DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, June 22, 2007

Note: **BM-17**

β-FUNCTION MEASUREMENT IN DAΦNE USING THE RESPONSE MATRIX

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1. Introduction

The standard procedure to measure the β-function in DAΦNE is to measure the variation of tunes as a function of quadrupole magnet strengths. This is possible, in DAΦNE, because almost all the quadrupole magnets have independent power supplies. A different technique [1,2], is the use of the response matrix of the machine. This procedure allows, in principle, measuring the β-function at each beam position monitor (BPM) and corrector (C) position. The accuracy of the measurement is limited by the accuracy of the response matrix measurement and of the BPMs and Cs calibrations.

The advantages of such technique with respect to the standard one are:

1) the big number of points in which the β-function can be measured. In DAΦNE, in fact, there are 47 BPMs and 27 C and, therefore, we can measure, in principle, the β-function in 74 points along the machine that are almost twice the number of quadrupoles.

2) the response matrix acquisition is completely automatic and fast: it requires about 15 minutes for each plane. This is, in particular, useful during a new start-up of the machine or when a new optics is installed.

2. Theory

In the following we report, for completeness, few basic concepts that are discussed in more detail in [2].

Let us suppose that, in the ring, there are M BPMs and N Cs. The element R_{ij} of the response matrix corresponds to the beam motion at the *i-th* BPM per unit angle of *j-th* corrector:

$$R_{ij} = \frac{\sqrt{\beta_i \beta_{cj}}}{2\sin(\pi\nu)}\cos\left(\Psi_i - \Psi_{cj} \pm \pi\nu\right) \tag{1}$$

where ν is the betatron tune of the machine, β_i and ψ_i are the β and phase functions at the BPM and β_{cj} and ψ_{cj} are those at the corrector.

If we know the response matrix the β and ψ functions at the *i-th* BPM are given by:

$$\begin{cases} \beta_i = \left(c_i^2 + s_i^2\right)\left[\sin(\pi\nu)\right]^2 \\ \psi_i = \arctan\left(\dfrac{s_i}{c_i}\right) + \pi\nu \qquad \left(0 < \psi_\mathrm{i} < 2\pi\nu \quad and \quad \psi_{\mathrm{i+1}} > \psi_\mathrm{i}\right) \end{cases} \tag{2}$$

where c_i and s_i are solutions of:

$$c_i \cos\psi_{cij} + s_i \sin\psi_{cij} = \frac{2}{\sqrt{\beta_{cj}}} R_{ij} \qquad \left(1 \le j \le N;\ \text{ø}_{cij} = \begin{cases} \text{ø}_{cj} & if \quad \text{ø}_{cj} < \text{ø}_i \\ \text{ø}_{cj}\text{-}2\partial\acute{\imath} \ \ if & \text{ø}_{cj} > \text{ø}_i \end{cases}\right) \tag{3}$$

Eq. (3) can be written in matrix form as:

$$\underline{\underline{A}} \cdot \underline{x} = \underline{a} \tag{4}$$

where:

$$\underline{\underline{A}} = \begin{pmatrix} \cos\psi_{ci1} & \sin\psi_{ci1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cos\psi_{ciN} & \sin\psi_{ciN} \end{pmatrix}, \qquad \underline{x} = \begin{pmatrix} c_i \\ s_i \end{pmatrix}, \qquad \underline{a} = \begin{pmatrix} \dfrac{2R_{i1}}{\sqrt{\beta_{c1}}} \\ \\ \dfrac{2R_{iN}}{\sqrt{\beta_{cN}}} \end{pmatrix} \tag{5}$$

The β and ψ functions at the C positions can be similarly obtained:

$$\begin{cases} \beta_{cj} = \left(c_{cj}^2 + s_{cj}^2\right)\left[\sin(\pi\nu)\right]^2 \\ \psi_{cj} = \arctan\left(\dfrac{s_{cj}}{c_{cj}}\right) + \pi\nu \qquad \left(0 < \psi_\mathrm{cj} < 2\pi\nu \quad and \quad \psi_{\mathrm{cj+1}} > \psi_\mathrm{cj}\right) \end{cases} \tag{6}$$

where c_{cj} and s_{cj} are solutions of:

$$\underline{\underline{B}} \cdot \underline{y} = \underline{b} \tag{7}$$

where:

$$\underline{\underline{B}} = \begin{pmatrix} \cos\psi_{1j} & \sin\psi_{1j} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cos\psi_{Mj} & \sin\psi_{Mj} \end{pmatrix}, \qquad \underline{y} = \begin{pmatrix} c_{cj} \\ s_{cj} \end{pmatrix}, \qquad \underline{b} = \begin{pmatrix} \dfrac{2R_{1j}}{\sqrt{\beta_1}} \\ \\ \dfrac{2R_{Mj}}{\sqrt{\beta_M}} \end{pmatrix} \tag{8}$$

A self-consistent set of solutions of systems (5) and (8) can be obtained by iteration. Starting with a set of initial values *(β_i, ψ_i)* and *(β_{cj}, ψ_{cj})* we calculate the matrices $\underline{\underline{A}}, \underline{\underline{B}}$ and the vectors $\underline{a}, \underline{b}$. Since the number of equations to determine $\underline{x}$ and $\underline{y}$ is larger than the number of unknowns, the problem is over-determined and the approximate solution can be found using, for example, the single value decomposition (SVD) or the QR decomposition [3]. The iterative procedure continues until the solutions converge for a given tune (the measured one). As pointed out in [2], inherent in this analysis are the ambiguities in the scaling of β-functions.

In fact in eq. (1), R_{ij} remain unchanged when β_i is multiplied by a constant and β_{cj} is divided by the same constant. Therefore, an extra constraint is necessary to determine the β-functions for BPMs and Cs. In the DAΦNE case we overcome this ambiguity by imposing that the β-function at some Cs near some BPMs are the same. In particular since the positions are not exactly the same we have considered few BPMs and Cs and we have averaged this condition over them.

3. Results and error analysis

The number of iterations to converge and the convergence of the solution itself depends, in general, on the choice of the iterative algorithm. One of the best algorithms to converge in few tens of iterations starting from the theoretical values of the β-functions is illustrated in Fig. 1. At the iteration *k* the β-functions found are re-entered as new input of the system for the iteration *k+1,* while the β-functions at the BPM are re-entered in a weighted sum with the β-functions of previous iteration.

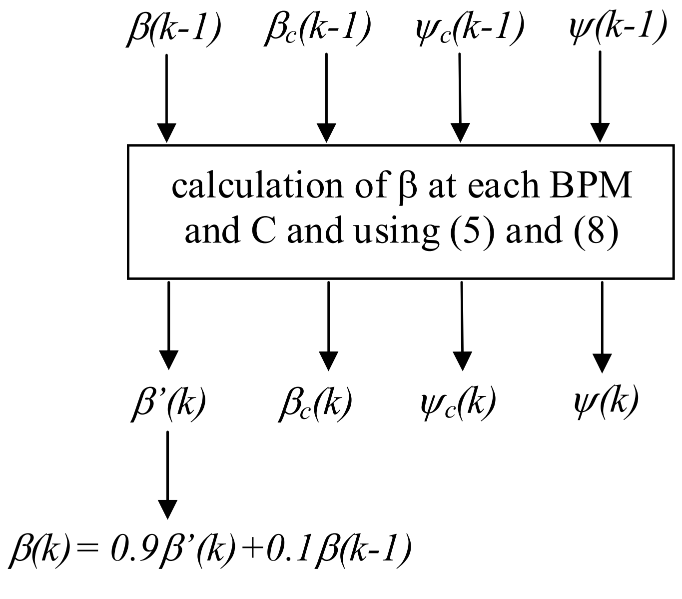

Fig. 1: Proposed algorithms to find iteratively the solutions of the system (5) and (8).

Once found the solution, it is possible to calculate for the i-th BPM (or h-th C) the following vectors: $E_{BPM\_i} = Ax-a$ (or $E_{C\_h} = By-b$). The *j-th* element of the vector E_{BPM_i} represents the "distance" between the found solution of β-function at the *i-th* BPM and those obtained using the β-function at the *j-th* C and the R_{ij} element of the response matrix. Similar considerations can be done for the *j-th* element of the vector E_{C_h}.

Therefore, if for all BPMs the *j-th* element of vectors E_{BPM} is always bigger than the other ones (or, more precisely, always few rms standard deviations away from the elements distribution) it means that there is a possible source of errors related to the *j-th* C. In this case we can eliminate the "suspected" elements (C and related response matrix line) from the final β measurement. Similar considerations can be done looking at the E_C vector.

From data analysis it is possible to observe that, even after this "cleaning", the equations defined by (5) or (8) do not give exactly the same solution. There are, in fact, the following sources of errors that give a residual "dispersion" of solutions:

1) BPMs non linearities;
2) errors in correctors strength calibration;
3) non-perfect cancellation of the ambiguity previously discussed in the amplitude of β-function because of the non-perfect superposition between the considered Cs and BPMs.

While in the vertical plane these sources of errors have almost the same weight, in the horizontal one the main source of error is given by (1).

To evaluate the error related to the measurement we proceed, therefore, as follow:

a) for a given BPM (or C) we calculate the β-function distribution using all possible pairs of equations in the system (5) (or (8));
b) we calculate the mean and rms value of the β-function distribution;
c) we eliminate iteratively the values of β-function until they are all in the range of ±4 standard deviation. It is possible to verify (se following paragraphs) that, within a negligible error, the mean of such "reduced" distribution is equal to the β-function calculated by the SV or QR decomposition. We assume that the standard deviation of each final distribution is equal to the standard deviation of each measuremen

4. Vertical β-function measurement results

The vertical β-function measurement is shown in the following plots considering, as example, the electron ring response matrix taken on 10/12/06. In the measurement, according to what illustrated in the previous paragraph we have eliminated few BPM whose errors were several rms standard deviations out from the other ones.

The differences between the sum of β-functions at each BPM and C at iteration k and those calculated at the iteration $k-1$ are reported in Fig. 2. From the plot it is easy to verify the convergence of the algorithm after few tens of iterations. The measurement of β-function compared with the MAD model of the machine is reported in Fig. 3. We used for the systems solution the QR decomposition algorithm. In each measure we have reported the error bar calculated as the ±1 rms standard deviation of the measurement distribution, as discussed in the previous paragraph. It is easy to observe that the error bars in each point are few percent the value of β-function at the same point.

The differences between the β-functions calculated by the QR decomposition algorithm and those calculated as mean of the β-function distribution are reported in Fig. 4. In each point the difference between the two values is a small fraction of the β-function at the same point.

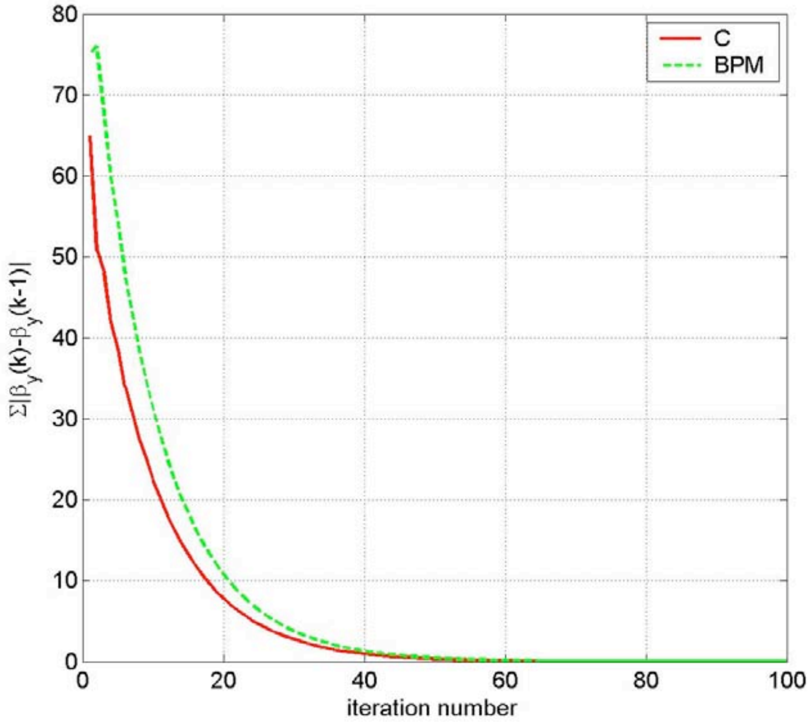

Fig. 2: Differences between the sum of β-functions at each BPM and C at iteration k and those calculated at the iteration k-1 (vertical plane).

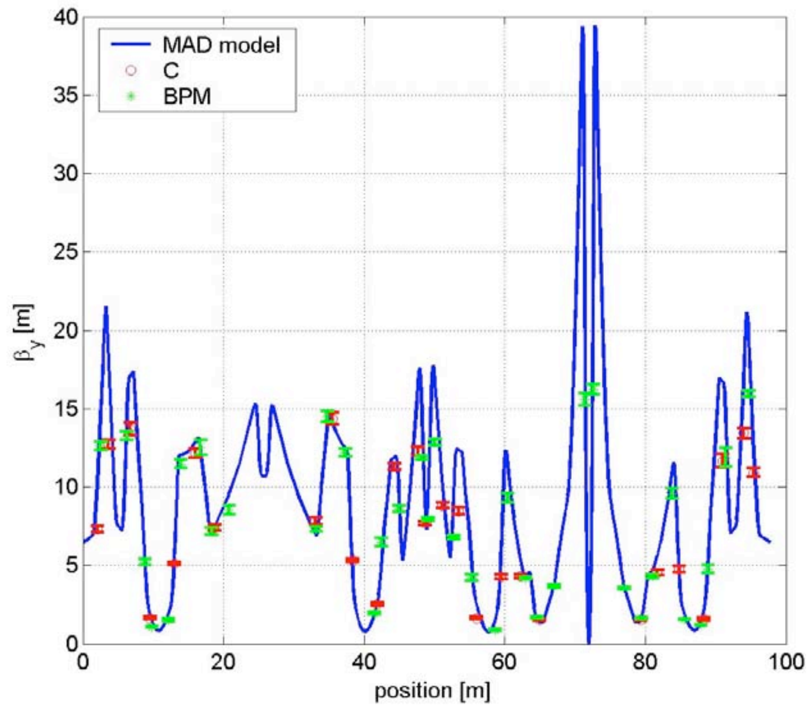

Fig. 3: Measurement of vertical β-function compared with the MAD model (electron ring).

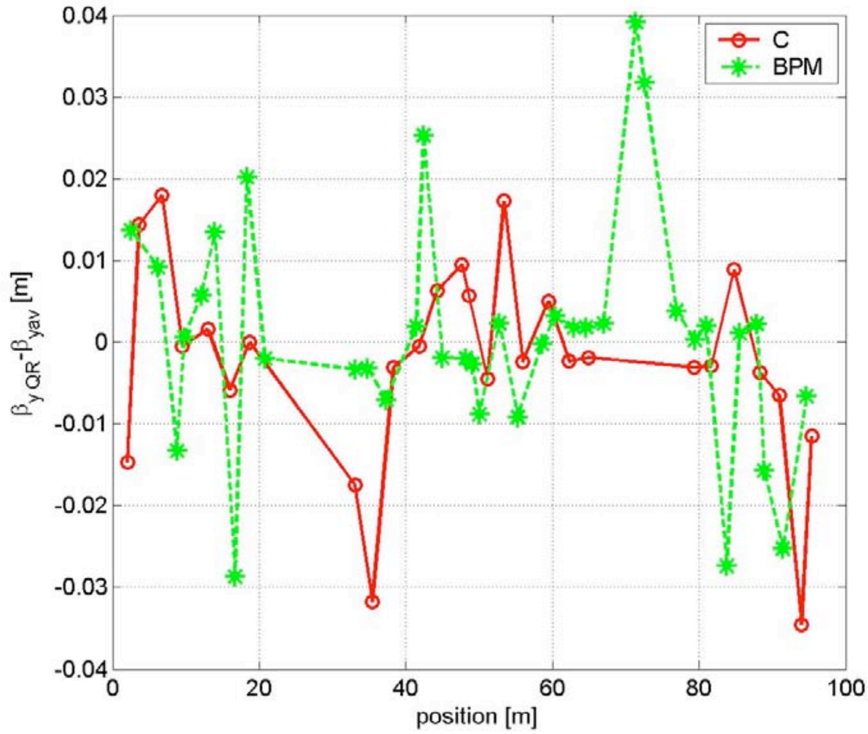

Fig. 4: Difference between the β-functions calculated by the QR decomposition algorithm and that calculated as the mean of the β-function distribution (vertical plane)

5. Horizontal β-function measurement

The horizontal β-function measurement is shown in the following plots. Also in this case we have eliminated few BPM whose errors were several rms standard deviations out from the other ones.

The differences between the sum of β-functions at each BPM and C at iteration k and those calculated at the iteration k-1 are illustrated in Fig. 5. The measured β-functions with the error bars are reported in Fig. 6 and are compared with the MAD model. In this case the rms standard deviations of each measure are bigger than the previous one because of the BPM non linearities.

Finally the differences between the β-functions calculated by the QR decomposition algorithm and that calculated as the mean of the β-function distribution are reported in Fig. 7. In each point the difference between the two values is a fraction of the β-function at the same point but bigger than in the vertical case.

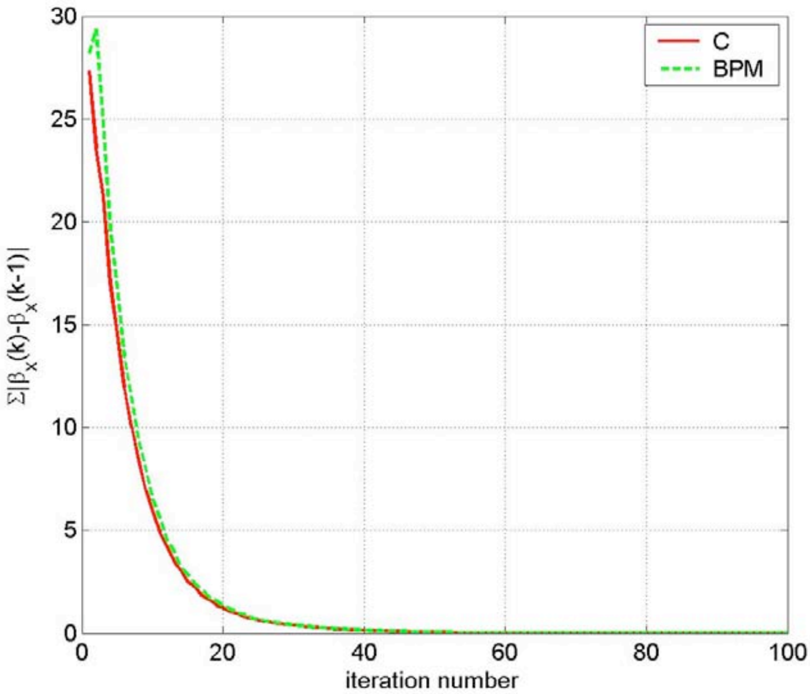

Fig. 5: Differences between the sum of β-functions at each BPM and C at iteration k and those calculated at the iteration k-1 (horizontal plane).

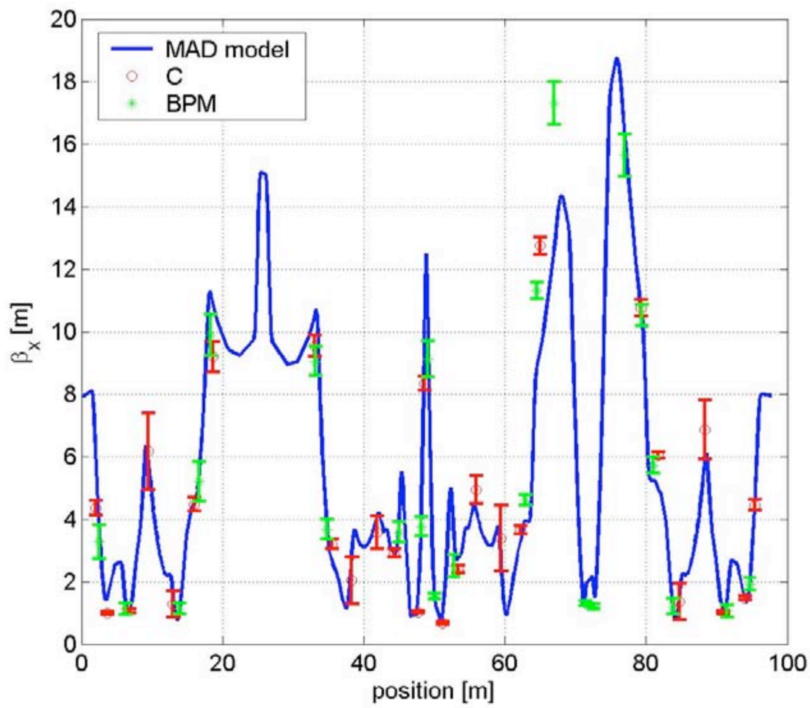

Fig. 6: Measurement of horizontal β-function compared with the MAD model (electron ring).

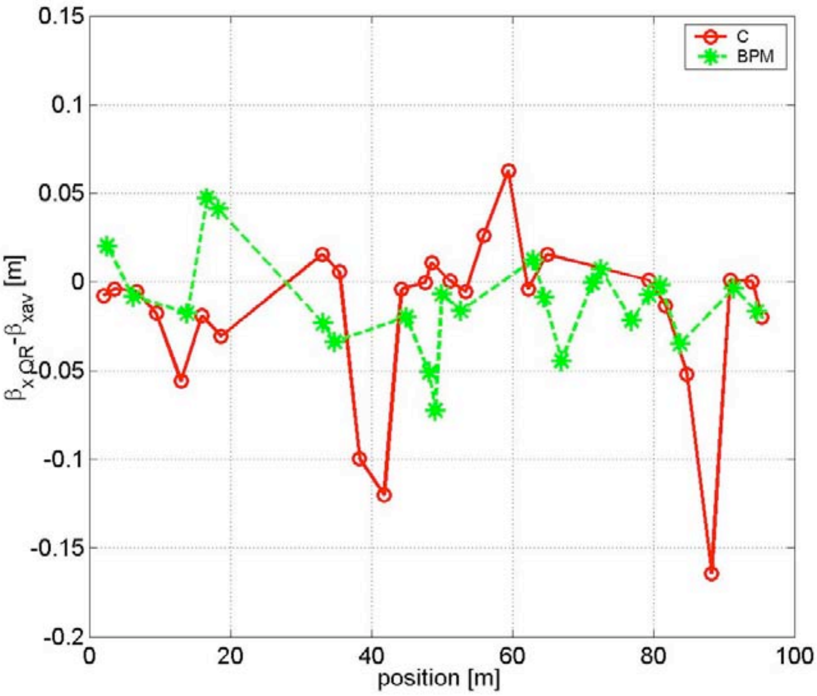

Fig. 7: Difference between the β-functions calculated by the QR decomposition algorithm and that calculated as mean of the β-function distribution (horizontal plane).

6. Conclusions

In the paper we have illustrated the β-function measurement results obtained in DAΦNE using the response matrix. The technique originally suggested by M. Harrison and S. Peggs. in 1987 allows, in principle, measuring the β-functions at each BPM and C position. The accuracy of the measurement is limited by the accuracy of the response matrix measurement and calibration of BPMs and Cs. In the paper we have illustrated the procedure we used to evaluate the errors of each measurement. The obtained results show that the procedure allows measuring the β-function in both planes with good accuracy, especially in the vertical one. It has also several advantages with respect to the standard one based on quadrupole strength variations as the larger number of points in which the β-function can be measured and the faster measurement itself, since the response matrix acquisition is completely automatic.

References

[1] M. Harrison and S. Peggs, “Global Beta Measurement from Two Perturbed Closed Orbits”, Proc. of PAC 1987.

[2] Y. Chung, G. Decker and K. Evans, “Measurement of Beta-Function and Phase Using the Response Matrix”, Proc. of PAC 1993.

[3] W. Press et al., “Numerical recipes”, Cambridge University Press, 2002.