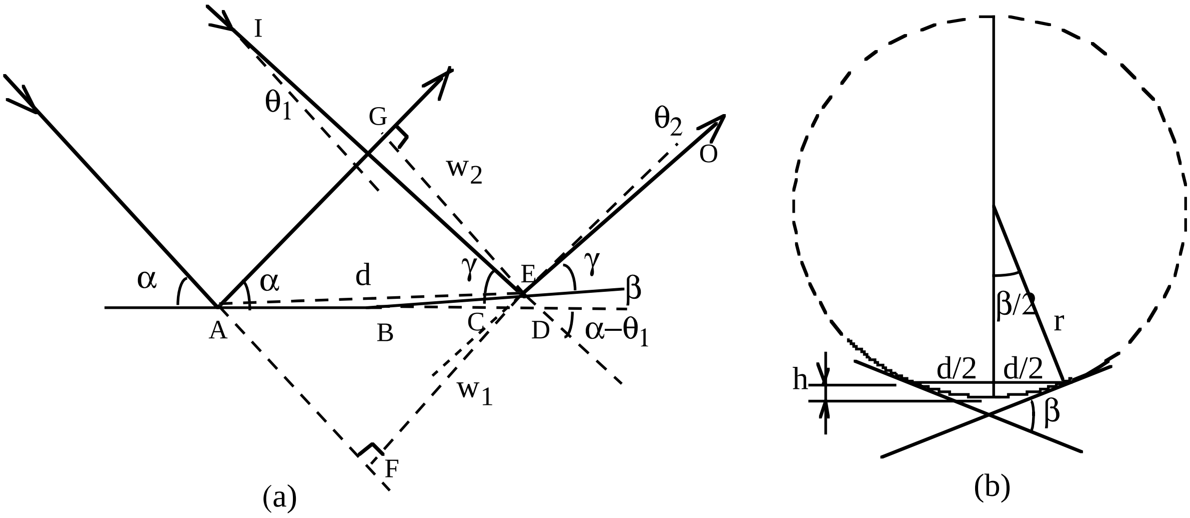

Figure 3 - Image distortion by a defected mirror

Figure 3(a) shows a bunch of light coming from a point source and having a radiation angle of θ_1. the source distance to the mirror is x. The two rays, separated by a distance w_1, reflected at two different points, which form an angle β on the distorted mirror change the radiation angle into θ_2. In Fig. 3(a), $\angle IEB=\gamma$, $\angle GAD=\alpha$, $\angle EBD=\beta$, $\angle IDB=\alpha-\theta_1$; $\angle IEB=\angle EBD+\angle IDB=\beta+\alpha-\theta_1$, i.e. $\gamma=\beta+\alpha-\theta_1$. On the other hand, $\angle OCD=\alpha-\theta_2$ and $\angle OCD=\angle EBD+\angle BEC=\beta+\gamma$, therefore, $\theta_2=\alpha-\beta-\gamma=\theta_1-2\beta$.

Because $x\approx w_1/\theta_1$, $x'\approx w_2/\theta_2$, $\angle EAF=\angle BAF+\beta/2=\alpha+\beta/2$,

$$w_1 = \overline{AE}\sin\angle EAF = d\sin(\alpha+\beta/2). \tag{3.1}$$

Similarly,

$$w_2 = d\sin(\alpha-\beta/2) \approx w_1-\beta d\cos\alpha \tag{3.2}$$

because of $\beta<<\alpha$, the longitudinal position distortion caused by the mirror will be:

$$\Delta x = x'-x = \frac{w_2}{\theta_2}-\frac{w_1}{\theta_1} = \frac{w_1(\theta_1-\theta_2)-\theta_1\beta d\cos\alpha}{\theta_1\theta_2} = \frac{2w_1\beta-\theta_1\beta d\cos\alpha}{\theta_1\theta_2} \tag{3.3}$$

for $\Delta x<<x'$ and x, then $\theta_2-\theta_1<<\theta_1,\theta_2$,

$$\Delta x = \frac{2x^2 w_1\beta - w_1 x\beta d\cos\alpha}{w_1^2} \approx 2\beta\frac{x^2}{d}\left(\frac{1}{\sin\alpha}-\frac{d\cos\alpha}{2x\sin\alpha}\right) \tag{3.4}$$

On the other hand, from Fig. 3(b), there is the relationship,

$$h = \frac{d}{2}\left[\frac{1}{\sin\beta/2} - \frac{1}{tg\beta/2}\right] = \frac{d}{2}\frac{1-\cos\beta/2}{\sin\beta/2} = \frac{d}{2}\frac{\left(\beta/2\right)^2\Big/2\left(1-\frac{\left(\beta/2\right)^2}{12}+\ldots\right)}{\left(\beta/2\right)\left(1-\frac{\left(\beta/2\right)^2}{6}+\ldots\right)} \approx \frac{d}{8}\beta \qquad (3.5)$$

From (3.4) and (3.5):

$$h = \frac{1}{16}\left(\frac{d}{x}\right)^2 \Delta x \left(\frac{1}{\sin\alpha} - \frac{d\cos\alpha}{2x\sin\alpha}\right)^{-1} \approx \frac{\sin\alpha}{16}\left(\frac{d}{x}\right)^2 \Delta x \qquad (3.6)$$

For M2, for example, if we suppose d/x = 50/1623 mm and sinα = sin45°, Δx = 1.17 mm, the value of the chromatic focus shift of F_1, than we get h = 0.0000491 mm = λ/12, where λ = 6000 Å; If we choose Δx = 4.65 mm, half source length, we will get h = 0.000195 mm ≈ λ/3.

If we treat the defected mirror with the paraxial Gauss Optics formulas for a simple estimation, the focus of the mirror is:

$$f = -\frac{r}{2} \approx -\frac{d^2}{16h}$$

then,

$$\Delta x = |x'| - |x| = \left|\frac{fx}{f+x}\right| - x = -\frac{x^2}{f+x} \approx -\frac{x^2}{f} = \frac{16x^2}{d^2}h, \quad \text{if } |f| >> x,$$

and,

$$h = \frac{1}{16}\left(\frac{d}{x}\right)^2 \Delta x \qquad (3.7)$$

Eq. (3.7) is the same as (3.6) when α = 90°.

Similarly, we discussed the quality requirement for the mirrors between the two lenses in Appendix C, which give us the suggested λ flatness.

Regarding for the irregular error of a mirror, the distortion on the image could be more serious. For example, if an irregular defect happens with a dimension of d/3, the limit of irregular depth "h" should be about one order smaller than the regular one because of $h \propto d^2$. The image of the beam may be more seriously influenced from the irregular surface error considering the following parameters: the size of the horizontal spot on the mirrors from a point source will only be several millimeters, the size of the slit, and the total vertical dimension of the spot on mirrors from the whole beam will be ~ 30÷40 millimeters. Unfortunately, we can not give an explicit limit for the irregular defect. However from the discussion above we know that the mirror flatness should at least better than the order of λ because of regular "spherical error". Perhaps better than λ/10 might be a basic requirement for the general flatness.

4. Mirror Size and the Second Lens Size

1. Size of the tilted mirror

We only discuss the size of M_3, because the incident angle on this mirror is large and therefore the size of mirror is large.

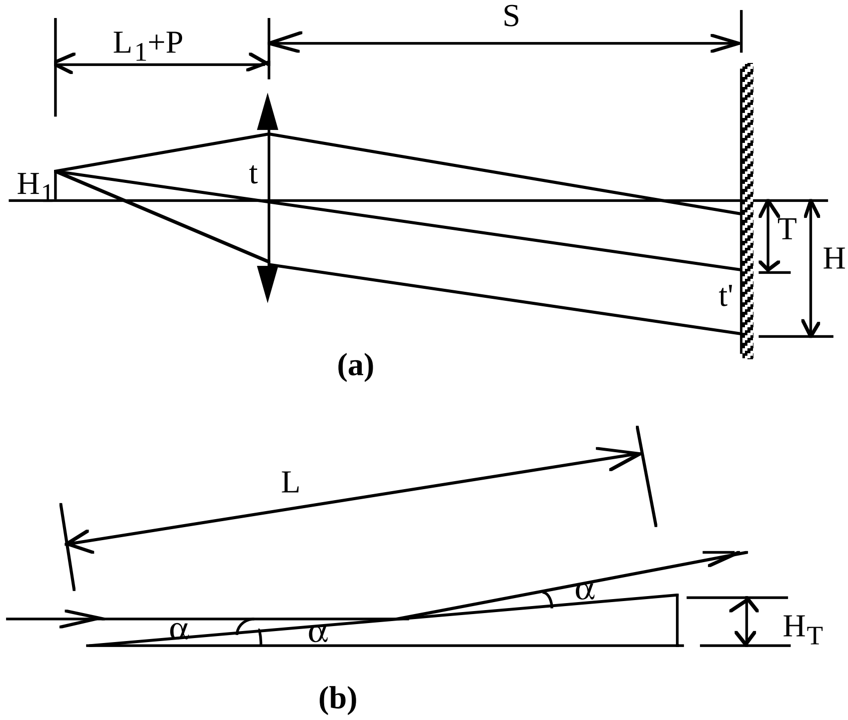

Figure 4 - The size of M_3 mirror.

Horizontally, in Fig. 4(a), t = a, the slit size, $H_1 = 2\delta_x$. The width of the mirror will be W = 2(T+t'), referring to eq.(A.14) about t', there will be:

$$W = 2H = 2\left[\frac{S}{L_1+P}H_1 + a\frac{L_1^2 - P(S-L_1)}{L_1(L_1+P)}\right] = 42.6mm \tag{4.1}$$

Vertically, we take $t = 1.5\psi_{typ}(L_1+P)$, for in Fig. 2 of Ref.[6] the synchrotron light will apparently exist till $\psi=1.5\psi_{typ}$ when $\varepsilon/\varepsilon_c=\lambda_c/\lambda=0.01$.(In DAΦNE, λ_c=62Å, and the light used in the monitor λ = 500 Å). $H_1 = 2\delta_y$.

$$H_T = 2H = 2\left[\frac{S}{L_1+P}(2\delta_Y) + t\frac{L_1^2 - P(S-L_1)}{L_1(L_1+P)}\right]$$

(4.2)

$$L = H_T/\sin\alpha = 2\left[\frac{S}{L_1+P}(2\delta_Y) + 1.5\,\psi_{typ}(L_1+P)\frac{L_1^2 - P(S-L_1)}{L_1(L_1+P)}\right]\Big/\sin\alpha = 235mm$$

Other data used in the calculation are: L_1=2250 mm, P=0, S=8127 mm, a=3 mm, δ_x=2.53 mm, δ_y=0.28 mm, ψ_{typ}=0.467E-2 rad, α=17.43°/2=8.715°. The result shows that the mirror could not be smaller than 42.6*235 mm. The minimum size of the mirror will be changed by the change of the beam parameters. We list the required mirror dimension against the beam parameters as following:

Mirror size vs beam coupling (P = 0, beam position: center)

k	0.01	0.05	0.1	0.2	0.4
ΔW (mm)	0	-0.7	-1.5	3	5.5
ΔL (mm)	0	31.7	54.0	82.5	116.3

Mirror size vs beam horizontal shift (P=0)

ΔX (mm)	1.0	2.0	5.0	10.0	20.0
ΔW (mm)	7.3	14.5	36.2	72.3	144.5

Mirror size vs beam vertical shift (P=0)

ΔZ (mm)	0.1	0.5	1.0	5.0	10.0
ΔL (mm)	4.7	23.8	47.6	238.3	476.7

Mirror size vs beam longitudinal position (horizontal position: center)

P (mm)	100.0	20.0	0.0	20.0	100.0
ΔW (mm)	2.6	0.6	0.0	-0.6	-2.6
ΔL (mm)	25.3	5.0	0.0	-5.0	25.3

The longitudinal position has little influence on the mirror size; We can increase M3 mirror size for possible larger beam coupling; We can remote control the longitudinal position of M2 to compensate beam vertical shift in order to limit the size of M3; By properly rotate M2 mirror, we can compensate beam horizontal shift. However, the rotation control should be a very precise one, for 0.1 degree rotation of M2 will be equal to 5.66 mm beam horizontal shift given the M2 being 1662.7 mm away from beam.

An alternative way to compensate the beam horizontal shift is to move the horizontal position of F1 lens and the horizontal slit simultaneously, as shown in Fig. 5. In this way the tangential point of the beam to the principal optical ray can keep unchanged. The increased field angle in this case could also be small. Move F1 vertically can also compensate beam vertical shift.

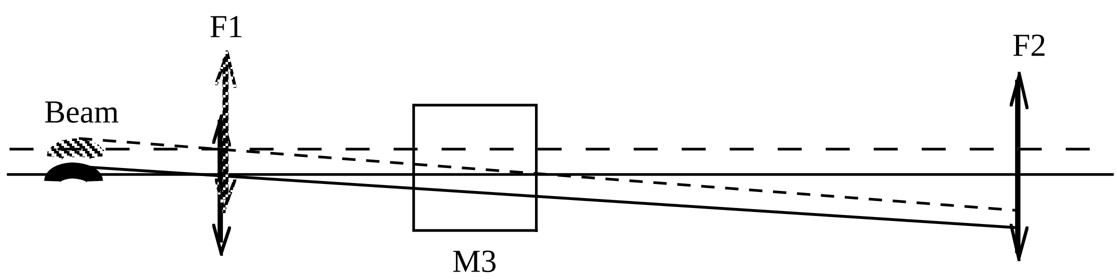

Figure 5 - Compensation of beam horizontal shift

Position control of M2 and F1 (or only F1) will be needed not only to limit the size of M3, but also to make sure that the image will not be cut by the second lens (see "the size of the second lens").

2. Size of the second lens

The distance between the lenses is S = L = 25000 mm; the horizontal and vertical size requirement will be: W = 118 mm and H_T = 44 mm, when P = 0 and t = 3 mm. The diameter of the second lens should ~ 118 mm.

If we choose the $L_2 = 1250/\Phi = 150$ mm lens, then maximum beam width that can be measured will be $2.51\delta_x$ for P = 0 mm and t = 3 mm.

Without position control of F1 or M2, with ~ 6.5 mm beam transverse position shift, the beam image will be cut by the second lens up to beam center. The large transverse beam shift must be compensated properly in our optical system.

5. Discussion

In order to set the detector precisely on the image surface, we will probably have to tune the detector longitudinally till the image of the beam is very clear or the size of the beam reaches its minimum value. But it probably happens that the horizontal size and the vertical size of the beam will not reach their minimum values at the same position when the detector slides along the optical axis because the image position is not the horizontal minimum size position.

In Fig. 6, if the detector is located at X_2'' instead of the right position of X'_{20}, and if $b_1<b_2$, then the image size will be smaller than H_2. On the other hand, if the image has a minimum size at image position, it needs the condition:

$$\psi'>\psi_2. \tag{5.1}$$

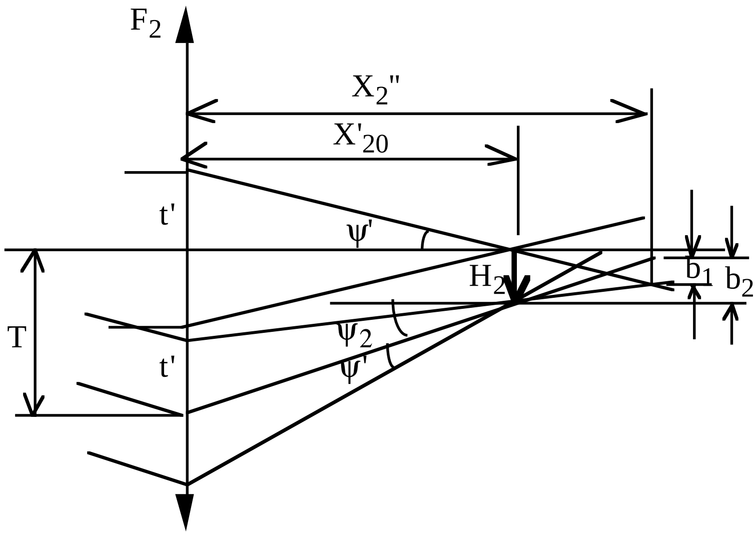

Figure 6 - The image with a longitudinal deviation

From Appendix B we know that,

$$\psi' = \frac{t'}{X'_{20}} = \frac{t}{|\beta_0|(L_1+P)}$$

$$\psi_2 = \frac{T-H_2}{X'_{20}} \approx \frac{T-H_{Q'}}{X'_{20}} = T\left(\frac{1}{L_2}-\frac{1}{L}\right) \approx \frac{T}{L_2} = \frac{L}{L_2(L_1+P)}H_1$$

then condition (5.1) means:

$$t > |\beta_0|\frac{L}{L_2}H_1 \qquad (5.2)$$

or:

$$\psi = |\beta_0|\psi' > |\beta_0|\frac{L}{L_2(L_1+P)}H_1 \qquad (5.3)$$

t and ψ are respectively the slit half width and the source divergence angle to the slit, H_1 is the beam transverse size. The typical data for our system could be: L = 25000 mm, L_2 = 1250 mm, L_1 = 2250 mm, and we may choose P = 0 and β_0 = 0.5556 for simplicity. In horizontal plane, H_1 = 2.529 mm, eqs. (5.2) and (5.3) require: t > 28.1 mm and ψ > 0.02248 rad = 1.3°. In vertical plane, H_1 = 0.2792 mm, they require: t > 3.1 mm and ψ > 0.00248 rad. Because in our system, in horizontal direction, the slit will be ~ 1 mm but in the vertical direction ψ will be the divergence of the synchrotron light itself which is ~ 0.004677 rad [1], we can expect a vertical minimum size image at the perfect image position but can not expect the minimum horizontal one there.

In Fig. 6, we can approximately take X_2'' as the minimum size position of the horizontal image, then there is,

$$X''_2 - X'_{20} \approx \frac{H_2}{\psi_2} \approx \frac{H_2(L_1+P)L_2}{L\,H_1} = |\beta_0|\frac{(L_1+P)L_2}{L} \qquad (5.4)$$

and the minimum spot size:

$$H'' = \psi'\left(X''_2 - X'_{20}\right) \approx \frac{L_2}{L}t$$

For P = 0, X_2'' - X_2' = 40 mm. That means that the minimum size positions for horizontal and vertical images can be separated by ~ 40 mm.

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Appendix A

The Image and its Aberration

In following calculation, the distance of an image, or source, will be positive if it is located on the right side of a lens, and will be negative if on the left side. X' is the image position of a source locating at X.

A.1. Image positions

For the first lens in Fig. 1,

$$X_1 = -(L_1 + P) \tag{A.1}$$

$$X_1' = \frac{f_1 X_1}{f_1 + X_1} \tag{A.2}$$

and for the second lens,

$$X_2 = X_1' - L \tag{A.3}$$

$$X_2' = \frac{f_2 X_2}{f_2 + X_2} = \frac{f_2\left[(L_1 + P)(L - f_1) - f_1 L\right]}{(L_1 + P)(L - f_1 - f_2) - f_1(L - f_2)} \tag{A.4}$$

With $f_1 = L_1 + \delta_1$, $f_2 = L_2 + \delta_2$, we get:

$$X_2' = \frac{L_2\left[L_1^2 - P(L - L_1)\right] + L_2(L + L_1 + P)\,\delta_1 + \left[L_1^2 - P(L - L_1)\right]\delta_2}{L_1^2 - P(L - L_1 - L_2) + (L + L_1 - L_2 + P)\,\delta_1 + P\,\delta_2}$$

with the assumption:

$$(L + L_1 - L_2 + P)\,\delta_1 << L_1^2 - P(L - L_1 - L_2) \tag{A.5}$$

$$P\,\delta_2 << L_1^2 - P(L - L_1 - L_2) \tag{A.6}$$

and keeping the first-order of δ_1, δ_2 during the expansion, we can get:

$$X_2' = \frac{L_2\left[L_1^2 - P(L - L_1)\right]}{L_1^2 - P(L - L_1 - L_2)} + \left[\frac{L_2(L_1 + P)}{L_1^2 - P(L - L_1 - L_2)}\right]^2 \delta_1 + \left[\frac{L_1^2 - P(L - L_1)}{L_1^2 - P(L - L_1 - L_2)}\right]^2 \delta_2 \tag{A.7}$$

Thus, the ideal image position without chromaticity is:

$$X'_{20} = X'_2(\delta_1, \delta_2 = 0) = \frac{L_2\left[L_1^2 - P(L - L_1)\right]}{L_1^2 - P(L - L_1 - L_2)} \tag{A.8}$$

and the longitudinal spot size caused by the residual chromatic aberration of the two achromats:

$$\Delta_L = \left[\frac{L_2(L_1 + P)}{L_1^2 - P(L - L_1 - L_2)}\right]^2 \delta_1 + \left[\frac{L_1^2 - P(L - L_1)}{L_1^2 - P(L - L_1 - L_2)}\right]^2 \delta_2 \tag{A.9}$$

A.2. Magnification

The transverse magnification of the system, under the assumptions of eq.(A.5) and (A.6), will be:

$$\beta = \frac{X'_2 X'_1}{X_2 X_1} = \frac{f_1 f_2}{(L_1 + P)(L - f_1 - f_2) + f_1(f_2 - L)}$$

$$= -\frac{L_1 L_2 + L_2 \delta_1 + L_1 \delta_2}{L_1^2 - P(L - L_1 - L_2) + P\delta_2 + (L - L_2 + L_1 + P)\delta_1} \tag{A.10}$$

$$\approx -\frac{L_1 L_2}{L_1^2 - P(L - L_1 - L_2)} + \frac{L_1(L - L_2)(L_1 + P)}{\left[L_1^2 - P(L - L_1 - L_2)\right]^2}\delta_1 - \frac{L_1\left[L_1^2 - P(L - L_1)\right]}{\left[L_1^2 - P(L - L_1 - L_2)\right]^2}\delta_2$$

and,

$$\beta_0 = \beta(\delta_1, \delta_2 = 0) = -\frac{L_1 L_2}{L_1^2 - P(L - L_1 - L_2)} \tag{A.11}$$

From eq.(A.8) we can get:

$$P = \frac{L_1^2\left(L_2 - X'_{20}\right)}{L_2(L - L_1) - X'_{20}(L - L_1 - L_2)}$$

and

$$L - L_1 - L_2 = \frac{L_1^2\left(L_2 - X_{20}^{'}\right) - L_2^2 P}{P\left(L_2 - X_{20}^{'}\right)},$$

inserting these into eq.(A.11) gives:

$$\beta_0 = -\frac{L_2(L - L_1) - X_{20}^{'}(L - L_1 - L_2)}{L_1 L_2} \tag{A.11a}$$

and

$$\beta_0 = -\frac{L_1\left(L_2 - X_{20}^{'}\right)}{L_2 P} \tag{A.11b}$$

Theoretically, with any two of the three parameters: P, L and X_{20}, the magnification can be calculated.

A.3. Transverse aberration size

The aberration spot center transverse shift shown in Fig. 1 will be:

$$\Delta_1 = |H_2| - |H_2^{'}| + \left(X_{20}^{'} - X_2^{'}\right)\frac{T - |H_2^{'}|}{|X_2^{'}|}$$

where $H_2 = \beta_0 H_1$, $H_2^{'} = \beta H_1$ and $T = LH_1/(L_1+P)$. With the further approximations: $\Delta_L << X_2^{'}$ and $|H_2 - H_2^{'}| << |H_2|$, and keeping the first order of δ_1 and δ_2, it can be obtained:

$$\Delta_1 = -(|\beta| - |\beta_0|)H_1 - \Delta_L \frac{\frac{L}{P + L_1} - |\beta|}{X_{20}^{'} + \Delta_L} H_1 \approx -(|\beta| - |\beta_0|)H_1 - \Delta_L \frac{\frac{L}{P + L_1} - |\beta_0|}{X_{20}^{'}}$$

$$= -\frac{L\left[L_1^2 - P(L - L_1)\right]}{L_2(P + L_1)\left[L_1^2 - P(L - L_1 - L_2)\right]} H_1 \delta_2$$

or,

$$\frac{\Delta_1}{H_2} = \frac{L\left[L_1^2 - P(L - L_1)\right]}{(P + L_1)L_1 L_2^2}\delta_2 \tag{A.12}$$

or even,

$$\Delta_1 = -\frac{T\,\Delta_L\left(\delta_1 = 0, \delta_2 \neq 0\right)}{X_{20}^{'}} \tag{A.13}$$

The physical meaning of Δ_1 described by (A.13) is more clear, as shown in Fig(A.1): Δ_1 is just the two rays vertical separation because of the second lens chromatic longitudinal image shift. When P=0, $\Delta_L(\delta_1=0)=\delta_2$, just the focus shift of the second lens.

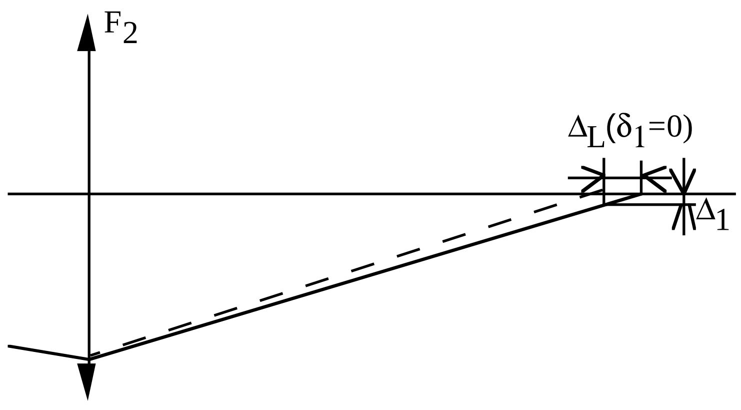

Figure A.1 - The equivalent meaning of Δ_1

On the other hand, with the assumptions $\delta_1 << L_1$, $L\,\delta_1 << L_1^2 - P(L - L_1 - L_2)$ and $\Delta_L << X'_2$,

$$\alpha = \frac{t}{X_1'} = t\left(\frac{1}{f_1} + \frac{1}{X_1}\right) \approx t\left[\frac{P}{L_1(L_1+P)} - \frac{\delta_1}{L_1^2}\right]$$

$$t' = t - \alpha L = \frac{L_1^2 - P(L - L_1)}{L_1(L_1+P)}t + \frac{L\,\delta_1}{L_1^2} \tag{A.14}$$

The aberration spot size will be:

$$\Delta_2 = \frac{\Delta_L}{X_2'}t' \approx \frac{\Delta_L}{X_{20}'}t'(\delta_1 = 0)$$

$$= \left\{\frac{L_2(L_1+P)}{L_1\left(L_1^2 - P(L - L_1 - L_2)\right)}\delta_1 + \frac{\left[L_1^2 - P(L - L_1)\right]^2}{L_1 L_2 (L_1+P)\left[L_1^2 - P(L - L_1 - L_2)\right]}\delta_2\right\}t \tag{A.15}$$

Because Δ_1 is proportional to the beam transverse dimension (H_1) and Δ_2 is proportional to the illuminated size (t) on the first lens, the Δ_1 error will be larger than D2 in the horizontal plane and D2 will be larger than D1 in the vertical plane.

In vertical plane, because $t = \Psi (L_1 = P)$, Δ_2 can also be written as:

$$\frac{\Delta_{2V}}{H_{2V}} = \left[\frac{(L_1+P)^2}{L_1^2}\delta_1 + \frac{\left[L_1^2 - P(L-L_1)\right]^2}{L_1^2 L_2^2}\delta_2 \right] \frac{\psi}{H_{1V}} \tag{A.16}$$

In horizontal direction, t=a, the half width of the entrance slit, it is also:

$$\frac{\Delta_{2h}}{H_{2h}} = \left[\frac{(L_1+P)}{L_1^2}\delta_1 + \frac{\left[L_1^2 - P(L-L_1)\right]^2}{L_1^2 L_2^2 (L_1+P)}\delta_2 \right] \frac{a}{H_{1h}} \tag{A.17}$$

A.4. Assumptions

In our system, the parameters have the inequality relationship: L>>L1,L2>>P. All the assumptions we have used above will then be equivalent to:

$$\begin{aligned}
&\delta_1 << L_1 \\
&\delta_2 << L_2 \\
&L\,\delta_1 << L_1^2 - P(L - L_1 - L_2) \\
&P\,\delta_2 << L_1^2 - P(L - L_1 - L_2) \\
&L_1^2 L_2 \delta_2 << \left[L_1^2 - P(L - L_1 - L_2)\right]\left[L_1^2 - P(L - L_1)\right]
\end{aligned} \tag{A.18}$$

In Fig. 1, we use a principal line going through the center of F1 to represent the central line of the light bunch and use this line to calculate the spot center deviation D1. This is exactly true for the horizontal case where t is determined by a slit whose center is also the lens center. However, for the vertical case, without any slit, the center line of a point source radiation bunch will be parallel to the optical axis before reaching F_1, and that will change Δ_{1V} a little bit by changing $T = LH_1/(L_1+P)$ to $T = LH_1/(L_1+P)-H_1$. Because $L>>L_1$, thus $T>>H_1$, we can expect that the final change on Δ_{1V} will be very small and all the results above can be used for the vertical image analysis without a large error.

A direct calculation of Δ_{1V} without such approximation leads to a similar result for P = 0 case:

$$\frac{\Delta_{1V}}{H_{2V}} = \frac{L - L_1}{L_2^2}\delta_2 - \frac{1}{L_1}\delta_1 \tag{A.19}$$

Appendix B

Connection Between Chromatic Errors and Geometric Errors

B.1. Δ_2 and the depth of field error

It is known that in the vertical direction Δ_2 is the dominant term of the image aberration. In the following we will prove that Δ_2 is equivalent to the depth of field error of a beam with image length Δ_L.

In Fig. B.1, $\Delta X_{DF} = \psi Z/2$ is the depth of field error of the source, and $\Delta X'_{DF} = \psi' Z'/2$ is the depth of field error of the image. For such image system, there are the relations:

$$Z' = Z\beta_0^2 \tag{B.1}$$

$$H_2 = H_1|\beta_0| \tag{B.2}$$

$$\psi' = \frac{t'}{X'_{20}} = \frac{L_1^2 - P(L-L_1)}{L_1(L_1+P)}t \Big/ \frac{L_2\left[L_1^2 - P(L-L_1)\right]}{L_1^2 - P(L-L_1-L_2)} = \frac{L_1^2 - P(L-L_1-L_2)}{L_1 L_2 (L_1+P)}t$$
$$= \frac{t}{|\beta_0|(L_1+P)} = \frac{\psi}{|\beta_0|} \tag{B.3}$$

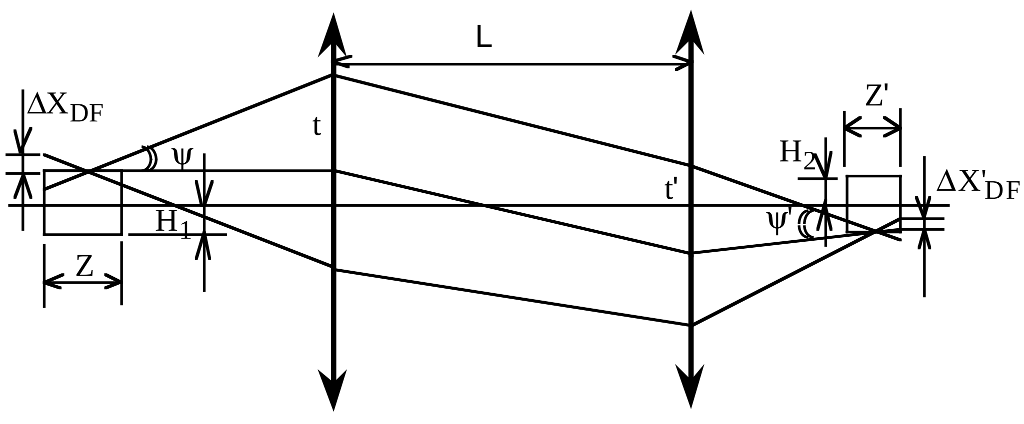

Figure B.1 - The image of the depth field error

Where β_0 , t, t' and X'_{20} have the same meanings as in Appendix A. From eqs.(B.1-B.3), it can be found that:

$$\Delta X'_{DF} = \psi' Z' /2 = |\beta_0| \psi Z / 2 = |\beta_0| \Delta X_{DF} \qquad (B.4)$$

or:

$$\frac{\Delta X'_{DF}}{H_2} = \frac{\Delta X_{DF}}{H_1} \qquad (B.5)$$

This means: (1) the image will have the same relative field error as the source; (2) When Z' equals the chromatic aberration longitudinal size Δ_L, the relative field error $\Delta Y_{DF}/H$ should equal the aberration value Δ_2 once we put the detector at the center of Δ_L, i.e.:

$$\frac{\Delta_{2V}}{H_{2V}} = \frac{\Delta_L}{Z'} \frac{\Delta X_{DF}}{H_{1V}}$$

B2. Δ_1 and the curvature error

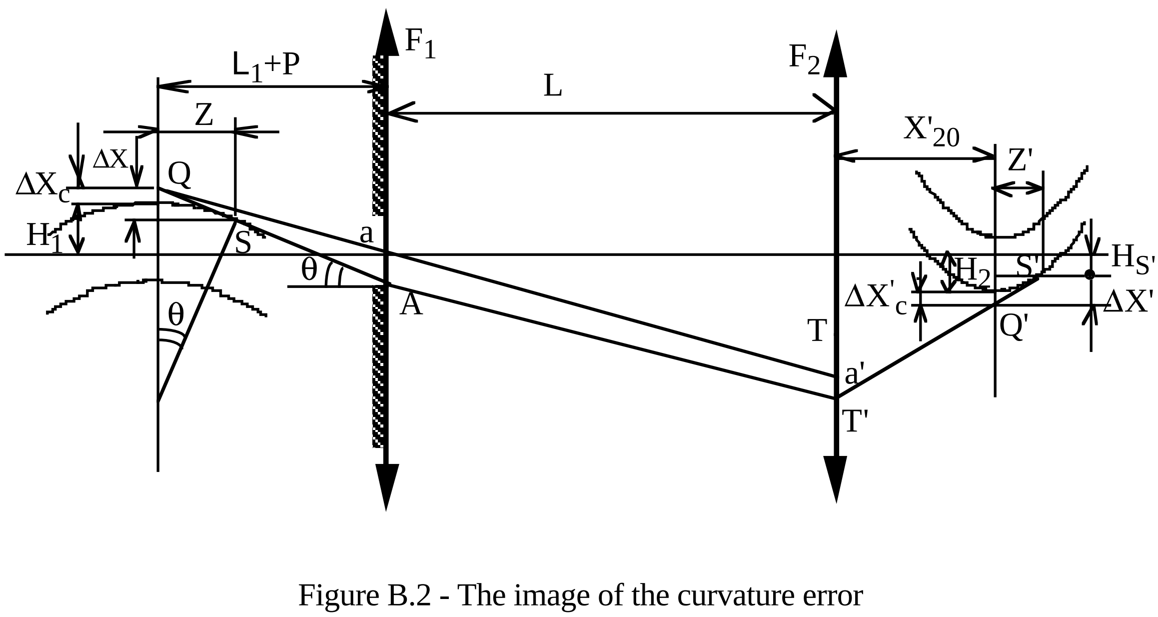

Figure B.2 - The image of the curvature error

Δ_1 is the dominant chromatic error in horizontal plane. In the following we will find the connection between Δ_1 and beam curvature error.

(i) First, we can see that $\Delta X'$ in Fig. B.2 is compatible to the Δ_1 of the chromatic error. As shown in Fig. B.2,

$$\Delta X' = \frac{T + a' - H_{S'}}{X'_{20} + Z'} Z' \approx \frac{T}{X'_{20}} Z' \qquad \text{(B.6)}$$

because $T = LH_Q/(L_1+P) >> H_{S'}$ and a', and $X'_{20} >> Z'$. Comparing with eq.(A.13), if Z' equals the chromatic shift $\Delta_L(\delta_1=0, \delta_2 \neq 0)$, $\Delta X'$ will be just the chromatic spot center vertical shift Δ_1:

$$\Delta_1 = \frac{\Delta_1(\delta_1 = 0)}{Z'} \Delta X' \qquad \text{(B.7)}$$

(ii) The connection between $\Delta X'$ and the curvature error ΔX_c.

On the left side of Fig. B2, when θ is small, $\Delta X = Z\theta = R\theta^2$, where R is the beam trajectory curvature radius. Meanwhile, $R = (\Delta X_c + R)\cos\theta \approx (\Delta X_c + R)(1 - \theta^2/2) \approx \Delta X_c + R - 0.5R\theta^2$, Then $\Delta X = 2\Delta X_c$. At the same time, there is also the relation:

$$\Delta X = \frac{(H_Q + a)Z}{L_1 + P}, \text{ } H_Q \text{ is the height of Q} \qquad \text{(B.7)}$$

With (B.6), (B.7) and $T = LH_Q/(L_1+P)$, we get:

$$\frac{\Delta X'/H_2}{\Delta X/H_1} \approx \frac{LZ' H_Q H_1}{X'_{20} Z (H_Q + a) H_2} = |\beta_0| \frac{H_Q}{H_Q + a} \frac{L}{X'_{20}} = \frac{H_Q}{H_Q + a} \frac{L L_1}{L_1^2 - P(L - L_1)},$$

or:

$$\Delta_1 / H_2 = \frac{\Delta_L(\delta_1 = 0)}{Z'} \frac{H_Q}{H_Q + a} \frac{2L L_1}{L_1^2 - P(L - L_1)} \left(\Delta X_C / H_1 \right),$$

or:

$$\Delta_1 / H_2 \approx \frac{\Delta_L(\delta_1 = 0)}{Z'} \frac{L L_1}{L_1^2 - P(L - L_1)} \left(\Delta X_C / H_1 \right) \quad \text{when } a \approx H_Q \qquad \text{(B.8)}$$

In our system, when P=0, equation (B.8) means:

$$\Delta_1/H_2 \approx 2.36\left(\Delta X_C/H_1\right) \qquad \text{(B.9)}$$

This is the relationship between Δ_1 and the relative curvature error.

Because the curvature error is very small, only ~ 0.024 times the total measurement error [1], we can expect that Δ_1 aberration error will be ~ 5 per cent of the total measurement error.

(iii) Although from (B.8) there will be $(\Delta X'/H_2)/(\Delta X/H_1) \approx 10$ for P=0, the image relative curvature error is the same as the source one. This can be explained as follows: the optical rays of SA, AT' are from point S; they should go to S'. Meanwhile SA, AT' can be treated as coming from Q and they should reach Q' too. So, T', Q' and S' are located at one line; $\Delta X'_C$ should be the curvature error caused by the ray to S'. Because S' and Q' are also the images of S and Q, $\Delta X'_C = |\beta_0| \Delta X_C$.

What is the reason for $(\Delta X'/H_2)/(\Delta X/H_1) \approx 10$ but $(\Delta X'_C/H_2)/(\Delta X_C/H_1) = 1$? This is because there are different magnifications at point Q and S and the image of the beam trajectory is no longer a circle at all.

Actually, with optical formulas we can prove that the image points S', Q' are located at the same line coming from T' although there is a big difference between ΔX and ΔX'. (Any one interested may read the verification).

On one hand,

$$\frac{T' - H_{Q'}}{X'_{20}} = \frac{T + a' - |\beta_0| H_Q}{X'_{20}}$$

Substitute a instead of a' with eq.(A.14),

$$\frac{T' - H_{Q'}}{X'_{20}} = \frac{T + \dfrac{L_1^2 - P(L - L_1)}{L_1(L_1 + P)}a - |\beta_0| \dfrac{T(L_1 + P)}{L}}{\dfrac{L_2\left[L_1^2 - P(L - L_1)\right]}{L_1^2 - P(L - L_1 - L_2)}} = T\left[\frac{1}{L_2} - \frac{1}{L}\right] + \frac{1}{|\beta_0|\left(L_1 + P\right)}a \qquad \text{(B.10)}$$

On the other hand, the vertical magnification at point S' will be:

$$\beta_Z = -\frac{L_1 L_2}{L_1^2 - (P-Z)(L-L_1-L_2)} \approx -\frac{L_1 L_2}{L_1^2 - P(L-L_1-L_2)} + \frac{Z L_1 L_2 (L-L_1-L_2)}{\left[L_1^2 - P(L-L_1-L_2)\right]^2}$$

or:

$$\Delta\beta = \beta_Z - \beta_0 = \frac{Z L_1 L_2 (L-L_1-L_2)}{\left[L_1^2 - P(L-L_1-L_2)\right]^2} = |\beta_0| \frac{Z(L-L_1-L_2)}{L_1^2 - P(L-L_1-L_2)} \qquad \text{(B.11)}$$

Then,

$$\frac{H_{Q'} - H_{S'}}{Z'} = \frac{|\beta_0| H_Q - |\beta_Z| \left(H_Q - \Delta X\right)}{Z'} = \frac{\Delta X |\beta_0| + H_Q \Delta\beta}{Z'}$$

$$\approx \frac{|\beta_0| Z \dfrac{(H_Q + a)}{L_1 + P} + |\beta_0| Z \dfrac{L - L_1 - L_2}{L_1^2 - P(L-L_1-L_2)} H_Q}{Z'}$$

$$= \frac{H_Q}{|\beta_0|}\left(\frac{1}{L_1 + P} + \frac{L - L_1 - L_2}{L_1^2 - P(L-L_1-L_2)}\right) + \frac{a}{(L_1 + P)|\beta_0|}$$

$$= \frac{H_Q (L - L_2)}{L_2 (L_1 + P)} + \frac{a}{(L_1 + P)|\beta_0|}$$

$$= T\left(\frac{1}{L_2} - \frac{1}{L}\right) + \frac{a}{(L_1 + P)|\beta_0|}$$

then:

$$\frac{H_{Q'} - H_{S'}}{Z'} = \frac{T' - H_{Q'}}{X'_{20}} \qquad \text{(B.12)}$$

This means T', Q' and S' are on the same line.

Appendix C

Flatness of the Mirror Between the Lenses.

The schematic drawing of the mirror distortion between the two lenses is shown in Fig. C.1. Because of the distortion by the mirror, the down-stream optical equipment feels the rays coming from point source Q as if coming from Q'.

First, let us look at the longitudinal image shift by the spherical curved mirror. From Appendix A, we know that the distance from the image of Q to the mirror will be:

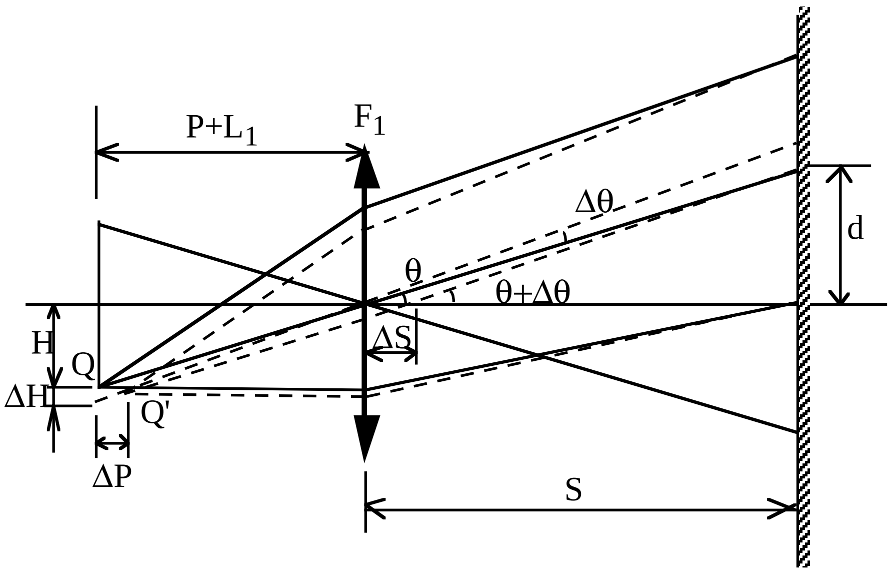

Figure C.1 - Distortion of the F_1 image

$$x = X_1' - S = \frac{f_1 X_1}{f_1 + X_1} - S \qquad \text{(C.1)}$$

where X_1=-(P+L_1). Because of the mirror distortion, the error on x will be (see eq.(3.6))

$$\Delta x = 16\left(\frac{x}{d}\right)^2 h \sin^{-1}\alpha \qquad \text{(C.2)}$$

When this error equals the source longitudinal shift ΔP,

$$\Delta x = \left[\frac{f_1(X_1 + \Delta P)}{f_1 + X_1 + \Delta P} - S \right] - \left[\frac{f_1 X_1}{f_1 + X_1} - S \right] = \left(\frac{f_1}{f_1 + X_1} \right)^2 \Delta P \qquad \text{(C.3)}$$

we can get,

$$h = \frac{\sin\alpha}{16}\left(\frac{d}{x}\right)^2 \left(\frac{f_1}{f_1 + X_1}\right)^2 \Delta P \approx \frac{\sin\alpha}{16}\left(\frac{d}{L_1}\right)^2 \Delta P \quad \text{, when } x >> S \qquad \text{(C.4)}$$

Secondly, for the vertical case, from Fig. C.1, there are the relations,

$$\theta = \frac{d}{S}$$

$$\theta + \Delta\theta = \frac{d}{S - \Delta S} \approx \frac{d}{S} + \frac{d}{S^2}\Delta S \Rightarrow \Delta\theta = \frac{d}{S^2}\Delta S$$

$$H = (L_1 + P)\theta$$
$$\Delta H = (L_1 + P)\Delta\theta$$

$$\Rightarrow \frac{\Delta H}{H} = \frac{\Delta\theta}{\theta} = \frac{\Delta S}{S}$$

From eq.(3.6), we know that,

$$h = \frac{\sin\alpha}{16}\left(\frac{d}{S}\right)^2 \Delta S$$

$$\Rightarrow h = \frac{\sin\alpha 1}{16}\frac{d^2}{S}\frac{\Delta H}{H} \qquad \text{(C.5)}$$

For instance, for M3 mirror, if the size is 200*400 mm, S = 8127 mm, α = 8.715°, ΔH/H = 0.026, the half vertical chromatic aberration, ΔP = 3 mm, one third of the beam length (remember the beam chromatic spot length is one third of the beam image length). Along the vertical direction: d = 400 mm, α = 8.715°, we get h = 0.0048 mm ≈ 8λ with (C.5) and h = 0.0009 mm = 1.5λ with (C.4); along the horizontal direction, d = 200 mm, α = 90°, ΔH/H = 0.005, the half horizontal chromatic aberration, we get h = 0.0015 mm ≈ 2.5λ with both (C.5) and (C.6). Here λ = 6000 Å.

For M5, we suppose the size will be 150*150 mm, S = 25000 mm, ΔP = 3 mm, α = 45° and ΔH/H = 0.026 in the vertical direction while α = 90° and ΔH/H = 0.005 in the horizontal direction. We can get h ≈ 1.5λ with (C.5) and h ≈ λ with (C.4) in vertical direction and h ≈ 0.5λ by (C.5) in the horizontal direction.

This suggests the "spherical" flatness of such mirrors should be better than λ.