$g - 2$

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1. The gyromagnetic ratio

By definition, the gyromagnetic ratio $g$ of a state of angular momentum $J$ and magnetic moment $\mu$ is:

$$g = \frac{\mu}{\mu_0} \sqrt{\frac{J}{\hbar}}.$$

For a particle of charge $e$ in a state of orbital angular momentum $L$ we have:

$$\vec{\mu} = \mu_0 L, \quad \mu_0 = \frac{e}{2m}, \quad g = 1.$$

For an electron $\mu_0 = \mu_B = 5.788 \ldots \times 10^{-11}$ MeV $\text{T}^{-1}$ ($\pm 7$ ppb).

The importance of $g$ in particle physics is many-fold. A gross deviation from the expected value, 2 for charged spin 1/2 Dirac particles, is clear evidence for structure.
Thus the electron and the muon ($g \sim 2.002$) are elementary particles while the proton, with $g_p = 5.6$ is a composite object. For the neutron $g$ should be zero, measurements give $g_n = -3.8$

Small deviations from 2, $\sim 0.1\%$, appear as consequence of the self interaction of the particles with their own field. Experimental verifications of the computed deviations are a triumph of QED.

We also define the anomaly, $a = (g - 2)/2$, a measure of the so called anomalous magnetic moment, $(g - 2)\mu_0$.

QED is not all there is in the physical world. The EW interaction contributes to $a$ and new physics beyond the standard model might manifest itself as a deviation from calculations.
2. Magnetic moment

The classical physics picture of the magnetic moment of a particle in a plane orbit under a central force is illustrated on the side. \( \vec{\mu} \) is along \( \mathbf{L} \), \( \mu_0 = q/2m \) and \( g=1 \). This remains true in QM. For an electron in an atom, \( \mu_B = e/2m_e \) is the Bohr magneton. \( \mathbf{L} \parallel \vec{\mu} \) is required by rotational invariance.

When we get to intrinsic angular momentum or spin the classic picture loses meanings and we retain only \( \vec{\mu} \parallel \mathbf{L} \). We turn now to relativistic QM and the Dirac equation.
2.1 $g$ for Dirac particles

In the non-relativistic limit, the Dirac equation of an electron interacting with an electromagnetic field ($p_\mu \rightarrow p_\mu + eA_\mu$) acquires the term

$$\frac{e}{2m} \vec{\sigma} \cdot \vec{B} - eA^0$$

which implies that the electron’s intrinsic magnetic moment is

$$\vec{\mu} = \frac{e}{2m} \vec{\sigma} \equiv g \frac{e}{2m} \vec{S} \equiv g\mu_B\vec{S},$$

where $\vec{S} = \vec{\sigma}/2$ is the spin operator and $g=2$.

The prediction $g=2$ for the intrinsic magnetic moment is one of the many triumphs of the Dirac equation.
3. Motion and precession in a B field

The motion of a particle of momentum $p$ and charge $e$ in a uniform magnetic field $B$ is circular with $p = 300 \times B \times r$. For $p \ll m$ the angular frequency of the circular motion, called the cyclotron frequency, is:

$$\omega_c = \frac{eB}{m}.$$ 

The spin precession frequency at rest is given by:

$$\omega_s = g \frac{eB}{2m}$$

which, for $g = 2$, coincides with the cyclotron frequencies.

This suggests the possibility of directly measuring $g - 2$. 

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For higher momenta the frequencies become

\[ \omega_c = \frac{eB}{m\gamma} \]

and

\[ \omega_s = \frac{eB}{m\gamma} + a\frac{eB}{m} \]

or

\[ \omega_a = \omega_s - \omega_c = a\frac{eB}{m} = a\gamma\omega_c \]
For $a = 1$ ($\gamma = 1$), spin rotates wrt momentum by $1/10$ turn per turn.
The rate of high energy decay electrons is time modulated with a frequency corresponding to the precession of a magnetic moment $e/m(\mu)$ or a muon with $g=2$. First measurement of $g(\mu)$!!

Also a proof that $P$ and $C$ are violated in both $\pi\mu\nu$ and $\mu \rightarrow e\nu\bar{\nu}$ decays.
S-p correlation fundamental to all muon anomaly experiments.

High energy positrons have momentum along the muon spin. The opposite is true for electrons from $\mu^-$. Detect high energy electrons. The time dependence of the signal tracks muon precession.
5. The first muon $g - 2$ experiment

**Shaped B field**

Performed in CERN, in the sixties. Need more turns, more $\gamma$.

Next step: a storage ring.
6. The BNL g-2 experiment

\((g-2)_\mu\) Experiment at BNL

Protons from AGS

Pions 
\[ p = 3.1 \text{ GeV/c} \]

\[ \pi^+ \rightarrow \mu^+ \nu_\mu \]

Inflector

In Pion Rest Frame

"Forward" Decay Muons are highly polarized

Storage Ring

Kicker Modules

Ideal Orbit

Injection Orbit
\[ \omega_a = a_\mu \frac{eB}{mc} \]

(exaggerated \sim 20x)

With homogeneous \( \vec{B} \), all muons precess at same rate
With homogeneous $\vec{B}$, use quadrupole $\vec{E}$ to focus and store beam

**Spin Precession with $\vec{B}$ and $\vec{E}$**

$$\vec{\omega}_a = \frac{e}{mc}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta} \times \vec{E}]$$

Choose “Magic” $\gamma = \sqrt{\frac{1+a}{a}} \approx 29.3 \rightarrow$ Minimizes the $\vec{\beta} \times \vec{E}$ term

- $\gamma \approx 29.3 \rightarrow p_\mu \approx 3.094$
- $B \approx 1.4T \rightarrow$ Storage ring radius $\approx 7.112m$
- $T_c \approx 149.2ns$  $T_a \approx 4.365\mu s$
- $\gamma T \approx 64.38\mu s$

(Range of stored momenta: $\approx \pm 0.5\%$)
**ωₘ Measurement**

- \( \mu^+ \rightarrow e^+\bar{\nu}_\mu \nu_e, \ 0 < E_e < 3.1 GeV \)
- Parity Violation \( \rightarrow \) for given \( E_e \), directions of \( \vec{p}_{e+} \) and \( \vec{s}_\mu \) are correlated.
  For high values of \( E_e \), \( \vec{p}_{e+} \) is preferentially parallel to \( \vec{s}_\mu \).
- Number of positrons with \( E > E_{threshold} \)
  \[ N(t) = N_0(1 + A(E) \cos(\omega a t + \phi)) \]
LP01  James Miller - (g-2)$_\mu$ Status: Experiment and Theory

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g-2 Magnet in Cross Section

- dipole correction coil
- pole piece
- inner coil
- wedge
- pole bump
- beam tube
- programmable current sheet
- NMR probes
- outer coils
- thermal insulation

Array of NMR probes moves through beam tube on cable car
Determination of Average B-field ($\omega_p$) of Muon Ensemble

Mapping of B-field

- Complete B-Field map of storage region every 3-4 days
  Beam trolley with 17 NMR probes
- Continuous monitor of B-field with over 100 fixed probes

Determination of muon distribution

- Fit to bunch structure of stored beam vs. time
Determination of Muon Distribution

![Graph showing muon radial distribution](image)

**Magic Radius**

**Matt Hare**
**Alfonso Lam Ng**

Muon Radial Distribution (cm)
Log plot of 1999 data ($10^9 e^+$)
149 ns bins
100 $\mu$s segments
Statistical error:
$$\frac{\delta \omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a \gamma \tau \mu A \sqrt{N_e}}$$

5-parameter function (used to fit to 1998 data)
$$N(t) = N_0 e^{-\lambda t}[1 + A \cos(\omega_a t + \phi)]$$
1,025 million e\(^+\) (E > 2 GeV, 1999 data)
7. Computing $a = g/2 - 1$

$\gamma \rightarrow e$ or $\mu$

$e \Rightarrow \mu, \tau; u, d, c, s, t, b; W^\pm \ldots$

$$a_e = \frac{\alpha}{2\pi} + \ldots c_4 \left(\frac{\alpha}{\pi}\right)^4 = (115965215.4 \pm 2.4) \times 10^{-11}$$

Exp, $e^+$ and $e^-$:

$$= (\ldots 18.8 \pm 0.4) \times 10^{-11}$$

Agreement to $\sim 30$ ppb or 1.4 $\sigma$. What is $\alpha$?
7.1 \( a_\mu \)

Both experiment and calculation more difficult.

\( a_\mu \) is \( \frac{m_\mu^2}{m_e^2} \sim 44,000 \) times more sensitive to high mass states in the diagrams above. Therefore:

1. \( a_\mu \) can reflect the existence of new particles - and interactions not observed so far.

2. hadrons - pion, etc - become important in calculating its value.

Point 1 is a strong motivation for accurate measurements of \( a_\mu \).

Point 2 is an obstacle to the interpretation of the measurement.
1. – New Physics

For calibration we take the E-W interaction

\[
\begin{align*}
\langle \phi \rangle &= 236 \text{ GeV} \\
M &\sim 90 \text{ GeV} \\
\delta a_\mu (\text{EW}) &= 150 \times 10^{-11}
\end{align*}
\]

\[
\begin{align*}
\mu &\rightarrow \nu + 389 \\
\mu &\rightarrow Z - 194 \\
\mu &\rightarrow H < 1
\end{align*}
\]

SUSY:

\[
\delta a_\mu (\text{SUSY}) \sim 150 \times 10^{-11} \times (100 \text{ Gev}/\tilde{M})^2 \times \tan \beta
\]
2. Hadrons

Need i.e. \( u, d, s \ldots \)

\[ \gamma \]

\( \mu \)

\( \gamma \ h \)

\( \gamma \)

Measure \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) and use dispersion relations:

\[ \Pi'(q^2) \]

\[ \sim \sigma_{\text{tot}}^{\text{had}}(q^2) \]

\( \delta a_\mu, \ (\text{hadr} - 1) \sim 7000 \times 10^{-11} \)

All these effects are irrelevant for \( a_e \)
\[ a_\mu = \frac{\alpha}{2\pi} + \ldots c_4 \left( \frac{\alpha}{\pi} \right)^4 = (116591596 \pm 67) \times 10^{-11} \]

Exp, \( \mu^+ \): \[ = (\ldots 2030 \pm 150) \times 10^{-11} \]

Measured-Computed = 430\( \pm \)160 or 2.6 \( \sigma \), \( \sim 3.7 \pm 1.4 \) ppm.
Standard Model Value for $a_\mu$ [1]

\begin{align*}
a_\mu(QED) &= 116584706(3) \times 10^{-11} \\
a_\mu(HAD; 1) &= 6924(62) \times 10^{-11} \text{ (DH98)} \\
a_\mu(HAD; > 1) &= -100(6) \times 10^{-11} \text{ (Except LL)} \\
a_\mu(HAD; LL) &= -85(25) \times 10^{-11} \\
a_\mu(EW) &= 151(4) \times 10^{-11}
\end{align*}

\textbf{TOTAL} \quad = 116591596(67) \times 10^{-11}


Used by the BNL $g-2$ experiment for comparison. Addition of above errors in quadrature is questionable.
8. $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$

$\delta a_\mu(\text{hadr} - 1) \sim 7000 \times 10^{-11}$

$\sigma(e^+e^- \rightarrow \text{hadrons})$ is dominated below 1 GeV by $e^+e^- \rightarrow \pi^+\pi^-$. Low mass $\pi^+\pi^- (\rho, \omega)$ contributes $\delta a_\mu \sim 5000 \pm 30$.

$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ or $(\gamma \rightarrow \pi^+\pi^-)$ is measured:

1. at $e^+e^-$ colliders, varying the energy
2. in $\tau$-lepton decays
3. at fixed energy colliders using radiative return
- 1. - Extensive measurements performed at Novosibirsk. Corrections for efficiency and scale plus absolute normalization (Bhabha, $e^+e^-\rightarrow e^+e^-$) are required for each energy setting. Data must also be corrected for radiation and vacuum polarization.

- 2. - $\tau$ data come mostly from LEP. To get $\sigma(\text{hadr})$ requires $I$-spin breaking, $M(\rho^\pm)-M(\rho^0)$, $I=0$ contrib... corrections. Radiative corrections are also required.

- 3. - The radiative return method is being used by the KLOE collaboration, spear-headed by the Karlsruhe-Pisa groups.
Can turn initial state radiation into an advantage.
At fixed collider energy $W$, the $\pi^+\pi^-\gamma$ final state covers the di-pion mass range $280 < M_{\pi\pi} < W$ MeV.
Correction for radiation and vacuum polarization are necessary.
All other factors need be obtained only once.
At low mass, di-muon production exceeds that of di-pion. ISR and vacuum polarization cancel.
Contribution to $a_{\mu}$ ($\times 10^{11}$)

$\sigma(\pi\pi)$, nb

\[ \sum(\ldots) = 5000 \]
\[ \sigma, \mu b \]

\[ s, \text{GeV}^2 \]

Amendolia et al. 1987

\[ \pi \pi \]

\[ \mu \mu \]
### Graph

**Graph Title:** $ee \rightarrow \pi\pi\gamma$

**X-axis:** $M(\pi\pi)$, GeV

**Y-axis:** $N/0.02$ GeV

**Data:** 20569 events
\[ \frac{d\sigma(ee \rightarrow \pi\pi\gamma)}{dM(\pi\pi)} \text{ nb/GeV} \]

\[ \delta a(\mu) \sim 2\% \]
Use small angle radiation, higher x-section but miss low $M_{\pi^+\pi^-}$.

All lost in beam pipe.
Unsatisfactory points:

1. $2.6 \sigma$ is not very compelling and is also author dependent.
2. M-C is $\sim 3 \times$ EW contribution. What about LEP, $b \rightarrow s \gamma$, $M_W$, $M_{\text{top}}$, $\Re(\epsilon'/\epsilon)$, $\sin 2\beta \ldots$
3. Hadronic corrections difficult, e.g. light-by-light
4. SUSY as a theory is not very precise at the moment. It has too many unknown, free parameters. There is no exp. evidence for it nor a prediction follows from the possible effect in the muon anomaly.

Soon better statistics and both signs muons. Still very exciting at present.