CTFF3 TECHNICAL NOTE

INFN - LNF, Accelerator Division

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Note: **CTFF3-009**

Particle tracking through the measured field of the CTF3 Delay Loop dipole

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1 - Introduction

The field of the EPA type II (HIP BHZ 20) has been measured in 1985 [1] and recently the measurement have been repeated by R. Chritin [2]. The data files have been made available to LNF Accelerator Division in the framework of the CTF3 collaboration.

Figure 1 (not in scale) shows the reference system used to track the beam through the measured field. The nominal trajectory of the beam consists of two straight sections (AB and CD) and a circle with radius 1.081774 m. The bending angle is 30°. The field has been measured along 15 straight lines parallel to the Z axis starting from –7 cm and ending at +7 cm in steps of 1 cm. On the left side the measurement are taken from z = -0.5105 m to +0.0095 m in steps of 1 cm while on the right side the range is from 0.0010 m to 0.5110 m.

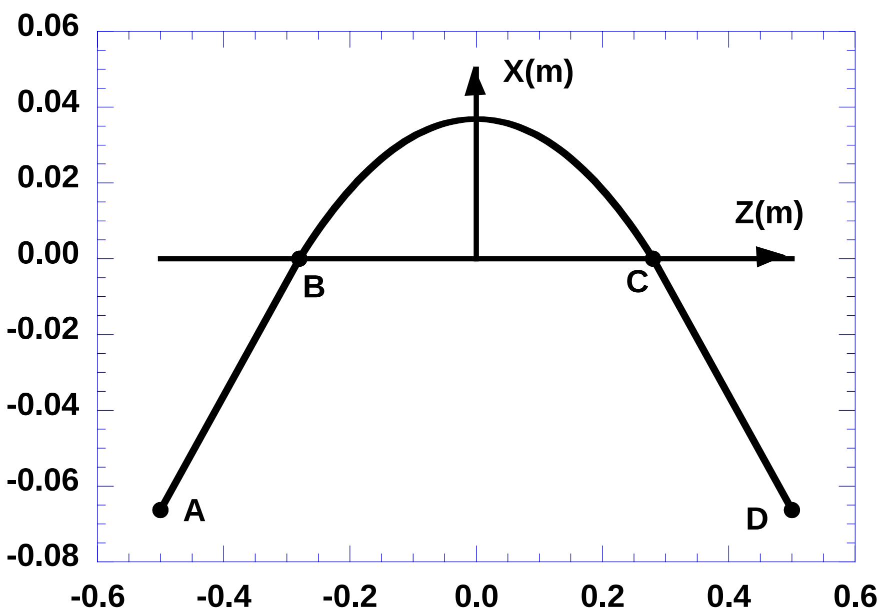

Figure 1: Reference system for the tracking code

The measured field along the dipole axis at 172.18A is shown in Fig. 2, while in Figs. 2, 3, 4 the measurements are taken along straight lines perpendicular to the dipole axis at the magnet center (z = 0.001 m) and at distances of -0.2805 m and 0.2810 m, roughly corresponding to points B and C in Fig. 1, where the field is approximately at half maximum. In these three figures the dots are the measured points, while the line is a polynomial:

$$B(Z,X) = b_0(Z) + b_1(Z)X + b_2(Z)X^2 + b_4(Z)X^4 + b_8(Z)X^8 \qquad (1)$$

which has been used to fit the measurements within ±6 cm from the axis before the fast drop near the pole boundary.

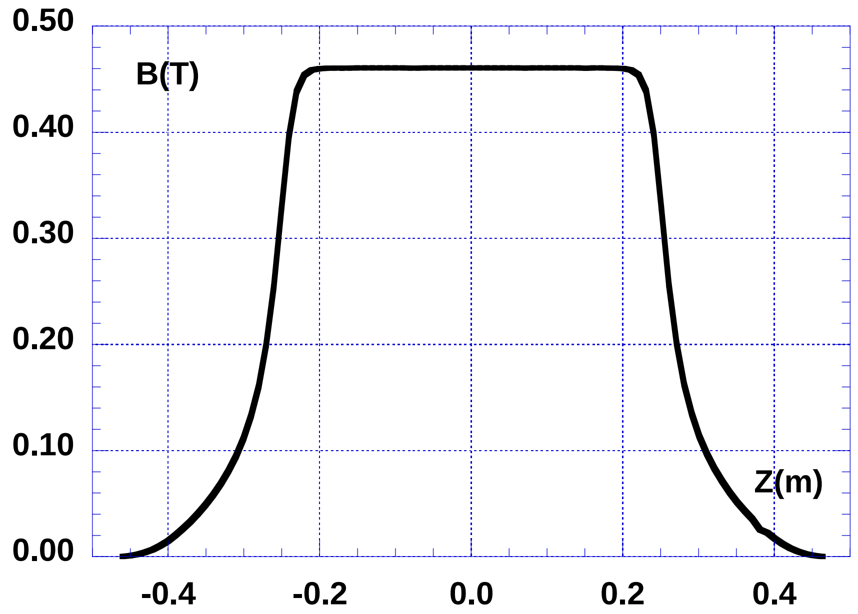

Figure 2: Vertical field component on the dipole axis

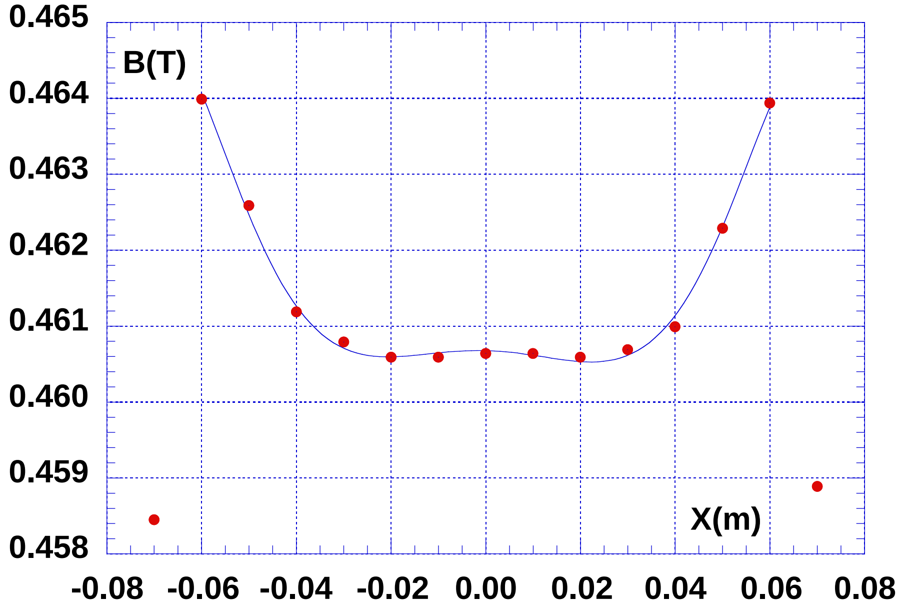

Figure 3: Vertical field component at magnet center (z=0.001 m) versus horizontal position

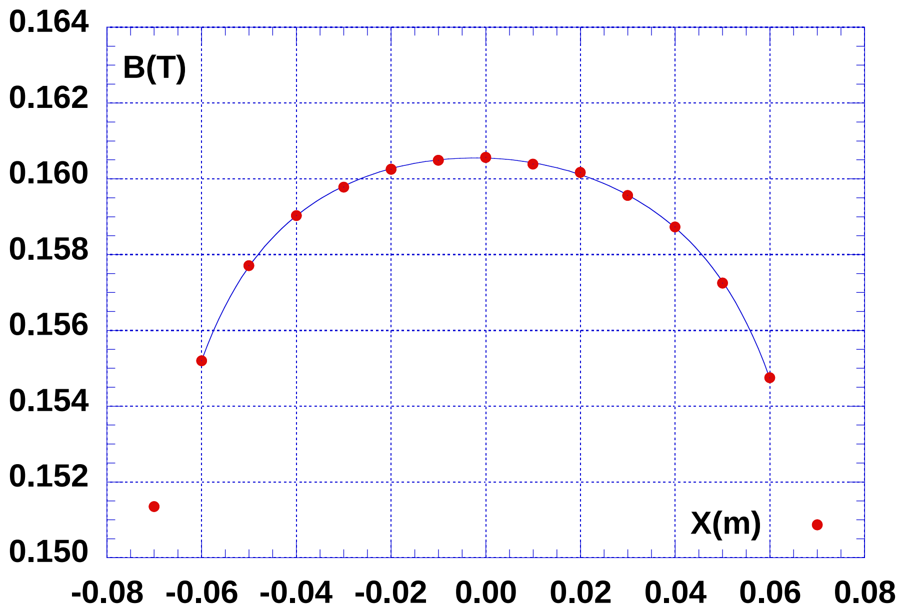

Figure 4: Vertical field component at z=-0.2805 m versus horizontal position

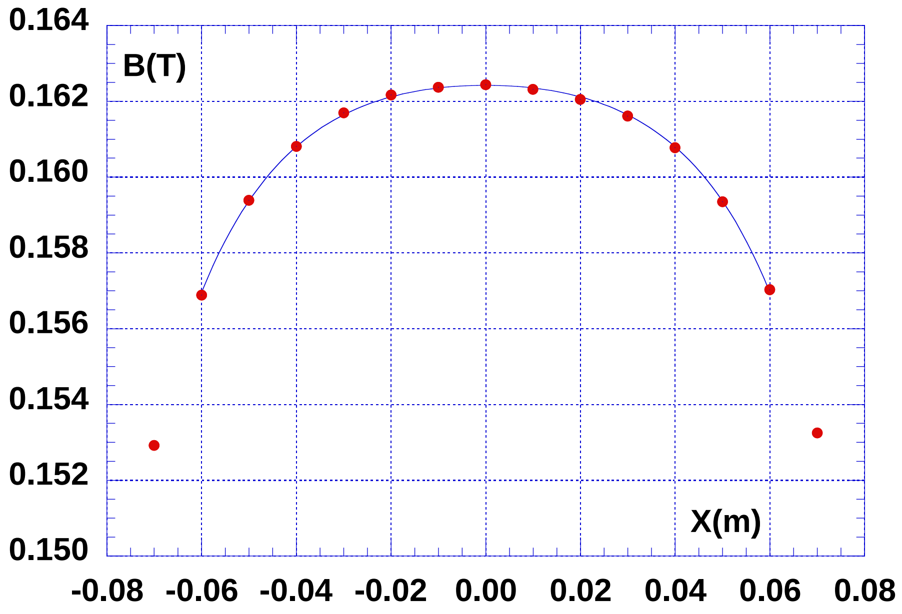

Figure 5: Vertical field component at z=0.281 m versus horizontal position

The field in the dipole is not symmetric in the longitudinal direction with respect to Z=0. Moreover, as mentioned before, the longitudinal positions at which the measurements have been taken are not symmetric as well. In order to estimate the amount of the asymmetry, the field points on axis on the left side have been reflected by just changing the sign of Z. Then the field on the right side has been interpolated between the three nearest points to each position on the left side with a second order polynomial to directly compare points at the same longitudinal distance from the center of the reference system. The difference between the points interpolated on the right side and those measured on the left one are shown in Fig. 6. The field on the right side is larger almost everywhere and exhibits a longer tail. The area between the points corresponds to a field integral of 6.2 Gm. At a beam energy of 150 MeV, the left side of the dipole will therefore deflect the beam by 0.6 mrad less than the nominal deflection, and the other way round for the right side. The anomalous point at 0.381 m is probably due to a wrong element in the data file on axis on the right side and will be ignored in the following.

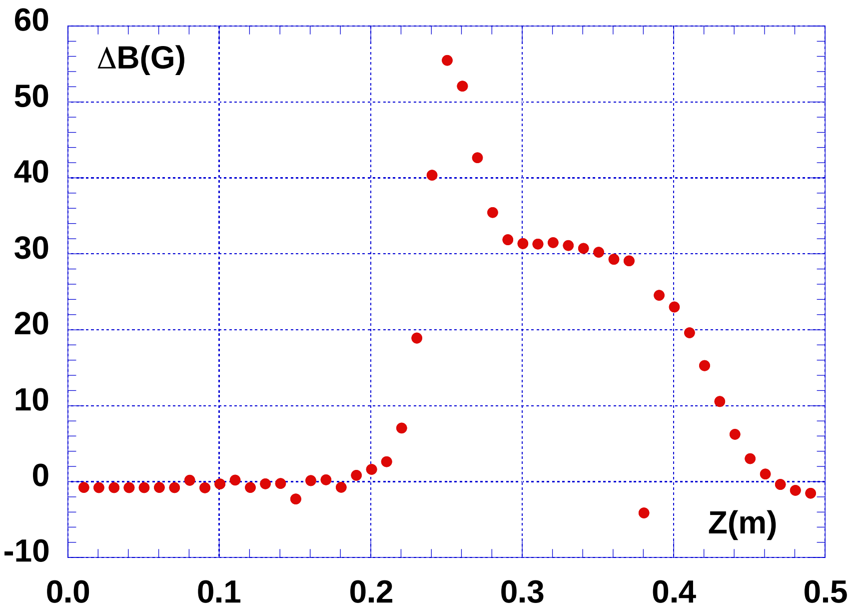

Figure 6: Difference between the fields on the right and left sides of dipole axis

2 - Transverse interpolation

The method followed to track the beam through the measured field is described in [3]. It allows to interpolate the vertical component of the field at any longitudinal and transverse position (Z,X) in the reference system of Fig. 1 and to calculate to first order the longitudinal and horizontal field components at any point in order to track the beam also in the vertical plane. However, due to the particular reference system and the peculiar shape of the field created by the shimming of the magnet (see Fig. 2), it has been necessary to use the high order fit (1) already defined in the introduction. The extension of the formalism described in [3] to this new function is straightforward. The following figures 7, 8, 9, 10, 11 show the behaviour of the fitting function coefficients calculated at each longitudinal position where the measurements have been taken. Here X is the transverse coordinate in the reference system of Fig. 1 centered on the dipole axis, and not the transverse position of the beam in the usual coordinate system of the beam centered on the nominal trajectory. The maximum difference between the measured and interpolated points is 2.6 G while the average difference is 0.5 G. The fluctuations from point to point of the high order terms, taking into account that the variation of the field versus the transverse position is very small (≈1% of the field at the magnet center) and that 4 even terms are used in the interpolation and they can cancel with each other, are consistent with the overall accuracy of the method.

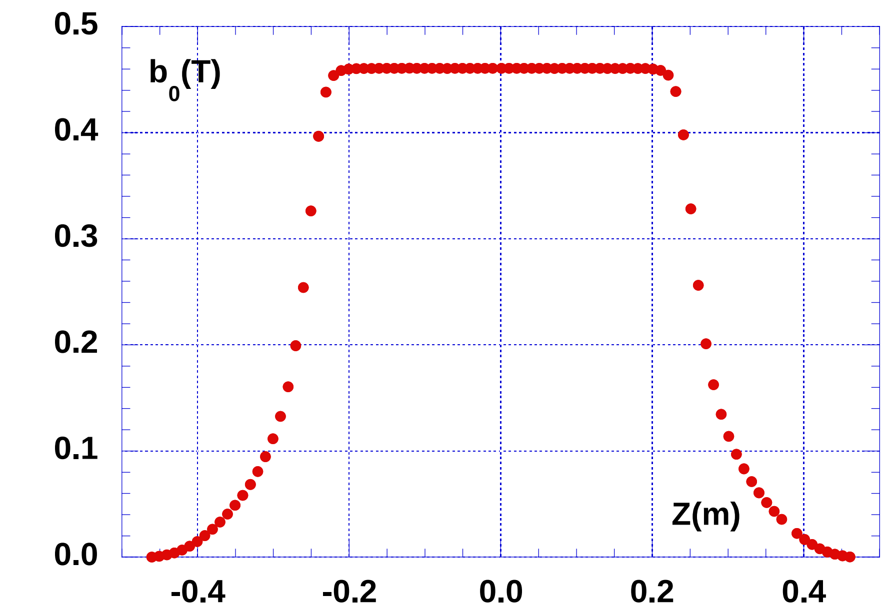

Figure 7: 0th order term of the polynomial fit versus longitudinal position

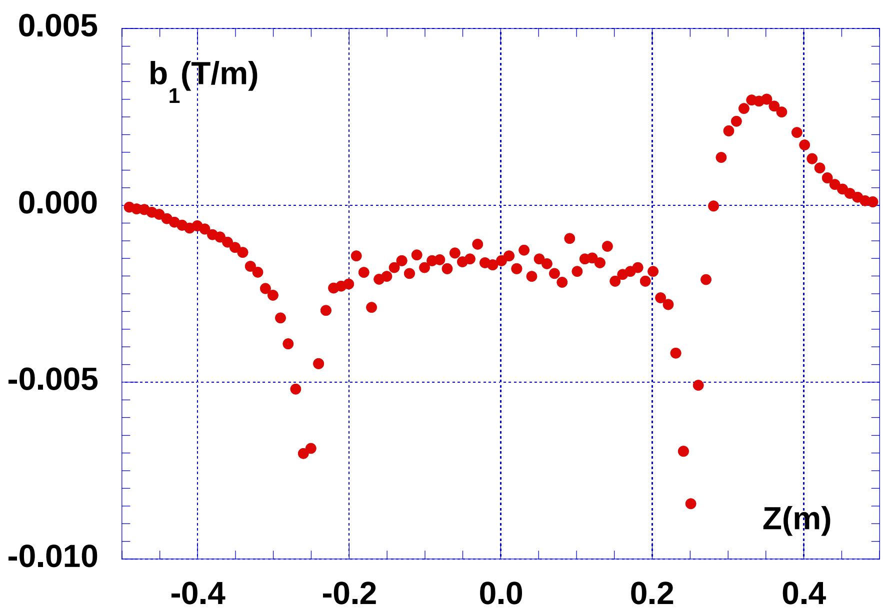

Figure 8: First order term of the polynomial fit versus longitudinal position

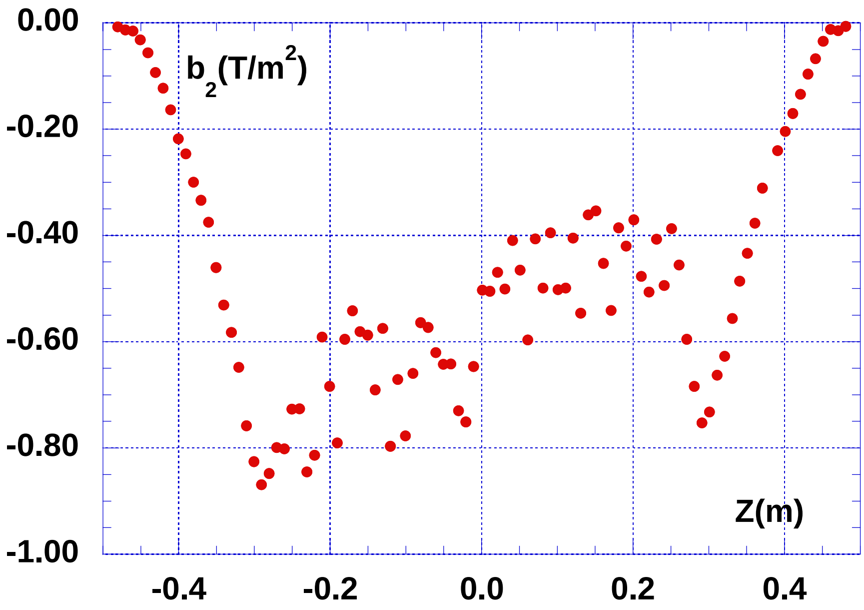

Figure 9: Second order term of the polynomial fit versus longitudinal position

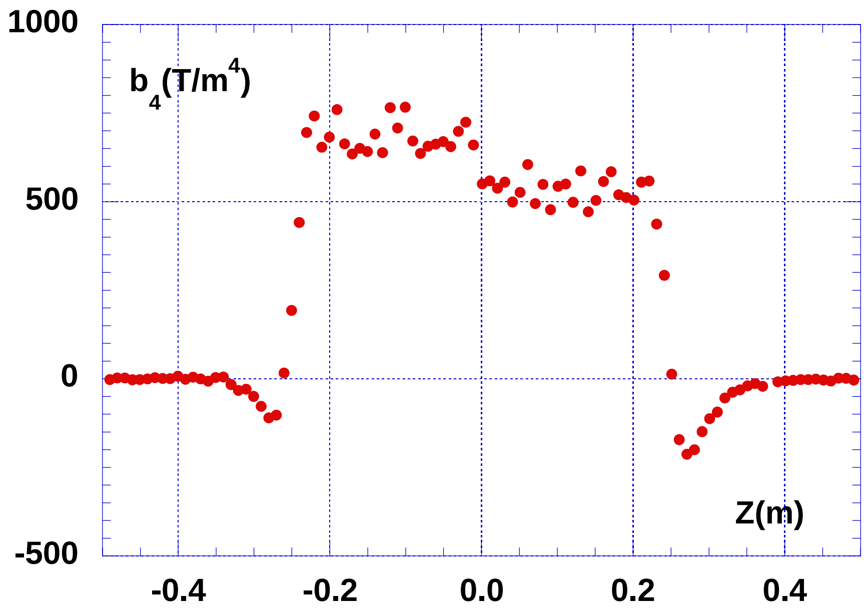

Figure 10: 4th order term of the polynomial fit versus longitudinal position

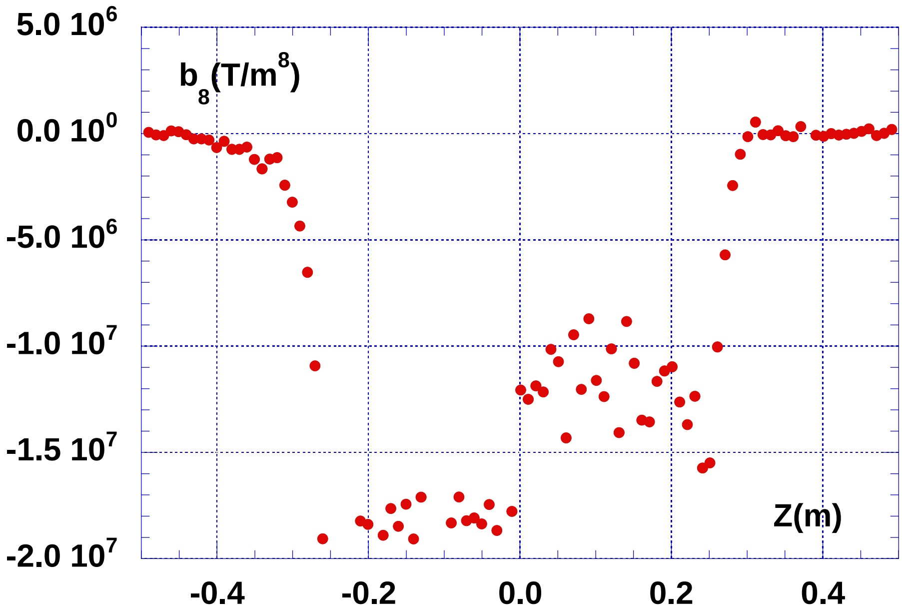

Figure 11: 8th order term of the polynomial fit versus longitudinal position

3 - Longitudinal interpolation

As explained in [3], each coefficient of the transverse interpolation is further interpolated in the longitudinal direction by means of a fourth order polynomial over six consecutive measured points. The maximum and average difference between the values of the coefficients b_i found at each longitudinal position and the corresponding longitudinal fit are given in Table I.

Table I – Accuracy of the longitudinal fit

coefficient	$\|b_i\|$ maximum	Δb_i average	Δb_i maximum
b_0 (T)	0.46	$0.78\ 10^{-4}$	$1.2\ 10^{-3}$
b_1 (T/m)	$8.4\ 10^{-3}$	$0.89\ 10^{-4}$	$4.2\ 10^{-4}$
b_2 (T/m^2)	2.1	$1.6\ 10^{-2}$	$7.8\ 10^{-2}$
b_4 (T/m^4)	$1.0\ 10^{3}$	8.8	40.9
b_8 (T/m^8)	$3.4\ 10^{7}$	$3.7\ 10^{5}$	$1.8\ 10^{6}$

4 - Tracking results

In order to establish the properties of the trajectory followed by the beam inside the dipole, from here on the usual coordinate system of particle motion inside a storage ring will be used: the horizontal displacement with respect to the nominal trajectory is indicated by x, the position on the trajectory with s and the vertical displacement with y, all in small caps to distinguish from the coordinate system of the field measurement inside the dipole, where the coordinates Z and X are in capital letters (see Fig. 1).

Starting at a distance of s=0.2 m from point B on the nominal trajectory, a test particle is tracked in 10^6 steps by calculating the Lorentz force due to the vertical field component interpolated between the measured points in the symmetry plane of the dipole and with the longitudinal and horizontal components obtained to first order from the derivatives of the vertical one [3]. The free parameter in the tracking program is the particle energy, which is changed until the particle is bent by the nominal bending angle (30°) at the corresponding position on the second straight section at 0.2 m from point C. Figure 12 shows the transverse displacement x of the beam trajectory from the nominal one as a function of the longitudinal coordinate Z of the dipole reference system.

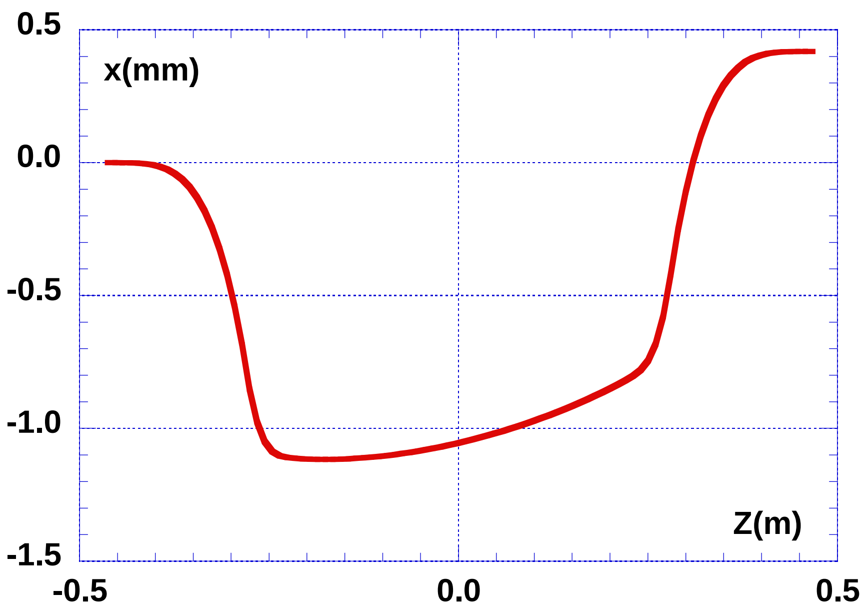

Figure 12: Horizontal displacement of the tracked particle with respect to the nominal trajectory.

As discussed in the first section, the field is not symmetric with respect to the center of the dipole, and this is the reason while the trajectory is not symmetric as well. The overall bending angle is the nominal one at an energy of 149.84 MeV and the displacement at the output is 0.42 mm. The angle between the particle trajectory at the magnet center and the nominal one is 0.6 mrad, as anticipated in Section 1. Since the displacement is mainly negative (on the inside of the nominal trajectory) the length of the path followed by the particle is shorter than the nominal length by 0.51 mm.

This asymmetry can be corrected by shifting the position of the dipole by 0.8 mm towards the left side in order to compensate for the larger field integral on the right one and avoid any harmful effect on the closed orbit in the ring. This displacement is simulated in the tracking program by simply shifting all the positions of the measured field points by the corresponding amount.

The energy for the nominal deflection is only 2 KeV lower but the trajectory is shorter, 0.62 mm less than the nominal one. The result in this condition is shown in Fig. 13.

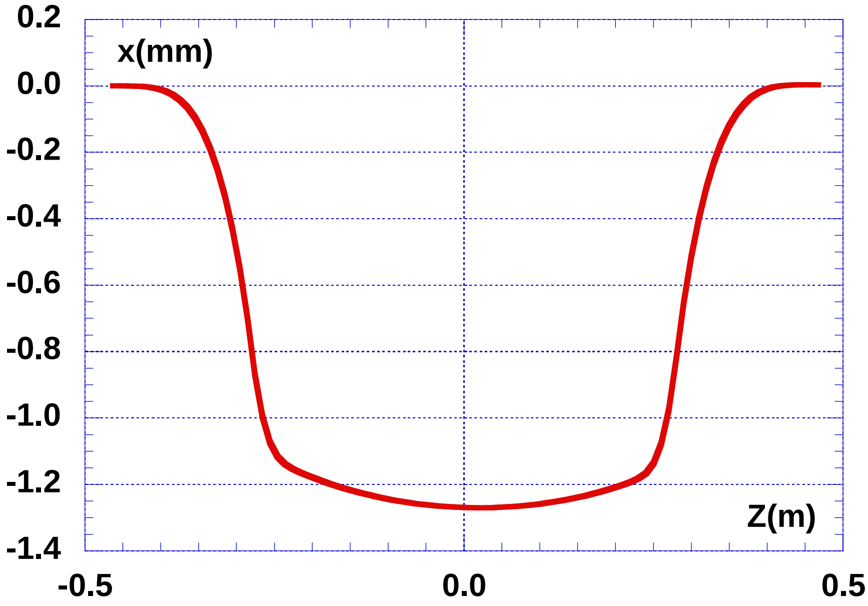

Figure 13: Horizontal displacement of the tracked particle with respect to the nominal trajectory after displacing the dipole by 0.8 mm on the left.

If the dipole axis passes through points B and C in Fig. 1 the beam trajectory is displaced by ≈36 mm at the magnet center; this means inside the magnet the beam is practically only on the external side with respect to the dipole axis and the good field region is not well exploited. In order to check the sensitivity of the most interesting parameters to an horizontal displacement of the dipole, the same method described for the longitudinal offset has been used, by simulating an horizontal displacement of the dipole by changing the horizontal position of the measurement points.

The behaviour of the energy matching the nominal deflection, the length shortening, the beam offset x at the end of the tracking with respect to the nominal position and the horizontal positions of the beam at points B and C and at the magnet center are shown respectively in Figs. 14, 15, 16, 17. A positive offset corresponds to a displacement of the dipole axis towards higher values of the X coordinate in Fig. 1, that is towards the external side of the ring. The first three figures indicate clearly that the dependence of the beam energy, output beam offset from the nominal trajectory and orbit length shortening is very weak. The last figure suggests that displacing the dipole by ≈15 mm towards the outside of the ring will give the best exploitation of the good field region.

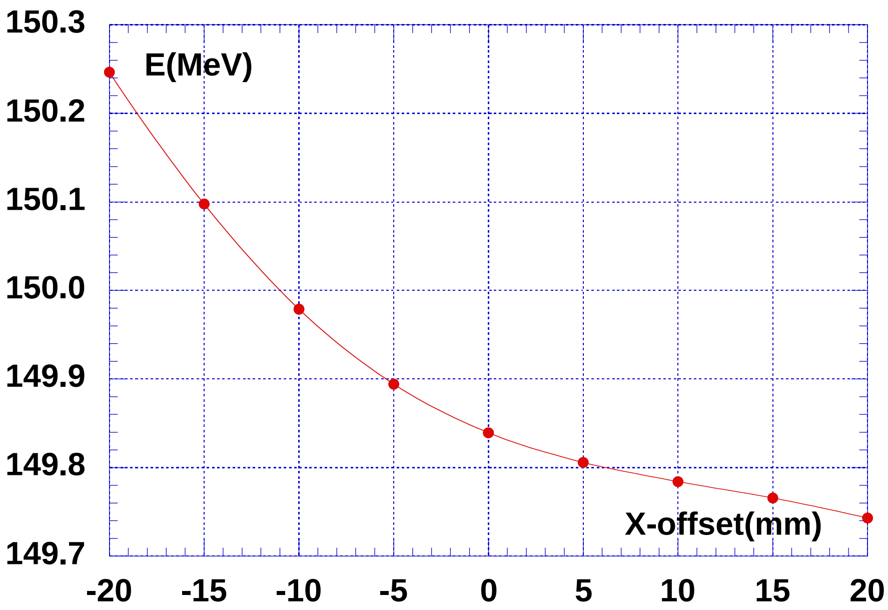

Figure 14: Beam energy for 30° deflection versus horizontal offset of the dipole

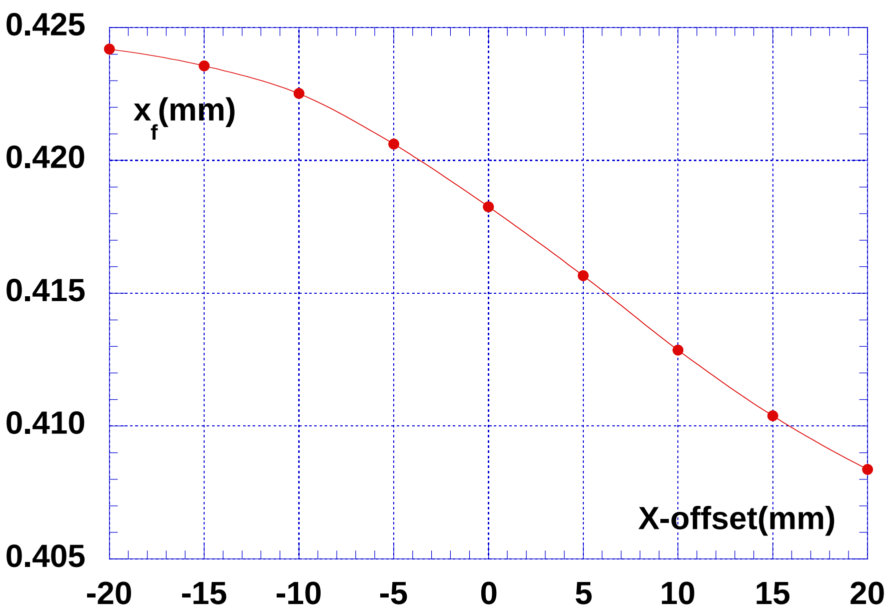

Figure 15: Beam displacement from the nominal trajectory at the end of the tracking versus horizontal offset of the dipole

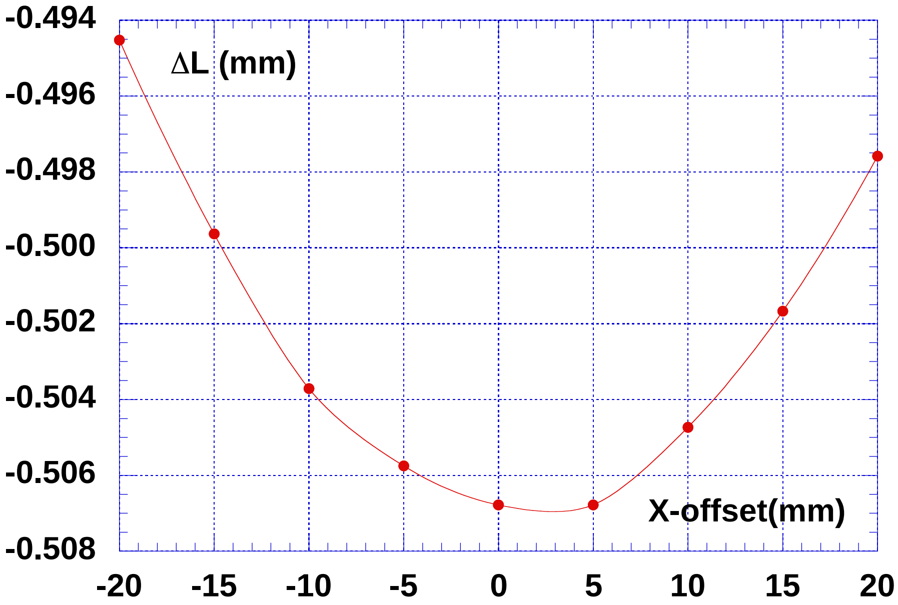

Figure 16: Shortening of the trajectory with respect to the nominal one versus horizontal offset of the dipole

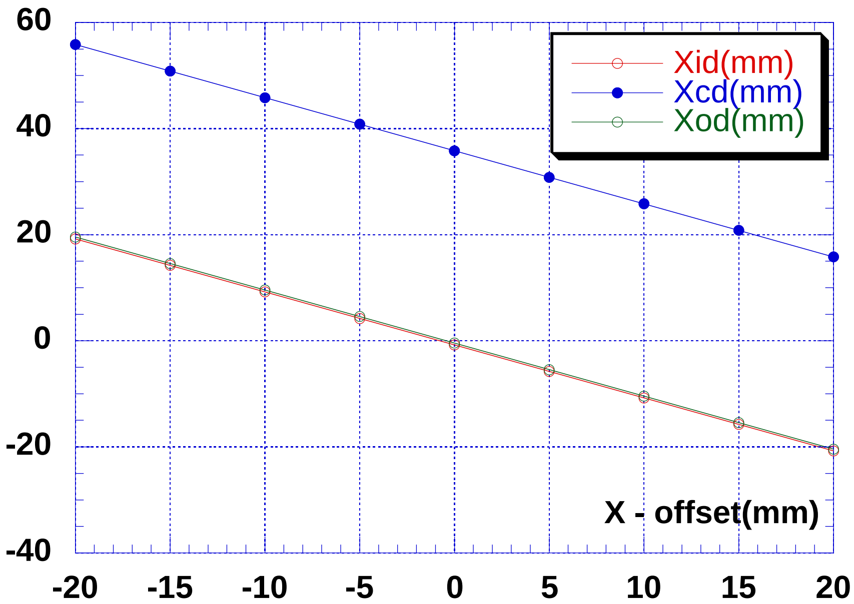

Figure 17: Horizontal distance of the beam from the dipole axis at magnet center (Xcd, full dots) and at field half-maximum (Xid and Xod, empty dots)

The 5x5 transfer matrix of the lattice section consisting of the first straight AB, the dipole and the second straight CD (see Fig. 1) can be found by changing the initial conditions of the test particle (horizontal position, horizontal angle, vertical position, vertical angle and energy) in the reference system of the nominal trajectory by small quantities. The result found from the tracking is:

$$M_{track} = \begin{matrix} 0.9852 & 0.9332 & 0.2505 & -0.0006 & -0.0005 \\ -0.0327 & 0.9843 & 0.5336 & -0.0020 & -0.0002 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.8032 & 0.9063 \\ 0 & 0 & 0 & -0.4047 & 0.8323 \end{matrix}$$

to be compared with the corresponding matrix for the ideal dipole in the hard edge approximation:

$$M_{ideal} = \begin{matrix} 1 & 0.9409 & 0.2521 & 0 & 0 \\ 0 & 1 & 0.5359 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.7676 & 0.8919 \\ 0 & 0 & 0 & -0.4606 & 0.7676 \end{matrix}$$

To better take into account the effect of the dipole fringing field, the MAD program [4], intensively used in accelerator design, introduces a vertical focusing parameter *FINT* defined as:

$$g * FINT = \int \frac{B_y(s)\left(B_0 - B_y(s)\right)}{B_0^2} ds \qquad (2)$$

where g is the dipole gap. By using the field measurement on axis the g**FINT* value for the dipole has been found to be 0.0496 m. Inserting the corresponding value into the dipole transfer matrix, the nominal matrix becomes:

$$M_{FINT} = \begin{matrix} 1 & 0.9409 & 0.2521 & 0 & 0 \\ 0 & 1 & 0.5359 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.8130 & 0.9066 \\ 0 & 0 & 0 & -0.3741 & 0.8130 \end{matrix}$$

which is in better agreement with the matrix obtained from the tracking. The horizontal determinant of the matrix is 1.000, while the vertical is 1.035.

5 - Conclusions

A tracking program has been used to examine the behaviour of the beam inside the EPA Type II dipole which will be used for the CTF3 Delay Loop. It has been found that a slight asymmetry of the field in the longitudinal direction can be cured by displacing the magnet by 0.8 mm towards the left. Horizontal displacements of the magnet do not have strong influence on the orbit parameters, and therefore the magnet can be displaced by ≈15 mm towards the outside of the ring in order to better exploit the good field region. The trajectory of the beam inside the dipoles is ≈0.5 mm shorter than the nominal one, leading to a 5 mm overall shortening of the ring circumference in the 10 dipoles of the ring. This shortening should be taken into account in the design of the ring layout in order to fully exploit the length adjustment provided by the wiggler.

References

[1] G. Suberlucq, M. Tardy, Mesure Magnetique du dipole EPA Type II. PS/PSR/Note 85-7 (24/6/1985)

[2] R. Chritin, Mesures magnetiques du champ de fuite d'un dipole EPA de type II (HIP BHZ 20), CERN Note AT-MTM-IN-2003-089, EDMS no: 425900 (8/12/2003)

[3] M. Preger, The wiggler transfer matrix, DAΦNE Technical Note L-34 (18/11/2003)

[4] http://project-mad9.web.cern.ch/project-mad9/